CONCEPT OF MICROWAVE PLASMA INTERACTION WITH REFERENCE TO DIFFERENT FIELD CONDITIONS

DR. A. B. GAUTAM

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Dr. A. B. Gautam
Associate Professor,
Dept. of Physics, S. B. A. N. College,
Darheta- Lari, Arwal

Kripa-Drishti Publications, Pune.

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Author By: **Dr. A. B. Gautam**

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Chapter 1

General Introduction

1.1 Introduction:

The study of electromagnetic wave propagation through conductors and semiconductor plasma is steadily increasing. It has a great diagnostic value and in the optical wavelength the wave interaction with the semiconductor plasma provides spectroscopes information. It is well known now that the solid state materials normally have a dense cloud of free electrons and the properties of this electron gas or electron ensemble or electron plasma are analogous to gaseous plasma with some characteristic differences. The electromagnetic wave propagation features in the solid state plasma and gaseous plasma also have close similarities. The electromagnetic wave propagating through good conductors interact strongly with the electron plasma and are excessively attenuated. Therefore, good conductors are generally referred to as dissipative or lossy materials. The sold state plasma supports many wave modes and these wave modes are known as excision. While discussing electromagnetic wave propagation through conductors, semiconductor and insulators one needs to develop certain macroscopic and microscopic models. It is well known that even at room temperature the conductors and semiconductor are characterized by a large number of free electrons. The electrons ensemble in conductors and electron-hole ensemble in semiconductors respond significantly to electromagnetic wave propagation. Thus for the sake of electromagnetic wave response and propagation, the materials are characterized in the following way:

Conductors = Immobile lattice ions + mobile and free electron gas.

Semiconductors = Immobile lattice ions + mobile and free electron and holes.

Insulators or Non - conductors. = Immobile lattice atoms + no electron gas

The ensemble of free electron, in this classification is known as electron gas or hole gas. The electron gas and hole gas mainly affect the electromagnetic wave propagation and extract maximum energy from the interacting or propagating waves.

The study of the propagation of electromagnetic wave through bounded magneto-plasma has become- increasingly important in recent years because of its application in areas like plasma diagnostic, beam plasma interaction, microwave circuitry etc. They have contributed recently in a great measure to progress in many fields of basic research and technological applications such as MHD power generation, fusion research, intense radiation sources, chemical reactions, material research, microelectronics etc. While the literature on the propagation of electromagnetic wave through unbounded magneto-plasma is extensive, the subject of bounded magneto-plasma has not received significant attention. Crawford (1971) presented somewhat a gloomy picture regarding the future of these devices and showed still a dearth of serious commercial plasma competitors to existing vacuum tube and solid state devices unless a major breakthrough is made in production of a simple quiescent plasma source. Plasma medium had drawn very much attention so given status of fourth state of matter i.e. plasma state in addition to solid, liquid and gaseous states. Ninety-nine percent of matter of universe exists in plasma state. It is characterized by a high mean particle energy in comparison to other states. Most important natural source of plasma around earth extends from 90 thousand of kilometers known as regions termed as ionosphere and magnetosphere. The energy available on earth from sun is due to existence of matter there in plasma state. The lightening which accompanies thunder storms is a vivid example of plasma. Solar flares, solar prominences and sun spots are the other spectacular plasma phenomena. In fact, all stars are themselves largely in the plasma state.

In most cases the readily accessible range of plasma parameters for the laboratory systems places the operation in the microwave band of the frequency spectrum. Numerically, plasma frequency $f_p \approx 900n^{1/2}$ Hz (n in electrons per cm^3) and the gyro frequency $f_b \approx 28B$ (B in Gauss) so that readily attainable electron densities ($\sim 10^{11}$ per cm^3) and magnetic field ($\sim 10^3$ Gauss) place these quantities in the microwave band.

The plasma devices such as phase shifters, attenuators, isolators, couplers hold out promise for the future development. The use of beam plasma interaction in the millimeter wave region has been demonstrated. Non-linear plasma properties have been tried in devices like harmonic generators, parametric amplifiers etc. But unfortunately none of these devices are able to compete at present with the established existing devices. However, an attempt is made in this dissertation to understand the interaction of electromagnetic waves with the plasma in the bounded system as a step towards exploring the feasibility of devices employing plasma as a working medium. Plasma and beams have an important role both in basic research and in technological applications. The interaction between plasmas and electromagnetic waves is one of the fundamental problems of plasma physics that has received considerable attention. The subject of microwave plasma interaction retains considerable interest for both the plasma heating as well as for the production of moderately high density (overdense) plasmas in the laboratory. Microwave discharges are also used for preionization in tokamaks, prior to ohmic heating of the plasma. In recent times, microwave produced plasmas have also begun to find application in the processing of semiconductor materials for devices fabrication. High density plasma production using different slow wave structures which facilitates the coupling of microwave energy to plasma electrons is in progress. The term "plasma' was first used to describe a collection of charged particles by Tonks and Longmuir in 1929 in their studies of oscillations in electric discharge. The term "fourth state of matter" often used to describe the plasma state, was coined by W. crooks in 1879 to describe the ionized medium created in gas discharge. Revised interest in plasma physics in united states began in 1952 with the attempts of a programme, known as project Sherwood (1958) to develop a controlled the monuclear fusion reactor. The presence of a controlled amount of background plasma inside microwave tubes can possibly lead to improvement in their characteristics beyond what is available in evacuated devices [Nusinovich et.al. (1998)]. In particular, recent results clearly demonstrated that the presence of plasma can significantly increase the band width, efficiency and power handling capabilities of non-relativistic microwave oscillators and amplifiers and allow operation without a guiding magnetic field. Plasma loaded microwave devices have the potential to advance the technological and scientific base of microwave tubes, and also to have an impact on commercial and industrial applications through the development of commercially variable technologies.

One of the earliest and most extensive investigations in this direction is the propagation of radio waves in the ionosphere. The mathematical formulation in that context is the magneto-ionic theory developed by Appleton (1930), Hartree (1931) and others and is discussed in detail by Ratclife (1959, 1972), Budden (1961) and Ginzburg (1964). The resonance and cut-off frequencies of an uniform plasma for different separations between emitting and receiving probes are measured by Mishra and Sahu (1990). Under this experimental condition plasma diagnostic such as electron density, temperature and collision frequency was performed. Sinha et.al. (1988) determined arc plasma parameters of Cu arc plasma using a moving Langmuir probe. Narayan et.al. (1990) show that use of moving double probe method reveals a parametric multiple structure showing variation of ion densities. The propagation of electromagnetic waves in unbounded stationary magneto plasma is considered by Haskell and Papa (1965), Holt and Haskell (1965), Strix (1964), Sturrock (1967) and many others. Ganapol et.al. (1989) evaluated the electric field generated by longitudinal plasma waves. The relativistic effect in plasma was considered by Chawla and Unz 91969, 1971). Articolo (1969) derived a general expression for the dielectric tensor for warm drifting non-relativistically and anisotropic unbounded plasma including the effect of velocity dependent collisions in the presence of the external magnetic field.

In order to produce plasma, it is necessary to free electrons that are normally bound in atoms. When the energy equal to ionization energy of any element is added, ionization takes place with the creation of an electron and ion. Ordinarily energy comes from collision events of one sort or another. On the addition of ionization energy to a fraction of atoms of a neutral gas, an ionized gas is formed. If sufficient energy is added, the gas may be completely ionized.

There are two ways usually to describe plasma properties. The first one is the microscopic approach based on the distribution function of positively and negatively charged particles. This description of plasma is the most rigorous one. Hence a large number of phenomena occurring in plasma is considered from a macroscopic point of view and is described in terms of average quantities like densities, pressure, temperature etc. This is equivalent to use fluid theory. Electrical property of plasma is described either in terms of dielectric constant or conductivity. In the dielectric model, electrons are treated as bound charges each being associated with a positives ion to form an electrical dipole.

The motion of electrons under influence of an oscillating electric field can be regarded as giving rise to a polarization current. In the conductor the electrons are considered as free charges whose response to applied fields is opposed by interactions with the other particles. The electrons moving under influence of oscillating electric field are considered to give rise to conduction current. However, the above two descriptions are equivalent. In general plasma dielectric constant also called relative permittivity and conductivity both are complex having resistive and reactive components. In case of a plasma medium being anisotropic, these parameters become tensors. The dependence of dielectric constant on frequency makes plasma quite attractive for many device applications.

Although the plasma is first of all a gas and nearly "ideal" as well, it has many properties in common with a conducting fluid exhibiting coherent motions i.e. waves, which reveal a great deal about the state and nature of plasma system. If all the particles in a small volume element feel essentially the same forces, they will move in the same way, even though they are not coupled to their near neighbours by collisions.

If a group of particles have widely varying thermal speeds, then of course, they will not tend to remain together as a fluid element, the results of fluid element, the results of fluid treatment for a plasma are usually referred to as waves in a cold plasma. The plasma state is enormously rich in wave phenomena, and a variety of classification schemes have been devised [Stix (1964)]. Only a broad categorization of plasma waves is made according to the state of the plasma that supports the waves because the normal modes of a plasma are so strongly influenced by the plasma and field configuration.

Only small amplitude waves are discussed here because the field equations can then be linearized in the wave quantities. Wave phenomenon in plasma is discussed on the basis of the orbit theory. Orbit theory gives insight into the physical phenomena that determine plasma behavior provided that particle interactions can be considered as playing only a minor role. The major limitation of the orbit theory approach to a study of wave propagation in plasmas is that it cannot adequately take into account the effect of the random thermal motion of the particles and antiparticles collisions. In order to include these effects, it is necessary to introduce the distribution function f(s) for each of the s constituents in the plasma medium.

This distribution function is defined as the density of points in the six-dimensional phase space made up of these components of the position of particles (configuration space) and three components of the velocity of particles (velocity space).

If dr is a small volume element in configuration space and dc is a small volume element in velocity space, then f(s) $dr^{(s)}$ $dc^{(s)}$ is the number of particles of species which are located in the volume element dr and have velocities in the velocity range dc.

The number density $n^{(s)}$ of particles of species s is found by integrating the velocity distribution function over all velocities

$$n^{(s)} = \int f^{(s)} dc^{(s)} \qquad (1.1)$$

If $\phi^{(s)}$ $(v_i)^{(s)}$ is some function of particle's velocity then the average value of $\phi^{(s)}$ denoted by $<\phi^{(s)}>$ is given by

$$<\phi^{(s)}> = \frac{1}{n^{(s)}} \int \phi^{(s)} f^{(s)} dc^{(s)} \qquad (1.2)$$

If the velocity distribution function is known then all of the average, or microscopic properties of the gas can be calculated. To do this, it is noted that $f^{(s)}$ satisfies the Boltzmann equation

$$\frac{\partial f^{(s)}}{\partial t} + V_i^{(s)} \frac{\partial f^{(s)}}{\partial t_i^{(s)}} + \left(\frac{e^{(s)}}{m^{(s)}} E_i - \in_{ijk} V_i^{(s)} \omega_b^{(s)} \right) \frac{\partial f^{(s)}}{\partial V_i^{(s)}} = \left(\frac{\partial f^{(s)}}{\partial t} \right)_{coll} \qquad (1.3)$$

Where $\in_{ijk}$ is the third order alternating unit tensor. The right hand side of the equation represents the rate of change of $f_{(s)}$ due to collisions with other particles. The first term on the left hand side, $\partial f^{(s)}/\partial t$ is the local variation of the distribution function. The second term $V_i^{(s)} \partial f^{(s)} / \partial x_i$ (s) is the variation of the distribution function from diffusion.

The third term is the variation of distribution function due to external forces acting on the molecules. Solution of Boltzmann equation yields information about waves in plasmas.

Some information regarding waves in plasma can be obtained without actually solving Boltzmann equation. Consider a plasma which consists of electrons and one species of singly ionized positive ions. The collision term on the right hand side of Boltzmann equation is neglected so that the only interaction between electrons and ions is due to the space charge fields. If the Boltzmann equation is multiplied by 1, $\left(m \pm V_i^2\right)$ and $\left(m \pm V_i^{\pm} V_j^{\pm}\right)$ and the resulting equations are integrated over all velocities, the three sets of moment equations [Holt and Haskell (1965)] are obtained. The first set is the continuity equations for electrons and ions and contain the unknowns $n_{\pm}$ and $< V_i^{\pm} >$. The second set is the momentum equation and contains an additional unknown quantity, the pressure tensor $\psi_{ij}^{\pm}$ given by

$$\psi_{ij}^{\pm} = n_{\pm} m_{\pm} < \left(V_i^{\pm} - < V_i^{\pm} >\right)\left(V_j^{\pm} - < V_j^{\pm} >\right) > \tag{1.4}$$

The third set of moment equation describe the time variation of the pressure tensor and $\psi_{ij}^{\pm}$ and contains an additional term $Q_{ijk}^{\pm}$, the heat flux tensor. There are always more unknowns then there are equations and it is necessary to truncate the set of moment equations. If the wave motion is adiabatic i.e. if the phase velocity of the wave is much larger than the perturbed thermal velocity of the charged particles, truncation can be achieved by neglecting the heat flux tensor. Then, the first three moment equations together with the electro-dynamic equations can be linearized and plane wave solutions can be obtained for the perturbed quantities. The results differ from those obtained from orbit theory in that they include first order temperature terms in the dispersion equations for both transverse and longitudinal waves. The effect is small for transverse waves except near resonance. However, for longitudinal waves, the effects of finite temperatures are important because they provide a mechanism for the propagation of longitudinal oscillations [Crawford (1968)]. The most general dispersion relation using the moment equations is obtained [Holt and Haskell, (1965)]. The moment equations thus obtained do not contain any information concerning the velocity distribution of the plasma particles because the Boltzmann equation has been averaged over velocity space. Thus in order to describe phenomena which depend upon the finite velocity spread of the particles, it is necessary to solve the Boltzmann equation for the various species. Methods for solving Boltzmann equation can be divided into two major categories. The first is a perturbation technique by means of which Boltzmann equation is linearized. This method has been used by many authors [Sitz, (1964); Bernstein, (1958) and Drummond (1958)] to describe the propagation of small amplitude waves in a collisionless plasma. The dispersion curves (Bernstein mode) thus obtained, coincide in the limit of long wavelengths of small wave number with those obtained from moment equations. As the wavelength decreases, the dispersion curves change drastically. The group velocity and phase velocity can have opposite signs. Resonances known as Bernstein modes appear at each harmonic of the cyclotron frequency approaching the harmonic frequencies from the high frequency side. These Bernstein modes are experimentally demonstrated by a number of workers [Landauer, (1991); Crawfor, (1968); Hiroe, (1973)].

The second major method for solving the Boltzmann equation lies somewhere between the perturbation method and moment equations. In this method, the distribution function is expanded in series of some orthogonal functions where the coefficients of the expansion are related to macroscopic quantities of the plasma. Several authors [Margenau. (1946), Allis et.al. (1963), Johnston, (1962)] have used a spherical harmonic expansion. The advantage of this method over the perturbation method is its ability to include the effects of close collisions with velocity dependent collision frequencies.

It was pointed out that one of the most attractive properties of the plasma is its frequency dependent dielectric constant. Based on this, a wide variety of device applications have been suggested. Most of these devices employing laboratory plasma bounded systems can be analysed only when the behaviour of electromagnetic fields is known at the boundary between the plasma and the walls of the container. Many important problems of both static and time varying fields have one medium directly adjacent to another.

The differential relations for each homogeneous medium relating space derivatives of the field to sources at the point and to properties of the medium, may be applied separately to each region. The question is that of joining or matching the solutions across the surface of discontinuity. The boundary conditions will be satisfied by time dependent electromagnetic field vectors $\overline{B}, \overline{E}, \overline{D}$ and $\overline{H}$ at the interface between two different media. For the correct description of the bounded plasma system it is necessary to know the behaviour of the electromagnetic fields at the boundary between the plasma and the conductor. The conditions to be satisfied by the field vectors at the interface separating a dielectric and a conductor are well known [Ramo et.al. (1970)]. Cold plasma whether uniform or non-uniform can be described as a dielectric medium obeying Maxwell's equations. Therefore, the boundary conditions at the interface between the cold plasma and a conductor are the same as the boundary conditions at the interface between the conductor and the dielectric since the plasma behaves as apolarizable medium with a strongly frequency dependent tensorial permittivity. These conditions for any dielectric can be described as:

a. Normal components of magnetic induction is continuous across the boundary.
b. Tangential components of electric field intensity is continuous across the interface.
c. Normal components of electric displacement is not continuous across the surface and changes by an amount equal to the free surface density at the interface.
d. The tangential components of magnetic field intersity is continuous across the surface separating the two dielectrics.

In case of boundary being perfectly conducting medium the tangential components of electrical field is zero at the boundary. Also the normal component of magnetic field vanishes at the boundary.

When the plasma is hot the problem is more involved. When a plasma is in contact with a solid surface, a sheath with a thickness of the order of several Debye radii is produced. Mos of the electrons are reflected by the sheath back into the plasma. A reasonable assumption is the the wall is perfectly reflecting [Vandepnplas, (1968)]. If one uses the perturbed scalar pressure, the additional boundary condition is that the mean perturbed velocity perpendicular to the wall is zero. On the other hand, if the perturbed pressure is a tensor then the off diagonal terms of the pressure tensor are also set equal to zero at the walls. If the wall is metalic it is an equapotential surface and a conduction towards it is possible, a sheath still exists and the properties of the sheath depend upon the potential of the wall with respect to the potential of the plasma outside the sheath. In some cases, it seems therefore that the assumption of a practically zero perturbed conduction current at the metallic wall might also be quite satisfactory. The sheath region close to a metal surface plays an essential role in the resonance properties of plasma system having such a metal surface.

The interest in bounded plasmas started ever since Tonks and Longmuir discovered high frequency electron oscillations in 1929. These oscillations have subsequently been investigated in considerable detail by Tonks and Longmuir (1929 a,b). Bohm and Gross (1949), Van Kampen (1957) and others. The closely related problem of space charge waves in electron beams have been studied by Hahn (1939) Ramo (1939), Birdsall and Whinnery (1953), Trivelpiece and Gould (1959). Bevc and Everhart (1962) and Trivelpiece (1967). Most of the analysis of waves in electron beams and plasmas in the presence of external magnetic field have been devoted mainly to the various modes of propagation. The electromagnetic wave propagation through conductors is affected by the charged particle system present in conductors and semiconductor plasma. Two approaches of modelling of solid state plasma are generally used:

- Electron gas Model
- Fermi Liquid Model

The electron gas in conductors is generally of high density. Therefore, the weak- coupling between charged particles in solid state plasma exists. The criterion for weak coupling is experessed in terms of characteristic lengths. However, in semi-metals and heavily doped degenerate semi- conductors the electron density is comparatively low as compared to metals and weak coupling generally holds good. The Fermi-liquid model was developed by Landau (1956) and later extended by Silin (1958) to account for the properties of metals. Fermi-liquid parameters have been developed which specify the electromagnetic response of metals rather exactly.

1.2 Characteristics and Parameters of Solid State Plasma:

1.2.1 Effective Mass of Charge Carriers:

The nature and distribution function of quasi-particles differs basically between gaseous plasma and the solid state. Certain wave mode excitations in solid state plasma with well-defined energy besides electrons and holes are known as quasi-particles namely, plasmon, helicons, phonons, polarons and magnons. Electrons and holes obey Fermi-Dirac statistics and plasmons, helicons and phonons obey Bose-Einstein statistics.

1.2.2 Screening length in Solid State Plasma:

The concept of Debye sheilding is good for plasmas obeying Maxwell- Boltzmann statistical distribution and characterizes ad distance to whicdh an external electrostatic field with penetrate in the plasma before being counter balanced by induced electric fields due to charge polarization of the medium. The concept of screening in degenerate and non-equilibrium plasmas changes. The natural generalization of Debye shielding distance in solid state plasma is obtained by replacing 3/2 KT by Fermi energy. This screening parameter is known as Fermi-Thomas length.

1.2.3 Plasma Oscillations:

The solid state plasmas also conform to harmonic oscillator and the plasma frequency in solid state plasmas is given by $\omega_p^2 = ne^2 / m^* \in_\upsilon$. This important frequency separates the low and high frequency response of the plasma. The quantum of plasma oscillation energy known as plasmons for most metals are of the order 10eV. This energy is far above the thermal energy, therefore, plasma waves are not normally excited in metals unless special conditions are met for their excitation. Under suitable conditions more than one mode of oscillations or waves are excited. Their degree of excitation and their predominance vary from one conductor to another. These waves in conductors are known as excitons.

1.2.4 Cyclotron Frequency or Gyrofrequency:

With thet exception of effective mass, the cyclotron frequency in case of solid state plasma is same as in the case of gaseous plasma. The cologron frequency is used as an important reference frequency for discussing electromagnetic wave propagation in solid state plasma.

1.2.5 Relaxation Time:

Gaseous plasma is characterized by binary collisions. In the case of solid state plasma, the current carriers invariably scatter from lattice defects, disorders and imperfections. The effect of this scattering process is particularly adverse when electromagnetic wave in the vicinity of cyclotron frequency are propagating through the solid state plasma.

The wave having frequency equal to cyclotron frequency propagates if the field vectors of the propagating electromagnetic wave and gyrating electrons make several gyrations before scattering of electrons which is analogous to the collision process in gaseous plasma which is governed by the inequality $\omega t >> 1$ where $\omega_{\in}$ is cyclotron frequency and τ is collision time or relaxation time. Inverse of this time gives the analogue of binary collision frequency in gaseous plasma.

1.2.6 Local and Non-Local Regimes:

The comparison of wave number and the inverse of mean free path classifies the solid state plasma into two clear cut regimes:

- Local regime $kl << 1$

and

- Non-local regime $kl >> 1$

The electron plasma behaves differently under these approximations and the propagating electromagnetic waves undergo characteristic changes. These regimes have certain implications in the propagation of electromagnetic waves in solid state plasma.

The interaction of a large number of electrons with self-created or externally imposed electromagnetic field is governed, in general, by a quantum statistical or quantum mechanical descriptions. In the long wavelength limit, the quantum mechanical description goes over to the classical description.

Therefore, for discussion of long wavelength propagation, the use of hydrodynamical model of plasma is quite valid and is extensively used. Various simplifying assumptions are made while using the hydrodynamic fluid model for evaluating the response of solid state plasma to external electromagnetic field perturbation:

a. The concept of effective mass of charge carriers in solid state plasma enables us to treat them as free electrons and ions in vaccum.

b. The energy and momentum changes per particle are small, so that the band to band transitions are generally ignored.
c. Interactions of electrons and holes with lattice vibrations and imperfections are accounted for by introducing constant phenomenological collision frequency which is inverse of the relaxation time of the solid state plasma.
d. The effect of distribution of interacting particles in velocity or momentum spaces is ignored.
e. The wavelength of external disturbance is much longer than the Debye length, so that one can treat the electron or the hole to be streaming hydrodynamically.

1.3 Macroscopic Equations of Solid State Plasma:

The general plasma approximations for gaseous plasma also hold good in the case of solid state plasma. The Maxwell's equations in the presence of charges and currents in the solid state plasma are written as-

$$\nabla . D = \rho$$
(1.5)

$$\nabla . B = 0 \qquad (1.6)$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

and

$$\nabla \times B = \mu J + \mu \frac{\partial D}{\partial t} \qquad (1.7)$$

here μ and $\in$ appear in place of μ_0 and $\in_0$ used for free- space. The source terms are charge density and current density in the solid state plasma and are written as

$$\rho(r,\tau) = \sum_i e\delta(r - r_i) \quad i = 1, 2, 3............. \qquad (1.8)$$

and

$$J\ (r,t) = \sum_i e v_i \delta(r - r_i) \quad i = 1, 2, 3............. \tag{1.9}$$

Where V_i is the velocity and r_i is the position of ith charge carrier denoting the movement from the initial position denoted by r.

The statistical description of solid state plasma, assumes the existence of a velocity distribution function *f* (r,v,t).

The behaviour of the velocity distribution function is given by the Boltzmann transport equation.

The average number density and the average velocity of electrons are defined by the zeroeth and first moment of the distribution function with respect to velocity

$$n(\bar{r},t) = \int_v f(\bar{r},\bar{v},t) d^3 r \tag{1.10}$$

$$\langle V \rangle = \frac{1}{n(r,t)} \int_v v f(\bar{r},\bar{v},t) d^3 r \tag{1.11}$$

The Maxwell's equations are supplemented by certain hydrodynamic equations such as momentum and continuity equations which hold good for a conducting fluid.

The electron plasma in solids conforms well with these equations and forms a basis for use of the hydrodynamic fluid model for studying some of the problems in solid state plasma.

The electromagnetic response of electron gas in solid state plasma characterises the properties of conductors and semiconducors. The fluid model is generally used to obtain the electrical conductivity of electron gas.

Except for the change of electronic rest mass to effective mass, the derivation of conductivity of conductors arens semiconductors are almost similar to that derived for gaseous plasma.

1.4 Longitudinal Plasma Oscillations in Solid State Plasma:

The electron gas is solids behaves exactly in the same way as in the gaseous plasma. For a cold and collisioniless electron gas which is free from diffusion, the dispersion eqaution for longitudinal oscillations is obtained by setting $\in_{33}$ element of dielectric tensor to be equal to zero. The Plasma oscillations are localized and non-radiative at signal frequency equal to plasma frequency. When realistic solid state plasma is taken account, we find that the dispersion equation implies the existence of two longitudinal modes for a given signal frequency. These wave modes are known as fast and slow wave modes. In the presence of phenomenological collisions, the effective plasma frequency in the solid state plasma is reduced to $\omega_{p.eff} = \omega_p \left(1 - \upsilon^2 / 4\omega_p^2\right)^{1/2}$. The dispersion equations appropriate to different forms of solid state plasma can be obtained from the general dispersion equation.

1.5 Helicon Waves in Solid State Plasma:

The single component electron plasma is capable of sustaining right- handed circularly polarized waves for which the electric field vector is written as $E_{RH}=E_x+jE_y$. The electron gyration in the presence of a static magnetic field has the same direction as the rotation of E_{RH}. Accordingly the electric field vector $E_{LH}=E_x-jE_y$ rotates in the opposite direction to the direction of the gyrating electron. The presence of such waves in metal plasma was first discovered by Konstantinov and Perel (1060) and in semiconductor plasma by Algrain (1960).

These waves are analogous to whistler mode waves first discovered by Storey (1953) in the earth's magnetosphere and are known as Helicon waves in solid state plasma. The dispersion equation for Helicon wave propagation can be obtained from the self- consistent solution of Maxwell's equation and the equation of motion of electrons. At low freqencies $\omega \ll \omega_v$, ω_p and υ. The magnetic field is large such that $\omega_e/\upsilon \geq 1$. In case of longitudinal propagation $\theta = 0$, the helicon waves (extra-ordinary mode) propagate along the static magnetic field with very low damping. Since ω_p is large and ω is small, the phase velocity of helicon wave becomes much smaller and interacts with the ambient electron gas and gives rise to growing or decaying Helicon wavesl

In case of general direction of propagation [Singh and Pandey, (1975)] which is rather important $\theta \neq 0$. the field component E_z appears along the direction of wave propagation which causes additional damping well known as Landau damping of helicon waves. The helicon wave propagation through solid state plasma changes significantly with changing wave and plasma parameters. Therefore, the helicon wave propagation is used as a diagnostic tool for study of electron gas in solid state plasma.

1.6 Waves in Two Component Plasma- Alfven Waves:

We find that the solid state plasma has close analogy with the gaseous plasma. The electron and ion system in gaseous plasma supports Alfven waves [Alfven, (1942)]. The electrons and holes system in solid state plasma also supports Alfven waves. In solid state plasma, we have $n_e = n_h = n$ and $\omega \ll \omega_p, \omega_c$. The Alfven waves are strongly damped unlike Helicon waves. The solution of dispersion equation yields two solutions, one corresponds to extraordinary Alfven wave which is a slow wave, and another solution corresponds to ordinary wave which is a fast wave.

The slow Alfven waves are analogous to helicon wave. We find that the slow Alfven wave propagates in different directions with different velocities. The fast Alfven wave however, has no directional dependence. The propagation properties of Alfven waves have been used as one of the diagnostic techniques for studying the nature of two components solid state plasma in semiconductors and semi-metals. In the local regime characterized by $kl \ll 1$, the cyclotron resonance take place $\omega = \omega_c$ at which the helicon waves is strongly absorbed. However, in the non-local regime the electrons drift along the static magnetic field and the resonance frequency is drastically decreased.

The maximum velocity of electron in a metal plasma is V_F, the Fermi velocity. Thus we find that the resonance in solid state plasma occurs at a reduced frequency $\omega = (\omega_c - kV_F)$ and the interacting wave is strongly damped. Study of these resoance provide diagnostic information about velocity distribution function of charged particles in metals and semiconductors. Some of these details are essential for evaluating the performance and response of metals and semiconducting materials as circuit elements subjects to various ranges of electromagnetic operations.

The thesis consists of six chapters. The first chapter serves as an introduction to the thesis. A short account is given of the basic concepts necessary to understand the behaviour of unbound and bounded gaseous and solid state plasmas. Basic features of the plasma existence, conducting waveguide, boundary conditions at the conducting surface, possible propagating wave modes and possible instabilities in solid state plasma are outlind. The chapter begins with a brief survey of existing literature on the unbounded gaseous and solid state plasma. A general solution for the propagating modes in waveguides completely or partially filled with moving inhomogeneous warm plasma in the presence of finite magnetic field has not yet been developed. Only ccertain simplified cases have been analysed in the literature. Bevc and Everhart (1962) classified modes of propagation in case of waveguide completely or partially filled with a stationary plasma column or a electron beam drifting with a uniform velocity and collimated with a magnetic field of finite magnitude. Formulae for cutoff frequencies as functions of the cyclotron frequency as well as some representative Brilllouin diagrams are given. Effect of anistropic plasma in a waveguide is studied by Samaddar (1962). Franklin and Oldfield (1969) have studied the properties of cylindrical waveguides filled with cold plasma in the presence of finite magnetic field in the direction of propagation including the effect of collisions. Oldfield and Franklin (1971) verified experimentally some of the results obtained by them at microwave frequencies using collision dominated weakly ionized plasma. Guided waves in a general class of waveguides filled with biisotropic medium which is the most general isotropic linear medium i.e. non-reciprocal chiral medium are studied by Kolvisto et.al. (1993). Field equations are derived first for arbitrary waveguide cross sections and then applied for a special circular cross section. The finite difference in the time domain method is used by Eduardo et.al. (1995) to study electromagnetic wave propagation in a rectangular cavity and in a rectangular waveguide containing magnetized plasma. The plasma is an anisotropic material and its permittivity tensor components are functions of frequency. A variational expression involving the transverse components of the electric field and their transverse derivatives is established by Cory et.al. (1996) for the propagation coefficient of an electromagnetic waver propagating along a lossless rectangular chirowaveguide with perfectly conducting walls. The solution providing the propagation coefficient of the dominant mode is given in terms of a very simple analytical expression enabling a tractable analysis of the device. The propagation coefficient of a second propagating mode is obtained simultaneously.

The present work in chapter II is an extension of that of Allis et.al. (1963) in which our aim is to study the effect of collision frequency with emphasis on phase velocities, phase and attenuation constants in case of dominant modes of electromagnetic waves propagating through a parallel plane and rectangular waveguide in the presence of external strong longitudinal and transverse magnetic field.

Propagation characteristics including relativistic effects in moving dielectrics inside the waveguide has been studied by many investigators. [Collier and Tai (1965). Du and Compton (1966), Shiozawa (1966), Daly (1967), Gruenber and Daly (1967) and Yee 91971)]. Jain et.al. (1975) discussed dispersion relation and cutt off frequencies for the TE and TM modes of propagation in case of temperate TMG uniaxial anisotropic relativistically moving plasma. Jain et.al. (1974) described the results of the theoretical study of electromagnetic wave propagation through a parallel plane waveguide containing relativistically moving temperate lossless plasma in the presence of strong transeverse static magnetic field. Propagation and cut off frequency for collisional warm relativistically moving transversely magnetized plasma filled waveguide along with some simplified cases have been analyzed in chapter III.

In the presence of transverse magnetic field, a growing surface wave occurs as a result of new type of interaction which has phase velocity close to drift velocity. A carrier waves in an infinite semiconductor such as InSb is regarded as a longitudinal space charge wave with a phase velocity close to the drift velocity, of electrons. Kino (1968) presented a theory regarding growing carrier waves in InSb which has a phase velocity near the electron drift velocity and is unstable with very large growth rate in the presence of a vernishers magnetic field. Burke and Kino (1968) observed growing surface waves in Indium Antimonide semiconductore where electrons are drifting througn holes in the presence of magnetic field perpendicular to direction of drift.

The dispersion properties of slow electromagnetic surface waves are studied by Girka et.al. 91966). Waves are propagating across external magnetic field in planar metallic waveguides with a dielectric coating and non-uniform plasma filling at the harmonics of electron cyclotron frequency. The analytical the numerical investigations of the surface modes are carried out using the kinetic description for plasma particles.

The transverse plasma cyclotron wave spectrum is studied. Plasma density in homogeneity in modeled by a set of plasma layers with uniform density. Maxwell equations system with boundary conditions is solved to obtain the dispersion equation. Dispersion properties of slow electromagnetic surface waves propagating on the harmonics of the electron cyclotron frequency across the external steady magnetic field are studied in the case when the magnetic field is parallel to the plasma metal interface [Azarenkov et.al. (1997)]. Theoretical investigation of these surface modes is carried out using the kinetic description.

The effect of the plasma density transverse in homogeneity of plasma particles concentration and of the external magnetic field value on the dispersion properties of the surface waves on electron cyclotron harmonics (SWCH) is researched. The effects of dispersion of slow electromagnetic surface waves propagating across an external magnetic field in planar metallic waveguides with dielectric coating and non-uniform plasma filling on the harmonics of the electron cyclotron frequency are studied by Girka et.al. (1997). The theoretical investigation of the surface modes is carried out using a kinetic description for the plasma particles and plasma boundary. The effect of the transverse plasma density in shomogeneity on the electron surface cyclotron-wave spectrum is investigated. It is shown by Gradov et.al. (1993) that the nonlinear surface waves on the boundaries of a thin plasma slabs can propagate as single wave solutions.

Lee and Cho (1995) investigated the electromagnetic surface waves propagating on the planar interface between un-magnetized warm two-fluid plasmas of different densities and between a magnetized warm plasma and free space. The most general forms of the dispersion relations are obtained in closed forms and various limiting cases are discussed.

In particular, the influence of a cold plasma on the surface ion acoustic wave in analysed. Girka and Zolotukoin (1994) showed that the dispersion properties of surface waves propagating in metal rectangular waveguides with n-semiconductor filling can be studied effectively analytically by the interactive method with using the results of azimuthal SW theory as a zeroth approximation. The interactive method is used for investigation of dispersion properties of SW propagating in direction perpendicular to external axial magnetic field in a metal rectangular waveguide filled n-semiconductor. Non-ordinary electromagnetic surface waves are shown to propagate in the considered waveguides.

Wave propagation in a 70 GHz transversely magnetized partially filled ring-form solid-plasma waveguide has been theoretically analyzed by Obunai (1996) assuming the plasma material to be an n-type InSb ring at liquid nitrogen temperature. Surface wave resonant characteristics of two possible propagation modes and their dependence upon the waveguide curvature have been numerically analysed. The results of the experiments are in good qualitative agreement with the calculated results for one of these modes. These findings both theoretically and experimentally confirm the existence of the slow surface wave resonance in the waveguide. Propagation characteristics of 70 GHz slow surface waves in a partially filled coaxial solid plasma waveguide in an γ-dependent azimuthal magnetic field have been theoretically analyzed by Obunai (1996).

Results of numerical analysis reveal that the non-uniformity of the magnetic field exerts no essential effect on the existence of the slow surface wave resonance, but that the magnetic field required for the propagation of the slow wave is considerably reduced. Wave propagation characteristics of a 70 GHz image guide consisting of a transversely magnetized p-InSb slab have been studied experimentally by Yodokawa and Obunai (1997). The results indicate the possibility of using ρ-InSb as a component of dielectric waveguides.

It has also shown that the propagation characteristics of this image guide can be controlled by light irradiation. The present wok in chapter V is an extension of work of Kino (1968). Phase constant and growth rate of a surface wave in a near intrinsic semiconductor has been computed and the result has been compared with that in case of infinite semiconductor.

Any amplifier can be converted to an oscillator by positive feedback and any oscillator can be represented as a one port device with a negative terminal resistance. We look at the system exhibiting the instability as a travelling wave amplifier and we describe the model for possible convective two stream instability in n-type GaAs. No negative- differential bulk resistance was observed in semiconductors like Ge and Si [Ryder (1953)] during high electric field experimental work. The difficulty ws principally due to not being able to get a sufficient number of electrons to populate the negative mass states. Successful steps in the search for negative differential resistance were given in independent theoretical papers by Ridley and Watkins (1961) and Hilsum (1962) involving two bands and the transfer of electrons from one band to other by means of a high electric field.

In n-type Gallium arsenide semiconductor at certain thereshold value of electric field transfer of electrons from lower valley to upper valley begins [Liao (1991)]. The microwave properties of InP/GaInAs heterojunction double avalanche region (DAR) diode have been explored by Mishra et.al. (2001) using some computer simulation program which is capable of estimating the integrated device properties as well as the dynamic negative resistance contribution from the individual space step of the diode. This, negative resistance distribution profile reveals many interesting features as the power generating mechanism of the IMPATT diode. Our objective in chapter VI is to study wave propagation in negative differential conducting medium and relate the resulting instabilities to the equivalent two-stream instabilities. This description will clarify of close relationship between the different ways of approaching the study of negative differential resistance media. Based on two stream instability approach variation of phase velocity and growth rate for different driff velocities under different- electric field conditions has been calculated.

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Chapter 2

Dominant Modes in Parallel Plane and Rectangular Waveguides Filled with Uniaxial Anisotropic Lossy Plasma

2.1 Introduction:

The nature of electromagnetic wave propagation through a system of parallel conducting plates has been discussed. It is shown that various wave modes exist in this configuration. Imposing two more conducting plates, the concept of the hollow rectangular waveguide has been developed. The concept of cavity and its resonance properties have been obtained by imposing two more conducting walls along the axis for closing the open ends of rectangular and cylindrical waveguides. It is showns that these cavities are very useful for the determination of dielectric constants of materials and even getting as output of different radiations such as LASER and MASER etc.

Singh (1991) considered electromagnetic wave propagation between two parallel conducting planes of infinite extent separated by a distance of the order of a wave length. The free space is assumed between the two conducting planes. Marques et.al. (1993) presented a rigorous and systematic method of analysis of the electromagnetic wave propagation in parallel plate waveguide with a multilayered bianisotropic medium. The method is applied to the numerical study of parallel plate waveguide with a multilayered medium including lossless ferrite layers magnetized at an arbitrary direction. Both the propagation constant and the transmitted power are computed. Girka and Pavlenko (1999) studied the dispersion properties of slow electromagnetic surface waves propagating across an external magnetic field in planer plasma filled dielectric coated ion cyclotron frequency waveguide. The theoretical analysis is carried out using the kinetic description of plasma particles. The influence of the transverse dimensions of the plasma layer permittivity of the dielectric coating of the waveguide walls and external magnetic field on the spectrum of ion cyclotron SWs (CSWs) are investigated.

The electromagnetic wave propagation through the system of parallel plates obey the Maxwell's field equations at all points on the conducting planes. Singh (1973) and Singh at.al. (1976) analysed the propagation characteristics such as phase constant, attenuation constant etc. of electromagnetic waves passing through gaseous plasma filled parallel plate waveguide under the influence of an external magnetic field. The propagation characteristics of microwaves in free space and in dielectric and ferrite filled waveguides are well known [Van Bladel and Higgins (1951), Collin (1960), Rosenbaum (1964), Chatterjee and Chatterjee (1965), Jordan and Balmain (1980). Ramo et.al. (1970). Gardiol (1968) and Gardiol and Vander Vorst (1969). The propagation characteristics of the electromagnetic waves in a waveguide filled with cold stationary and homogeneous plasma are duscyssed in considerable detail by Allis et.al. (1963). They taking into account the dielectric model of the plasma considered the effect of strong longitudinal and transverse magnetic fields such that $\omega_h >> \omega_p$ on the propagation characteristics of waveguide containing cold plasma in terms of (ω–B) diagrams. They concluded that the characteristics of a free space waveguide is modified by the presence of the plasma medium. Only electron component of plasma takes part in the phenomena of interest. Bevc and Everhart (1962), Tuan (1969), Kalluri (1970), Gruenber and Daly (1967) and Gonzalez (1971) discussed electromagnetic wave propagation in waveguides containing gaeous plasma. Novel features of phase and attenuation characteristics in case of propagation of electromagnetic wave in a colalitional plasma column magnetized in the axial direction and enclosed in a conducting cylinder are outlined by Biswas and Basu (1987). The propagation characteristics of waveguides partially and completely filled with solid state plasma in the presence of transverse magnetic field have been discussed by many investigators [Barlow and Koike (1963), Toda (1964), Hirota (1964), Engineer and Nag (1965), Rahman and Gunn (1969), Arnold and Rosenbaum (1971), Nejib and Ruduski (1973a) and Gupta et.al. (1973)] Takeda (1994) calculated the dispersion relation and the wavelength of microwaves required to produce [ECR] plasmas in waveguides by approximate and exact equations. A wave can also propagate through a high density magnetized plasma in a waveguide with an extremely small diameter. Methods of quasi-static approximation and rigorous calculations of the phase constant variation of an electromagnetic wave in a rectangular waveguide with gas discharge plasma in dielectric tubes disposed along the longitudinal axis of the waveguide are considered [Belous et.al. (1996)].

It is shown that, for negative values of the plasma permitivity, there are qualitative and quantitative discrepancies between phase constant dependence on the redii of dielectric tubes calculated by these two methods. The wave dispersion property in completely plasma-filled waveguide with definite magnetic field is analysed [Zhu and Liu (1997). A revised dispersion equation is derived and calculated. A new division of wave modes is given. Compared with the old division, this is more suitable. The propagation properties of a plasma dielectric waveguide immersed in an external magnetic field are presented [Hu and Ruan (1988)]. The field component expressions, eigenvalues and characteristic equations have been obtained.

The variations of the propagation properties with plasma parameters, external magnetic field and frequency are discussed in detail on the basis of the method of separation of variables combined with the technique of Muller's calculating roots. The study of cut off frequency of rectangular waveguide filled with dielectric and plasma medium has been studied by Du and Compton (1965) and Singh (1993). In this chapter the propagation characteristics in case of parallel plane and rectangular waveguides filled with homogeneous uniaxial anisotropic collisional plasma is considered for parameters appropriate to laboratory plasma in parallel plane waveguide the propagating dominant ET, TM and TEM modes are analysed with an emphasis on phase refractive index variation with waveguide dimension and plasma parameters. In rectangular waveguide filled with cold, homogeneous lossy plasma under influence of strong longitudinal and transverse magnetic fields the dominant TE and TM modes are analysed with an emphasis on study of phase and attenuation constants with varing appropriate plasma parameters. Whole computations in this chapter have been done by taking electron number density $n \approx 10^{14}$ per cubic meter (appropriate to laboratory plasma), the waveguide dimensions along X-direction, d =1 = 5.0cm. and along Y-direction b = 2.5cm. (appropriate to x-band of frequency). The corresponding plasma frequency ω_p, free space waveguide cutoff frequency $\omega_\in$ and transverse wave number along X-direction $\left(i.e. k_d = k_1 = \frac{\pi}{d} = \frac{\pi}{1} \right)$ come out to be 56.4 GHz, 42.13 GHz and 62.8 rad/m respectively. The single conductor waveguides do not allow the propagation of TEM waves which is true in the case of rectangular and cylindrical waveguides.

2.2 Effect of Collisions on Electromagnetic Wave Propagation:

The dispersion equation for electromagnetic wave propagation can be derived with the help of the collisional conductivity of the plasma equation. The conductivity of the plasma in response to the imposed electromagnetic field is written as

$$\sigma = \frac{j}{(\omega - j\upsilon)} \sum_p \frac{n_p q_p}{m_p}, \qquad p = 1,2,3 \tag{2.1}$$

Where, n_p and q_p are number density, mass and charge of species p. ω and υ are wave and collision frequencies. The conductivity of plasma is imaginary. The electron current in a collisionless plasma lags behind the electric field vector by π/2 radians.

Therefore, the electron current is said to be inductive. However, in the presence of collisions, the phase lag of π/2 is disturbed. The real part of E.J shows that a part of the ern energy of the wave is dissipated into the collisional plasma where E is electric field and J is current density.

The conventional dispersion equation in the presence of binary collisions between electron-neutral and electron-ion is written as

$$c^2 k^2 = \omega^2 - \frac{\omega_p^2}{\left(1 - \frac{j\upsilon}{\omega}\right)} = \omega^2 - \frac{\omega_p^2}{(1 - jZ)} \tag{2.2}$$

Where Z-υ/ω, is the dimensionless collision parameter. For longitudinal plasma waves, the dispersion equation modified and is written as

$$\omega^2 = \frac{\omega_p^2}{(1 - jZ)} \tag{2.3}$$

We find that the principal effect of collision is to produce damping of longitudinal as well as of transverse waves. For longitudinal waves, the solution of equation (2.3) is easily obtained

$$\omega = \frac{j\upsilon \pm \sqrt{4\omega_p^2 - \upsilon^2}}{2} = \omega_r + j\omega_i \qquad (2.4)$$

The frequency of plasma waves thus obtained is complex, therefore, the wave which varies as $e^{-j\omega 4}$ now has an additional damping term. The electric field wave vector is thus written as

$$E_e = E_0 e^{j\omega_r t} e^{-\omega_i t} \qquad (2.5)$$

We rationalize the dispersion equation (2.2) and write as

$$\gamma^2 c^2 = \omega^2 - \frac{\omega_p^2}{(1+Z^2)} - \frac{j\omega_p^2 Z}{(1+Z^2)} \qquad (2.6)$$

We find that due to collisional effect, the dispersion equation acquires a complex term.

Therefore, the analysis of the dispersion equation can be carried out either by assuming ω to be real and γ to be complex or by assuming γ real and ω complex. The complex wave number can be written as

$$\gamma = \alpha + j\beta \qquad (2.7)$$

With, α and β both real. The form of the harmonic wave propagating along the Z-axis can be written as

$$E_x = E_o e^{(ij\omega t - yz)}$$

$$= E_o e^{j(\omega t - \beta z)} e^{-\alpha z} \qquad (2.8)$$

The positive values of α show damping of the electromagnetic wave as it propagates along the positive Z-direction.

The negative values of α for waves propagating along the positive Z-direction show the exponential growth of the wave.

Similarly, with real γ, the complex frequency can be written as

$$\omega = \omega_r + j\omega_i \tag{2.9}$$

The negative value of ω_i shows the growth of the wave as it propagates along the positive Z-axis whereas positive values of ω_i show the decay of the wave propagating along the positive Z-axis.

In the propagation band defined by $\omega > \omega_r$ the attenuation of the wave increases with increasing values of υ and reaches a maximum value and thereafter falls off to zero asymptotically. The maximum attenuation of the wave caused by either α or ω_i is found for

$$\upsilon^2 = \omega^2\left(1 - \frac{2\omega_u^2}{3\omega^2 - \omega_u^2}\right) \tag{2.10}$$

Where $\omega_u^2 = \frac{3}{4}\omega_p^2$, In the limit of a very large collision frequency equation (2.6) reduces to

$$\lim_{\upsilon\to\infty} it \qquad K^2c^2 = \omega^2 \tag{2.11}$$

That is, in the limiting case of a very large collision frequency, the plasma behaves like a free-space.

This behaviour of plasma is understandable because infinite collisions make the plasma highly non-conducting; therefore, the electromagnetic wave propagates without any loss of energy. This behaviour of the plasma is analogous to wave propagation through free- space.

2.3 Boundary Conditions:

In previous chapter boundary conditions at the interface of two media have been outlined. However the same conditions are derived here in case of an interface consisting of a good conductor as used in parallel plane or rectangular waveguide.

Current carrying conductors, satisfy ohm's law. For good conductors carrying finite current, the electric field within the conductor everywhere is zero [Ram et.al. (1970)].

The corresponding magnetic field within the perfectly conducting planes is also zero.

These two conditions are consistent with two Maxwell's equations namely.

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.12}$$

and

$$\nabla . B = O \tag{2.13}$$

Emerging from these equations, one has tangential components of E zero everywhere in conductor i.e.

$$n \times \overline{E} = 0 \tag{2.14}$$

and the continuity of normal component of B requires that

$$n, B = 0 \tag{2.15}$$

where n is surface normal.

In the absence of external currents, the magnetic field outside the conductor is given by

$$\nabla \times B = \mu \in \frac{\partial E}{\partial t} \tag{2.16}$$

One can obtain derived boundary conditions from the two boundary conditions given by equations (2.14) and (2.15).

Substituting of E from Maxwell's equations in equation (2.14) one obtains.

$$n \times (\nabla \times B) = 0 \tag{2.17}$$

and likewise substituting for B form Maxwell's equation in equation (2.15), one obtains.

$$n \times (\nabla \times B) = 0 \tag{2.18}$$

These conditions imply that the normal component of the electric field vector is not necessarily zero, since there can be some charges on the conducting surfaces, and likewise the tangential component of the magnetic field vector is not necessarily zero, since there can be some surface currents in the perfectly conducting plates.

2.4 LMG parallel plane waveguide

2.4.1 Development of Theory:

We consider a waveguide consisting of two parallel perfect conducting planes separated by a distance d along X-direction and extending infinitely in the Y and Z directions. The space between two planes is assumed to contain homogeneous cold lossy plasma. The propagation direction of signal inside the parallel plane waveguide is along Z-axis. The plasma and wave frequencies are negligibly small in comparison to gyro-frequency because of externally applied strong static magnetic field. In longitudinally magnetized (LMG) parallel plane waveguide case the strong static magnetic field is applied along Z-axis.

The dielectric constant of uniaxial anisotropic lossy plasma which is a tensor of rank two is given by Singh (1973) as

$$\overline{\overline{\in}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \in_{33} \end{bmatrix} \tag{2.19}$$

Where $\in_{33} = 1 - \dfrac{\omega_p^2}{\omega(\omega - j\upsilon)}$, plasma frequency $\omega_p = \left(\dfrac{ne^2}{m \in_0}\right)^{1/2}$ and υ is collision frequency.

All field vectors are assumed to very as exp(jωt–γz) where γ is the propagation constant, β and α are phase and attenuation constants.

Now Maxwell's crul equations can be written as

$$\gamma H_y = -j\omega\mu_0 H_x \qquad \gamma H_y = j\omega \in_0 E_x$$

$$-\gamma E_x - \frac{\partial E_x}{\partial x} = -j\omega\mu_0 H_y \qquad -\gamma E_x - \frac{\partial H_x}{\partial x} = -j\omega \in_0 H_y \qquad (2.20)$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu_0 H_y \qquad \frac{\partial H_y}{\partial x} = j\omega_{0\in_{33}} H_y$$

Where μ_0 and $\in_0$ are free space permeability and permittivity. From the above equations, one obtains the followin linear differential equations for transverse field components.

$$E_x = \frac{\gamma}{\gamma^2 + k_0^2}\frac{\partial E_z}{\partial x}$$

$$H_x = \frac{\gamma}{\gamma^2 + k_0^2}\frac{\partial H_2}{\partial x} \qquad (2.21)$$

$$E_x = \frac{j\omega\mu_0}{\gamma^2 + k_0^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{j\omega\mu_0}{\gamma^2 + k_0^2}\frac{\partial H_z}{\partial x}$$

In equations (2.20) and (2.21) the fields are assumed invariant because of no boundary in Y-direction i.e. $\frac{\partial}{\partial y} = 0$. The field configurations that can exist inside parallel plane waveguide satisfying the above set of equations and boundary conditions fall into following classes.

The first one is the transverse electric (TE) wave in which electric vector is transverse to the direction of propagation while the magnetic vector has both transverse and longitudinal components.

The second one is the transverse magnetic (TM) wave in which magnetic vector is entirely transverse to the direction of propagation whereas the electric field has both transverse and longitudinal components. The third one is transverse electric and magnetic (TEM) wave in which electric and magnetic vectors has only transverse components.

2.4.2 Transverse Electric (TE) Waves:

For TE waves, put E_z =0. From equations (2.20) one obtains

$$\frac{d^2E_{y0}}{dx^2} = h_0^2 E_{y0} \tag{2.22}$$

where

$$K_0^2 = \gamma^2 + K_0^2, K_0^2 = \frac{\omega^2}{c^2} \tag{2.23}$$

and

$E_y = E_{yo} \exp(-\gamma z)$

Equation (2.22) is a second order differential equation whose solution is of the form

$$E_{yo} = C1 \sin k_d X + C_2 \cos k_d X \tag{2.24}$$

Where C_1 and C_2 are constants

Applying boundary conditions which state that tangential components of electric field is zero at the boundary of conducting walls i.e. E_y = 0 and d, one obtains

$E_y = C_1 \sin k_d x \exp(-\gamma z)$

where $k_d = \frac{m_1\pi}{d}$, m_1 is the wave number in X-direction and is a positive integer. Its each value specifies a particular field configuration or moder and the wave associated with the integer m_1 is designated as the $TE_{m1,0}$ wave. Phase refractive index is obtained using equation (2.23) as

$$\mu_p = \frac{c}{V_p} = \sqrt{1 - \frac{k_d^2 c^2}{\omega_p^2}} \quad (2.26)$$

where phase velocity $V_p = \frac{\omega}{k}$

2.4.3 Dominant TE mode:

It will be noticed that the smallest value of m_1 that can be used in equation (2.26) in m_1=1, because m_1 = 0 makes all the fields identically zero.

Thus for dominant transverse electric ($TE_1 0$) mode, putting m_1=1 and $k_d = \frac{\pi}{d}$ in equation (2.23) we have

$$\gamma^2 = \left(\frac{\pi}{d}\right)^2 - k_0^2 \quad (2.27)$$

Putting in general $\gamma = \alpha + j\beta$ where α is attenuation constant and β is phase constant. In this case attenuation does not exist i.e. α=0. Form equation (2.27) one obtains

$$\beta^2 = k_0^2 - \left(\frac{\pi}{d}\right)^2$$

$$\beta^2 c^2 = \omega^w - \frac{\pi^2 c^2}{d^2} \quad (2.28)$$

phase refractive index for dominant $TE_{1,0}$ mode is given by

$$\mu_p = \sqrt{1 - \frac{\pi^2 c^2}{d^2 \omega 2}} \quad (2.29)$$

Making use of equations (2.21) and (2.25) and putting E_z=0, the expression for field components between parallel planes are

$E_v = C_1 \sin k_d x \exp\left(-jk_0\mu_p\right)z$

$$H_z = \frac{k_0\mu_p}{\omega\mu_0} C_1 \sin k_d x \exp\left(-jk_0\mu_p\right)z \tag{2.30}$$

$$H_z = \frac{k_d}{j\omega\mu_0} C_1 \cos k_4 x \exp\left(-jk_0\mu_p\right)z$$

The above transverse field components for dominant TE modes are obtained as

$$E_x = C_1 \sin\frac{\pi x}{d} \exp\left(-jk_0\mu_p\right)z$$

$$H_x = -\frac{k_0\mu_p}{\omega\mu_0} C_1 \sin\frac{\pi x}{d} \exp\left(-jk_0\mu_p\right)z \tag{2.31}$$

$$H_z = -\frac{\pi}{j\omega\mu_0 d} C_1 \sin\frac{\pi x}{d} \exp\left(-jk_0\mu_p\right)z$$

2.4.4 Transverse Magnetic (TM) Waves:

Putting H_z = 0 for TM modes. equation (2.20) yields the following differential wave equation for H_γ as

$$\frac{\partial^2 H_v}{\partial x^2} = -k_d^2 \in_n H_i \tag{2.32}$$

The solution of above equation can be written as

$H_x = (C_2 \sin k_d x + C_4 \cos k_4 x) \exp(-\gamma z)$

However from equation (2.20, the expression for E_z can be obtained as

$$E_z = \frac{k_4}{J\omega \in_\upsilon \in 11}\left[\left(C_1 \cos k_d x - C_4 \sin k_d x\right)\exp(-\gamma z)\right] \tag{2.33}$$

Applying boundary conditions which state that E_z =0 at x = 0 and d, one obtains for dispersion relations as

$$\left(\gamma^2 + k_0^2\right)\in_{33} = k_d^2 \tag{2.34}$$

Separating real and imaginary parts and eliminating α we have

$$4\beta^4 + 4\beta^2\left\{k_4^2\left[\frac{\omega^2\left(\omega^2-\omega_p^2\right)+\omega^2\upsilon^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right] - k_0^2\left[\frac{\upsilon\omega\omega_p^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right]^2\right\} = 0 \tag{2.35}$$

2.4.5 Dominant TM Mode:

For dominant TM mode put m_i= 1 i.e. $k_d = \frac{\pi}{d}$, the equation (2.35) yields

$$4\beta^4 + 4\beta^2\left\{\frac{\pi^2}{d^2}\left[\frac{\omega^2\left(\omega^2-\omega_p^2\right)+\omega^2\upsilon^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right] - \frac{\omega^2}{c^2}\right\} - \frac{\pi^4}{d^4}\left[\frac{\upsilon\omega\omega_p^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right]^2 = 0 \tag{2.36}$$

which gives the relation for phase refractive index as

$$\mu_p = \frac{1}{\sqrt{2}}\left|-\left\{\frac{\pi^2c^2}{d^2}\left[\frac{\omega^2-\omega_p^2+\upsilon^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right]-1 \pm \sqrt{\left\{\frac{\pi^2c^2}{d^2}\left[\frac{\omega^2-\omega_p^2+\upsilon^2}{\left(\omega^2-\omega_p^2\right)^2+\omega^2\upsilon^2}\right]-1\right\}^2 + \frac{\pi^4c^4\upsilon^2}{d^4\omega^2\left[\left(\omega^2-\omega_{[}^2\right)^2+\omega^2\upsilon^2\right]^2}}\right\}\right|^{1/4} \tag{2.37}$$

The field components for dominant TM mode for uniaxial anisotropic lossy plasma can be expressed in terms of μ_p. as;

$$H_y = C_4 \cos\frac{\pi x}{d}\left(-jk_0\mu_p\right)z$$

$$E_x = \frac{k_0\mu_p}{\omega \in_0} C_4 \cos\frac{\pi x}{d}\exp\left(-jk_0\mu_p\right)z$$

$$E_z = \frac{j\pi x}{d\omega \in_0 \in_{33}} C_4 \sin\frac{\pi x}{d}\exp\left(-jk_0\mu_p\right)z \tag{2.38}$$

For collisionless case i.e. υ =0, equation (2.37) simplifies to

$$\mu_p = \left[1 - \left(\frac{\frac{\pi^2 c^2}{d^2}}{\omega^2 - \omega_{[}^2}\right)\right]^{1/2} \tag{2.39}$$

2.4.6 Dominant TEM Mode:

Putting ....in dispersion relation equation (2.35) on has

$$\mu_p = \frac{c}{V_p} = 1$$

and field components are written as

$$E_x = \frac{C_4}{c \in_0}\exp\left(-jk_p\right)z$$

$$H_x = C_4 \exp(-jk_0)z \tag{2.40}$$

The equation (2.40) is independent of guide geometry and magnetic field and fields are entirely transverse.

2.4.7 Results and Discussion:

The lowest order TE mode that can exist is the TE_{10} mode Usual TE_{m0} mode equation (2.26) shows that the phase refractive index is independent of the presence of plasma and strong magnetic field and has the characteristics of free space waveguide with only dependence on wave number k_d in X-direction. The propagation of wave takes place for frequencies greater than k_dc and with velocity of light at very high frequencies with negligible effect of even separation between planes. Since the value of μ_p is either real of imaginary but not complex of a particular wave frequency the wave ceases of propagate below a frequency k_dc. Equation (2.29) has been computed for $\frac{\pi c}{d}$ =1885 GHz and variation of phase refractive index μ_p with microwave frequencies has been shown in Figure (2.1). It is concluded from Figure (2.1) for dominant TE (i.e. TE_{10}) mode that the value of μ_p increases fastly for values of ω greater but close to 18.85 and then approaches unity. It means propagation of signal is possible for frequencies greater then $\frac{\pi c}{d}$ and at very high e.m. wave frequencies the effect of plasma presence and hence collision frequency is not observed. It becomes almost like free space wave propagation. Below cut off frequency , $\frac{\pi c}{d}$ e.m. wave does not propagate because refractive index becomes imaginary and is called evanescent mode. The ratio of velocity of light in free space to the phase velocity of microwave signal in LMG uniaxial anistropic parallel plane waveguide i.e. the index of refraction μ_p has been computed for dominant TM_n mode from equation (2.37) for $\frac{\pi c}{d}$ =18.85 GHz taken suitably for X-band waveguide, plasma frequency ω_p = 56.4 GHz and collision frequencies υ = 1 and 10GHz suitable for laboratory gaseous plasma. For comparison, curve for collisionless case υ =0 has also been depicted in Figure (2.2).

The positive values of μ_p^2 indicate propagation of wave and negative value the evanescence. The value μ_p =1 indicates that phase velocity of the wave is equal to speed of light. The collisionless curve (υ=0) has got two branches: first is the slow wave branch having phase velocity less than velocity of light and other fast wave branch having phase velocity higher than c and in between the two branches there is no propagation of electromagnetic wave. From the Figure (2.2) it is observed that the range of evanescence is 56.4 GHz < ω < 59.46 but for collisional plasma one observes no such evanescence: the propagation of wave is possible over all frequencies down to zero. In the low frequency region of the curve the value of phase velocity goes on decreasing with increases of frequency upto certain frequency.

With further increase of frequency the phase velocity increases and approaches the value of the velocity of light and the propagation is similar to that of free space. Peak values of μ_p decrease with decrease in collision frequency as seen from the curve.

In case of dominant TEM mode here, TM mode dispersion relation reduces to free space propagation. Phase velocity of electromagnetic wave is equal to the velocity of light and is independent of frequency as well as the distance between the two parallel planes.

Also there is no cutoff in case of TEM mode i.e. electromagnetic waves of all frequencies can propagate along the guide. This means that behaviour of the parallel plane waveguide is similar to that of free space and thus the electromagnetic field is entirely transverse as seen in equation (2.40).

2.5 TMG Parallel Plane Waveguide

2.5.1 Development of Theory:

Let s consider two pallel sheets of a perfect conductor infinite in extent, parallel to the Y, Z plane and separated by a distance d along X-axis. This parallel plane waveguide contains homogeneous, lossy temperature plasma. In transversely magnetized (TMG) case, the magnetic field is applied in the direction of plate separation i.e. X-axis.

The plasma frequency and wave frequencies are negligibly small in comparison to electron cyclotron frequency because of externally applied strong static magnetic field. The direction of propagation of signal is along Z-direction.

The dielectric tensor appropriate for the case of uniaxial anisotropic transversely magnetized (TMG) lossy plasma in given by [Singh (1973)];

$$\bar{\bar{\in}} = \begin{bmatrix} \in_{11} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.41}$$

Where $\in_{11} = 1 - \frac{\omega_p^2}{\omega(\omega - j\upsilon)}$

Under this condition Maxwell's equations can be written as;

$$\gamma E_y = -j\omega\mu_0 H_x \qquad \gamma H_y = j\omega \in_0 \in_{11} E_x$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y \qquad -\gamma H_x - \frac{\partial E_z}{\partial x} = -j\omega \in_0 E_y \tag{2.42}$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu_0 H_z \qquad \frac{\partial E_y}{\partial x} = -j\omega \in_0 E_z$$

The above equations can be combined to yield expressions for E_x E_y, H_x and H_y in terms of axial field components E_z and H_z as:

$$E_x = -\frac{\gamma}{\gamma^2 + k_0^2 \in_{11}} \frac{\partial E_z}{\partial x}$$

$$E_y = -\frac{j\omega\mu_0}{\gamma^2 + k_0^2} \frac{\partial H_z}{\partial x} \tag{2.43}$$

$$H_x = -\frac{\gamma}{\gamma^2 + k_0^2} \frac{\partial H_z}{\partial x}$$

$$E_y = -\frac{j\omega \in_0 \in_{11}}{\left(\gamma^2 + k_0^2 \in_{11}\right)} \frac{\partial E_z}{\partial x}$$

From the wave equation it can be seen that propagating waves are easily classified into TE, TM and TEM modes with respect to the direction of propagation.

2.5.2 Usual and Dominant TE Modes:

For this case put $E_z = 0$. Following the same procedure as in case of dominant TE modes of LMG parallel plane waveguide one finds that the expressions for phase velocity and field components are the same as those in case for LMG parallel plane waveguide and are given by equations (2.29) and (2.31) which characterize the wave propagation in free space parallel plate waveguide.

2.5.3 Usual and Dominant TM Modes:

From equations (2.42) with $H_z = 0$, the differential equation for H_y is

$$\frac{\partial^2 H_y}{\partial x^2} + \left(\frac{\gamma^2}{\in_{11}} + k_0^2 \right) H_y = 0 \tag{2.44}$$

Proceeding in the same manner as the TM modes for the case of LMG parallel plane waveguide, equation (2.44) can be solved with the appropriate boundary conditions to obtain the dispersion relation as:

$$\gamma^2 + \in_{11} \left(k_0^2 + k_d^2 \right) = 0 \tag{2.45}$$

Putting $\gamma = \alpha + j\beta$ and separating real and imaginary parts we have expressions for phase and attenuation constants for dominant TM mode as

$$\beta^4 + \left[\frac{\pi^2 c^2}{d^2 \omega_p^2} - \frac{\omega^2}{\omega_p^2} + \frac{\omega^2 - \frac{\pi^2 c^2}{d^2}}{\omega^2 + \upsilon^2} \right] \frac{\omega_p^2}{c^2} \beta^2 - \frac{\upsilon^2 \omega_p^4}{4\omega^2 c^4} \left(\frac{\omega^2 - \frac{\pi^2 c^2}{d^2}}{\omega^2 + \upsilon^2} \right)^2 = 0 \tag{2.46}$$

and

$$\alpha^4 + \left[\frac{\pi^2 c^2}{d^2 \omega_p^2} - \frac{\omega^2}{\omega_p^2} + \frac{\omega^2 - \dfrac{\pi^2 c^2}{d^2}}{\omega^2 + \upsilon^2}\right]\frac{\omega_p^2}{c^2}\alpha^2 - \frac{\upsilon^2 \omega_p^4}{4\omega^2 c^4}\left(\frac{\omega^2 - \dfrac{\pi^2 c^2}{d^2}}{\omega^2 + \upsilon^2}\right)^2 = 0$$

From equation (2.46) one obtains expression for the phase refractive index as

$$\mu_p = \frac{\omega_p}{\sqrt{2\omega}}\left\{\left[\frac{\omega^2}{\omega_p^2} + \frac{\pi^2 c^2}{d^2(\omega^2+\upsilon^2)} - \frac{\pi^2 c^2}{d^2 \omega_p^2} - \frac{\omega^2}{\omega^2+\upsilon^2}\right]\right.$$

$$\left. \pm \sqrt{\left[\frac{\omega^2}{\omega_p^2} + \frac{\pi^2 c^2}{d^2(\omega^2+\upsilon^2)} - \frac{\pi^2 c^2}{d^2 \omega_p^2} - \frac{\omega^2}{\omega^2+\upsilon^2}\right]^2 + \upsilon^2\left[\frac{\omega^2}{\omega^2+\upsilon^2} - \frac{\pi^2 c^2}{d^2(\omega^2+\upsilon^2)}\right]}\right\}^{1/2}$$

(2.47)

Equation (2.47) give the phase refractive index for a transversely magnetized waveguide filled with weakly ionized plasma. For a lossless plasma i.e. when collisions between electrons and neutrals are ignored i.e. $\upsilon = 0$, equation (2.47) reduced to

$$\mu_p = \frac{\omega_p}{\omega}\left\{\left[\frac{\omega^2}{\omega_p^2} + \frac{\pi^2 c^2}{d^2 \omega^2} - \frac{\pi^2 c^2}{d^2 \omega_p^2} - 1\right]\right\}^{1/2} \qquad (2.48)$$

The field components for dominant TM modes can be determined from equation (2.43) with $H_z = 0$ as

$$E_x = \frac{k_0 \mu_p}{\omega \in_0 \in_{11}} E_0 \cos\frac{\pi x}{d}\exp(-jk_0\mu_p)z$$

$$E_z = \frac{\pi}{j\omega \in_0 d} E_0 \sin\frac{\pi x}{d}\exp(-jk_0\mu_p)z \qquad (2.49)$$

$$E_y = H_0 \cos\frac{\pi x}{d}\exp(-jk_0\mu_p)z$$

where E_0 and H_0 are amplitudes of electric and magnetic vectors.

2.5.4 Dominant TEM Modes

Putting value of $m_1 = 0$ i.e. kg = 0 in equation (2.45) and following the same procedure the expression for refractive index is obtained

$$\mu_p = \frac{\omega_p}{\sqrt{2}\omega}\left\{\left[\frac{\omega^2}{\omega_p^2} - \frac{\omega^2}{\omega^2+\upsilon^2}\right] + \sqrt{\left[\frac{\omega^2}{\omega_p^2} - \frac{\omega^2}{\omega^2+\upsilon^2}\right]^2 + \frac{\upsilon^2\omega^2}{\left(\omega^2+\upsilon^2\right)^2}}\right\}^{1/2} \quad (2.50)$$

As it is a particular case of equation (2.47), μ_p is independent of guide geometry. Distance between the parallel planes do not influence the propagation characteristics but is possible at all frequencies. There is no propagation through lossless plasma filling TMG parallel plane waveguide having frequency less than the plasma frequency.

The field components possible in dominant TEM mode of TMG parallel plane waveguide filled with uniaxial anisotropic lossy plasma are

$$E_x = \frac{k_0\mu_p}{\omega \in_0 \in_{11}} E_0 \exp\left(-jk_0\mu_p\right)z \quad (.251)$$

$$H_y = H_0 \exp\left(-jk_0\mu_p\right)z$$

They are independent of guide geometry, magnetic field and are entirely transverse.

2.5.5 Results and Discussion:

In case of dominant transverse electric mode for TMG parallel plane waveguide the propagation of microwave is similar to that in LMG parallel plane waveguide case and is shown in Figure (2.1)

In order to study the effect of collisions one the propagation characteristics in dominant TM ($TM_{1,1}$) mode of TMG parallel plane waveguide, phase refractive index is computed from equation (2.47) for different values of microwave signal frequencies with collision frequency υ = 1 and 10GHz as running parameter for a fixed value of $\frac{\pi c}{d}$ =18.85GHz and plasma frequency ω_p =56.4GHz. The results are shown in Figure (2.3).

For comparison curve for collision less case $\upsilon = 0$ has also been shown. Each $\upsilon \neq 0$ has also been shown.

Eavh $\upsilon \neq 0$ curve has two branches isolated from each other. In the first branch μ_p decreases from ∞ (i.e. resonance condition) to 0 (i.e. cut off condition) as microwave signal frequency ω varies from zero to $\frac{\pi c}{d}$ = 18.85GHz. This means that the phase velocity increases from zero to infinity. In the second branch μ_p varies from 0 to 1 asymptotically as ω is increased from $\frac{\pi c}{d}$ = 18.85GHz. and onward. This means that the phase velocity decrease from infinity to the velocity of light. In collision less case $\upsilon = 0$, the region of evanescence extends between 18.85GHz < ω < 56.4GHz where propagation of signal is not possible as shown in Figure (2.3). Here also first branch has variation of refractive index from ∞ for $\omega = 0$ to 0 for $\omega = \frac{\pi c}{d}$ = 18.85GHz. and second branch has value of $\mu_p = 0$ again for ω = 564 GHz and reaches the value of $\mu_p = 1$ asymptotically.

Equation (2.50) has been computed for υ = 1 and 10 GHz with value of plasma frequency appropriate to laboratory plasma and (ω–μ_p) curve for dominant TEM mode in case of parallel plane TMG plasma waveguide has been shown in Figure (2.4).

Propagation in TEM mode for TMG parallel plane waveguide is independent of guide geometry and is possible for all frequencies but for collision less case the curve has a branch with cutoff at plasma frequency.

It is observed from figure (2.4) that for very low frequencies phase velocity of wave is relatively higher for highly collisional plasma but the order changes beyond some frequency for each curve.

Finally the phase velocity for each curve reaches asymptotically the speed of light, curve for each collision frequency has got some minimum value of μ_p which moves towards high frequency side with increase in collision frequency. There is no propagation through lossless plasma filling TMG parallel plane waveguide having frequency less than the plasma frequency.

2.6 Longitudinally Magnetized (LMG) Rectangular Waveguide:

2.6.1 Development of Theory:

Propagation characteristics of electromagnetic waves for dominant modes in a rectangular waveguide containing cold, homogeneous lossy plasma under the influence of strong magnetic field in the direction of propagation can be discussed with the help of Maxwell's equation and equation (2.19) for dielectric tensor.

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu_0 H_x$$

$$\frac{\partial Ez}{\partial y} + \gamma H_y = j\omega \in_0 E_x$$

$$-\gamma E_x = \frac{\partial Ez}{\partial x} = -j\omega\mu_0 H_y$$

$$-\gamma H_x = \frac{\partial Ez}{\partial x} = j\omega \in_0 E_y \tag{2.52}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 Hz$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = j\omega \in_0 \in_{33} E_z$$

Where a variation of type exp $(j\omega t - \gamma z)$ has been assumed. The transverse components of the field vectors can be related to the longitudinal components by the equations

$$E_{\mathrm{T}} = -\frac{\gamma}{\gamma^2 + k_0^2}\nabla_{\mathrm{T}} E_z + \frac{j\omega\mu_0}{\gamma^2 + k_0^2}\hat{z} \times \nabla_{\mathrm{T}} H_z$$

$$H_{\mathrm{T}} = -\frac{j\omega \in_0}{\gamma^2 + k_0^2}\hat{z}\nabla_{\mathrm{T}} E_z + \frac{j\omega\mu_0}{\gamma^2 + k_0^2}\nabla_{\mathrm{T}} H_z \tag{2.53}$$

Where $\hat{z}$ is unit normal vector along Z-direction. The solution can be obtained by assuming either E_z =0 (TE modes) or H_z = 0 (TM modes).

2.6.2 Dominant TE Mode:

For obtaining the characteristics of electromagnetic waves in the TE mode, put E_z = 0 equation (2.52) may be manipulated to yield.

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = -\left(\gamma^2 + k_0^2\right)H_z \tag{2.54}$$

This equation can be solved by using Maxwell's equations and appropriate boundary conditions to give

$$\gamma^2 + k_0^2 = k_c^2 \tag{2.55}$$

Which gives expressions for phase constant as

$$\beta^2 c^2 = \omega^2 - \omega_c^2 \tag{2.56}$$

Here $k_c = \left(k_1^2 + k_2^2\right)^{1/2} = \left[\left(\frac{m_1\pi}{1}\right)^2 + \left(\frac{m_2\pi}{b}\right)^2\right]^{1/2}$ corresponds to cutoff frequency of free space rectangular waveguide. This equation is similar to the equation for the free space waveguide with cut at ω_c.

The wave inside the waveguide travels with velocity of light of higher frequencies.

For frequencies below ω_c, the fields d not propagate but oscillate everywhere along Z-axis in phase. This is called evanescent mode.

Insertion of the assumed H_z value in equation (2.52) and (2.53) with $E_z = 0$ permits the TE field components to be written as

$$E_x = \frac{j\omega\mu_0}{k_c^2} k_2 H_0 \quad \cos k_1 x \ \sin k_2 y$$

$$E_y = \frac{j\omega\mu_0}{k_c^2} k_1 H_0 \quad \cos k_1 x \ \sin k_2 y \tag{2.57}$$

$$H_x = \frac{\gamma}{k_c^2} k_1 H_0 \quad \cos k_1 x \ \sin k_2 y$$

$$H_y = \frac{\gamma}{k_c^2} k_1 H_0 \quad \cos k_1 x \ \sin k_2 y$$

$$H_z = H_o \qquad \cos k_1 x \ \sin k_2 y$$

It can be seen from field equations that wave of $TE_{m1,0}$ type are possible except for the $TE_{0,0}$ mode in which case fields go to zero.

The fields travel unattenuated through uniaxial anisotropic lossy plasma filling rectangular waveguides overall frequencies down to zero.

2.6.3 Dominant TM Mode:

For TM mode put Hz = 0 one can obtain from equations (2.52) the differential equation E_z is

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} = -\in_{33} \left(\gamma^2 + k_0^2\right) E_z \tag{2.58}$$

The partial differential equation (2.58) can be solved by the usual technique of assuming a product solution of the type

$$E_z = E_o \sin k_1x \sin k_2y \exp(-\gamma z) \sin \omega t \qquad (2.59)$$

By putting the appropriate boundary conditions for E_z at the wall of guide, one can show that $k_1 = \frac{m_1\pi}{1}$ and $k_2 = \frac{m_2\pi}{b}$ where transverse wave number m_1 and m_2 are integers designating the various modes. The value of E_z may then be substituted in equation (2.58) to give

$$\in_{33}\left(\gamma^2 + k_0^2\right) = k_c^2 \qquad (2.60)$$

Where propagation constant is complex $\gamma = \alpha + j\beta$. The equation (2.6) for propagation constant contains real and imaginary parts.

After eliminating α, these equations yield expressions for phase constant for the plasma filled waveguide as

$$\beta^4 + \frac{\omega^2}{c^2}\left\{\frac{\omega_c^2\left(\omega^2 - \omega_p^2 + \upsilon^2\right)}{\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2} - 1\right\}\beta^1 - \frac{\omega^2\upsilon^2\omega_p^4\omega_c^4}{4c^2\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2} = 0 \qquad (2.61)$$

For the case of dominant TM mode which moves down the waveguide predominantly, put $m_1 = 1$ and $m_2 = 1$ equation (2.61) yields

$$\beta^4 + \omega^2\left\{\frac{\frac{\pi^2\left(l^2 + b^2\right)}{l^2b^2}\left(\omega^2 - \omega_p^2 + \upsilon^2\right)}{\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2} - \frac{l}{c^2}\right\}\beta^2 - \frac{\pi^4\left(l^2 + b^2\right)^2\omega^2\upsilon^2\omega_p^4}{4l^4b^4\left[\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2\right]^2} = 0$$

(2.62)

Similarly the equation of attenuation constant for dominant TM ($M_{1,1}$) mode is given as

$$\alpha^4 + \omega^2 \left\{ \frac{\frac{\pi^2\left(l^2 + b^2\right)}{l^2 b^2}\left(\omega^2 - \omega_p^2 + \upsilon^2\right)}{\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2} - \frac{l}{c^2} \right\}\alpha^2 - \frac{\pi^4\left(l^2 + b^2\right)^2 \omega^2\upsilon^2\omega_p^4}{4l^4 b^4\left[\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2\right]^2} = 0$$

(2.63)

Thus the phase and attenuation constants for TM_{11} mode depend upon the guide dimension, collision and plasma frequencies.

These are the fourth degree equation in phase or attenuation constants of waveguide filled with lossy anisotropic plasma.

Inclusion of collision frequency changes the dispersion relation for propagation mode by an additional term $\frac{\omega^2\upsilon^2\omega_p^4\omega_c^4}{4c^2\left[\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\upsilon^2\right]^2}$ and some modification in second term totally dependent on collision frequency parameter.

When collisions are not taken into account i.e. collision frequency is allowed to go to zero $\upsilon = 0$ the equations (2.62) and (2.63) reduce to

$$\beta^2 + \omega^2 \left\{ \frac{\frac{\pi^2\left(l^2 + b^2\right)}{l^2 b^2}}{\omega^2 - \omega_p^2} - \frac{1}{c^2} \right\} = 0 \tag{2.64}$$

and

$$\alpha^2 + \omega^2 \left\{ \frac{\frac{\pi^2\left(l^2 + b^2\right)}{l^2 b^2}}{\omega^2 - \omega_p^2} \right\} = 0 \tag{2.65}$$

Using value of E_z in equation (2.53), the complete field components for $TM_{1,1}$ mode can be written as [exp(−γz) sin ωt omitted]

$$E_x = \frac{\gamma l b^2 \in_{33}}{\pi\left(l^2 + b^2\right)} E_0 \cos\frac{\pi}{l} x \sin\frac{\pi}{b} y$$

$$E_y = \frac{\gamma l^2 b \in_{33}}{\pi\left(l^2 + b^2\right)} E_0 \sin\frac{\pi}{l} x \cos\frac{\pi}{b} y \qquad (2.66)$$

$$E_z = E_0 \sin\frac{\pi}{1} x \sin\frac{\pi}{b} y$$

$$H_x = \frac{j\omega \in_0 1^2 b \in_{33}}{\pi\left(1^2 + b^2\right)} E_0 \sin\frac{\pi}{1} x \cos\frac{\pi}{b} y$$

$$H_y = \frac{j\omega \in_0 1 b^2 \in_{33}}{\pi\left(1^2 + b^2\right)} E_0 \cos\frac{\pi}{1} x \sin\frac{\pi}{b} y$$

These expressions show the variations of each components of electric and magnetic field with x and y. It is seen that if either m_1 or m_2 is zero, all fields are zero. Thus $TMm_{1,0}$ or TM_0m_2 mode of propagation cannot exist in the rectangular waveguide.

2.6.4 TEM Modes:

When transverse electromagnetic (TEM) waves are field into rectangular waveguide, the bounding surfaces give rise to non-zero E and B fields along the axis of the guide. The basic character of the TEM wave is changed into various modes of TE and TM waves.

The propagation of the TEM wave requires the B field to be in the transverse plane and satisfies the divergence equation for the magnetic field. Therefore, if a TEM mode wave exists inside the waveguide, the B lines will be closed loops in the plance perpendicular to axis. From Maxwell's curi equation, one finds that the magnetomotive force around each of these must be equal to the axis current flowing through the loop. In the case of rectangular waveguide, there is no axial conductor to sustain the axial current.

Therefore, the single conductor waveguide do not allow the propagation of TEM waves which is true in the case of rectangular and cylindrical waveguide.

2.6.5 Results and Discussion:

Propagation characteristics for TEX mode in LMG rectangular waveguide is independent of presence of plasma thus of plasma frequency and collision frequency i.e. same as that of the free space waveguide. Using equations (2.62) and (2.63) propagation and attenuation constants have been computed for different values of collision frequency 1GHz and 10GHz ω_p = 56.4GHz, 1=5.0cm, and b = 2.5cm and the result has been presented in figs. (.25) and (.26). For comparison collisionless $\upsilon = 0$ case has also been depicted in Figure As seen from Figure (2.5) collisionless curve has got two branches : one having resonance at $\omega = \omega_p$ and other branch has lower cutoff at $\sqrt{\omega_p^2 + \omega_c^2}$. Figure (2.5) indicates that the effect of collision frequency is to alter the general characteristic feature of the waveguide in that it does exhibit neither resonance at $\omega = \omega_p$ nor cutoff at $\omega = \sqrt{\omega_p^2 + \omega_c^2}$. The inclusion of collision frequency makes propagation possible for all frequency ranges in contrast to that for $\upsilon = 0$. The curve exhibits a peak at a certain value of ω which shifts towards lower frequency side as the collision frequency increases. The curve for higher υ values cut the curve for lower υ value twice and finally again higher values which is in keeping with the characteristic curves for homogeneous isotropic plasma. At high frequency the values of β are not significantly changed by the collision frequency.

Figure (2.6) shows that variation of α with ω for different values of collision frequency υ = 1GHz and 10GHz for parameters appropriate to laboratory plasma and for waveguide dimensions corresponding to X-band frequencies. For comparison a curve indicating characteristics of lossless plasma $\upsilon = 0$ is also given. In this figure it is observed that attenuation for lower values of collision frequency is small and curves are relatively flattened in comparison to those of high frequency cases. There is a shift in maxima in (ω–α) curve towards lower frequency side with increasing collision frequency. The curves for higher collision frequency cut the lower collision frequency curves twice and finally atatins the corresponding free space values. For lossless plasma, wave propagates without attention in the propagation region $\omega > \sqrt{\omega_p^2 + \omega_c^2}$. Unlike the case of parallel plane waveguide, a rectangular or cylindrical waveguide does not support TEM modes.

2.7 Transversely Magnetized (TMG) Rectangular Waveguide

2.7.1 Development of Theory:

In this case the direction of propagation and magnetic field are in Z and X axis respectively. The modes of wave propagation are considered with respect to the direction of magnetic field. For TMG using equation (2.41) for dielectric tensor the Maxwell's equations can be written as

$$\frac{\partial Hz}{\partial y} + \gamma H_y = -j\omega\mu_0 H_x$$

$$\frac{\partial Hz}{\partial y} + \gamma H_y = -j\omega \in_0 \in_{11} E_x$$

$$-\gamma E_x - \frac{\partial E_x}{\partial x} = -j\omega\mu_0 H_y$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = -j\omega \in_0 E_y \qquad (2.67)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -j\omega \in_0 E_z$$

From equation (2.67) transverse field components are expressed in terms of longitudinal components E_x and H_x as

$$\left(\frac{\partial^2}{\partial x^2} + k_0^2\right)E_y = \frac{\partial^2 E_x}{\partial x \partial y} + j\omega\mu_0\gamma H_x$$

$$\left(\frac{\partial^2}{\partial x^2} + k_0^2\right)E_x = -\gamma\frac{\partial^2 E_x}{\partial x \partial y} + j\omega\mu_0\gamma\frac{\partial H_x}{\partial y}$$

$$\left(\frac{\partial^2}{\partial x^2}+k_0^2\right)H_y = j\omega \in_0 \gamma E_x + \frac{\partial^2 H_x}{\partial x \partial y}$$

$$\left(\frac{\partial^2}{\partial x^2}+k_0^2\right)H_z = -j\omega \in_0 \frac{\partial E_x}{\partial x} - \gamma \frac{\partial H_x}{\partial y} \tag{2.68}$$

Where the longitudinal fields E_x and H_x are found to satisfy the following equations

$$\left(\in_{11} \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} = \gamma^2 + k_0^2 \in_{11}\right)E_x = 0$$

(2.69)

and

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} = \gamma^2 + k_0^2\right)H_x = 0$$

(2.70)

Once again the propagation of electromagnetic waves in the waveguide can be split into TE and TM modes. It can be shown that TE modes are not affected by the presence of the plasma or the magnetic field i.e. It is same as that found in case of free space waveguides.

2.7.2 Dominant TM Mode:

For TM modes set H_x =0, the dispersion relation is obtained as

$$\gamma^2 + \in_{11}\left(k_0^2 - k_1^2\right) - k_2^2 = 0$$

Separating real and imaginary parts of complex propagation constant in equation (2.71) one gets expressions for phase and attenuation constants as

$\beta^4 + X\beta^2 + Y = 0$

and

$\alpha^4 - X\alpha^2 + Y = 0$

where

$$X = \frac{\omega_p^2}{c^2}\left(\frac{\omega_c^2}{\omega_p^2} - \frac{\omega^2}{\omega_p^2} + \frac{\omega^2 - k_1^2 c^2}{\omega^2 + \upsilon^2}\right)$$

and

$$Y = \frac{\omega_p^4 \upsilon^2}{4\omega^2 c^4}\left(\frac{\omega^2 - k_1^2 c^2}{\omega^2 + \upsilon^2}\right)^2$$

For dominant modes i.e. $TM_{1,1}$ mode

$$\omega_c^2 = \frac{\pi^2 c^2 \left(l^2 + b^2\right)}{l^2 b^2} \text{ and } k_1^2 = \frac{\pi^2}{l^2}$$

Thus equations (2.72) and (2.73) for phase and attenuation constants in case of $TM_{1,1}$ mode can be written as

$$\beta^4 + \frac{\omega_p^2}{c^2}\left[\frac{\pi^2 c^2\left(l^2 + b^2\right)}{l^2 b^2 \omega_p^2} - \frac{\omega^2}{\omega_p^2} + \frac{\left(\omega^2 - \frac{\pi^2 c^2}{l^2}\right)}{\omega^2 + \upsilon^2}\right]\beta^2 - \frac{\omega_p^4 \upsilon^2}{4c^2\omega^2}\left[\frac{\omega^2 - \frac{\pi^2 c^2}{l^2}}{\omega^2 + \upsilon^2}\right] = 0 \quad (2.74)$$

and

$$\alpha^4 + \frac{\omega_p^2}{c^2}\left[\frac{\pi^2 c^2\left(l^2 + b^2\right)}{l^2 b^2 \omega_p^2} - \frac{\omega^2}{\omega_p^2} + \frac{\left(\omega^2 - \frac{\pi^2 c^2}{l^2}\right)}{\omega^2 + \upsilon^2}\right]\alpha^2 - \frac{\omega_p^4 \upsilon^2}{4c^2\omega^2}\left[\frac{\omega^2 - \frac{\pi^2 c^2}{l^2}}{\omega^2 + \upsilon^2}\right]^2 = 0 \quad (2.75)$$

The propagation of electromagnetic waves is possible only in the ranges of parameters for which the values of β^2 is positive and real. The cutoff frequency ω_{c0} in this case is found to be equal to k_1c i.e. $\frac{\pi c}{1}$ For lossless plasma the equations (2.74) and (2.75) reduce to the same expression for phase and attenuation constants as given by Allis et.al. (1963) for transversely magnetized plasma waveguide. Like, LMG uniaxial anisotropic plasma rectangular waveguide case TEM mode is not supported in this case also.

2.7.3 Results and Discussion:

The Propagation constant for dominant TM mode from equation (2.74) has been computed for different values of collision frequencies υ= 1GHz and 10GHz, l = 5.0cm and b = 2.5cm appropriate to x-band waveguide and the result has been presented in Figure (2.7) taking appropriate values of plasma frequency, ω_p = 56.4GHz. The (ω–β) curve in collisional TMG uniaxial plasma has two branches: a low frequency branch which extends in range $0 < \omega < \frac{\pi c}{l}$ and 18.85GHz is the critical frequency which characterizes the propagation and cutoff properties of a TM mode. In the Figure the (ω–β) diagram corresponding to $\upsilon = 0$ (no collisions) is also shown for the sake of comparison.

One concludes the following general behavior: When $\omega = 0$, the value of β comes out to be infinite and it decreases with increase of frequency and attains zero value at ω = 18.85GHz i.e. a cutoff. When $\omega > \frac{\pi c}{l}$ i.e. at high frequency side β increases with increase of frequency and finally attains asymptotically the free- space values. It is also concluded from the graph, that at very low frequencies the value of β are maximum for lower υ value curves. For the curve corresponding to $\upsilon = 0$ the value β falls rapidly with ω than those curves corresponding to finite values of υ and the slope decrease with increase of collision frequency.

In the low frequency side, due to decrease in slope with increase of collision frequency, the curves intersect each other and intersection points of curves of higher υ shifts slightly towards high frequency side.

The curve for collisionless ($\upsilon = 0$) case, has also got two separate branches and showing no propagation in the frequency range 11.42GHz < ω<69.43 GHz. In the very high frequency range, the values of propagation constant approach those values for curves corresponding to $\upsilon = 0$. It is also found from the curves for collisional case that the cut-off frequency is not affected by variation of collision frequency, but wholly depends upon the wave number in the X-direction.

Equation (.275) has been computed for appropriate laboratory plasma parameters and the resulting (ω–α) curve has been depicted in Figure (2.8) for collision frequencies $\upsilon = 1$ GHz and 10GHz. For comparison collisionless ($\upsilon = 0$) curve has also been shown. This curve encloses a region outside which propagation of wave is possible. Because of finite loss one observes that attenuation is present for the whole frequency range in contrast to $\upsilon = 0$ case. In Figure (2.8) the attenuation of wave for no collision takes place in the region 11.42GHz < ω< 69.43 GHz with a peak value $\alpha = 176$ neper/ radian at $\omega = 30$GHz.

In very low frequency range, the value of attenuation is more, lower the value of collision frequency. This order is also maintained in the region enclosed by $\upsilon = 0$ curve. In higher frequency side one observes more attenuation for higher collision frequency. With assumed values of E_x and H_x corresponding to corresponding to equations (2.69) and (2.70) and by use of equation (2.67) expressions for E_y, H_y, E_z and H_z are obtained for TE and TM modes respectively for transversely magnetized waveguide filled with lossy uniaxial plasma.

2.8 Conclusion:

The propagation characteristics of electromagnetic waves in waveguides filled with uniaxial anisotropic plasma including the effect of collisions are discussed. The effect of collisions in general is to introduce attenuation in the propagating modes.

The usual characteristic feature of a free space waveguide is the existence of a pass band and the attenuation band. The inclusion of the effect of collisions in the formulation is to make the propagation possible at all frequencies. At very high frequencies the effect of collision is generally negligible because electrons are not able to respond to high frequency waves and thus there is no interaction between electromagnetic waves and electrons.

At low frequencies the phase velocity shows a rapid change with wave frequency. In general the inclusion of collision alters the propagation characteristics in the Transverse Magnetic ™ modes without affecting the Transverse Electric (TE) modes because there is no component of electric field in the direction of strong magnetic field and thus interaction between electrons and the electric field is not possible.

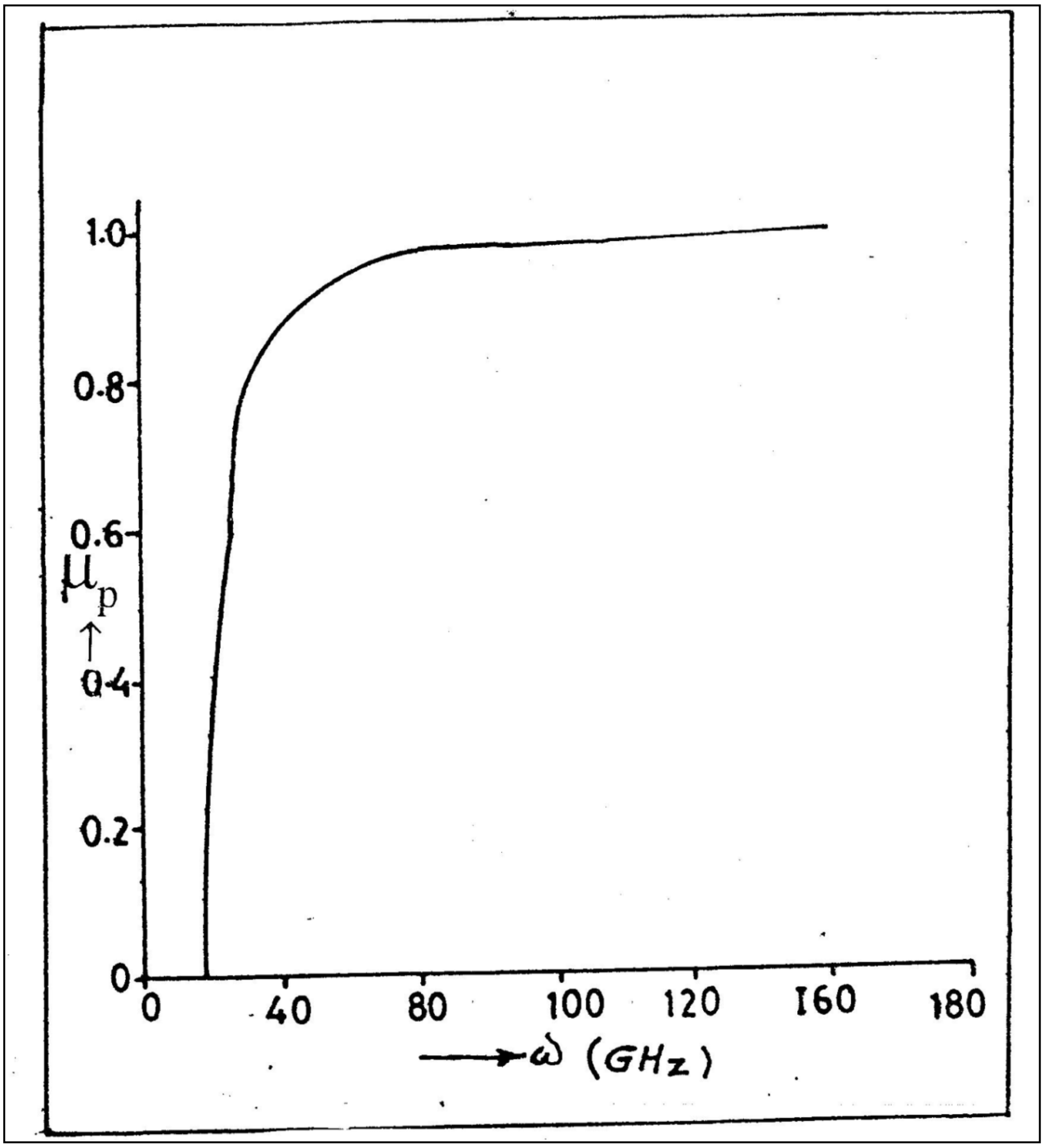

Figure 2.1: Variation of phase refractive index with microwave frequency for dominant $TE_{1,0}$ mode in LMG and TMG parallel plane plasma waveguide

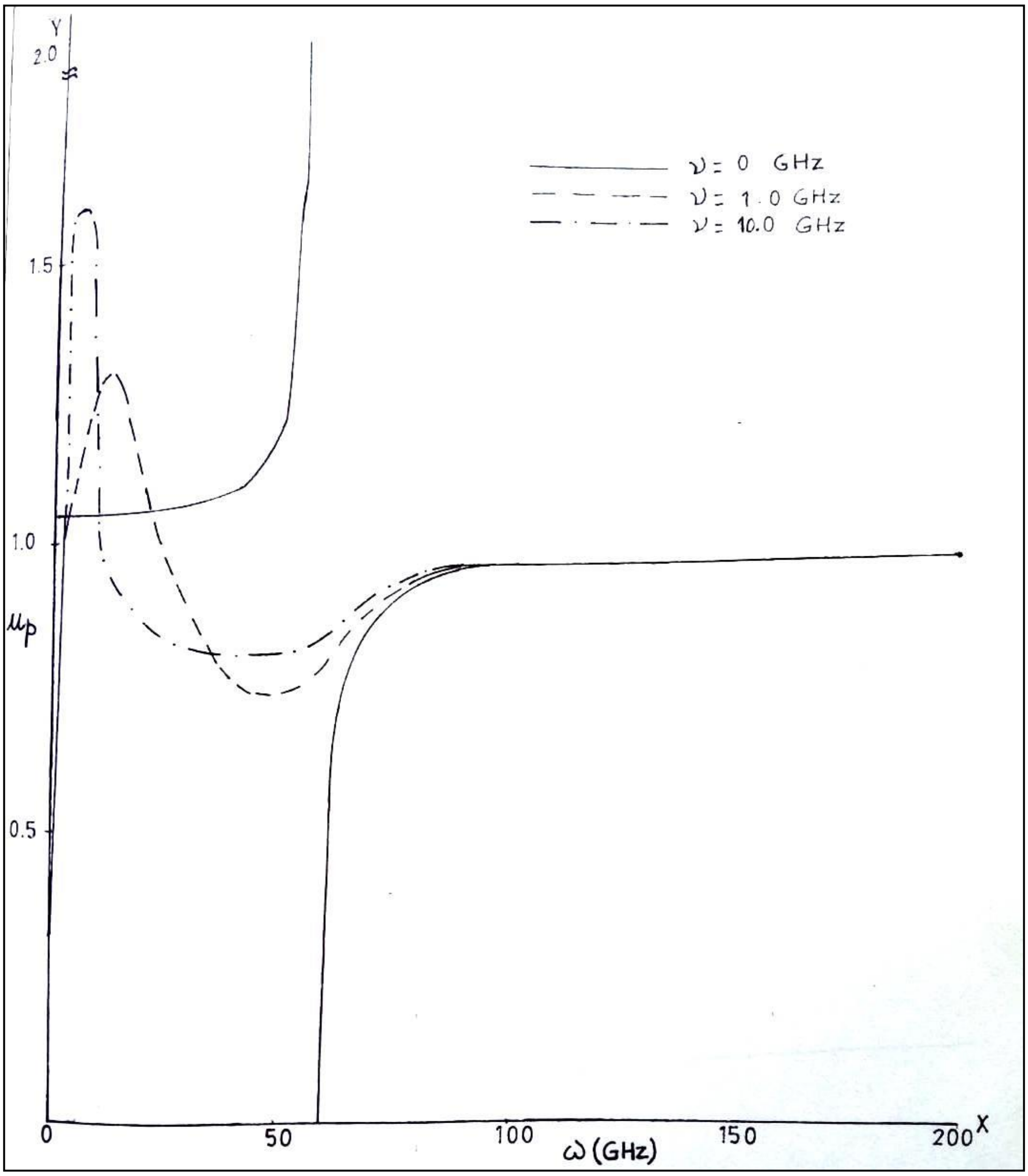

Figure 2.2: Variation of Phase Refractive Index with Microwave Frequency for Dominant $TM_{1,1}$ Mode in Uniaxial Anisotropic LMG Parallel Plane Plasma Wave Guide.

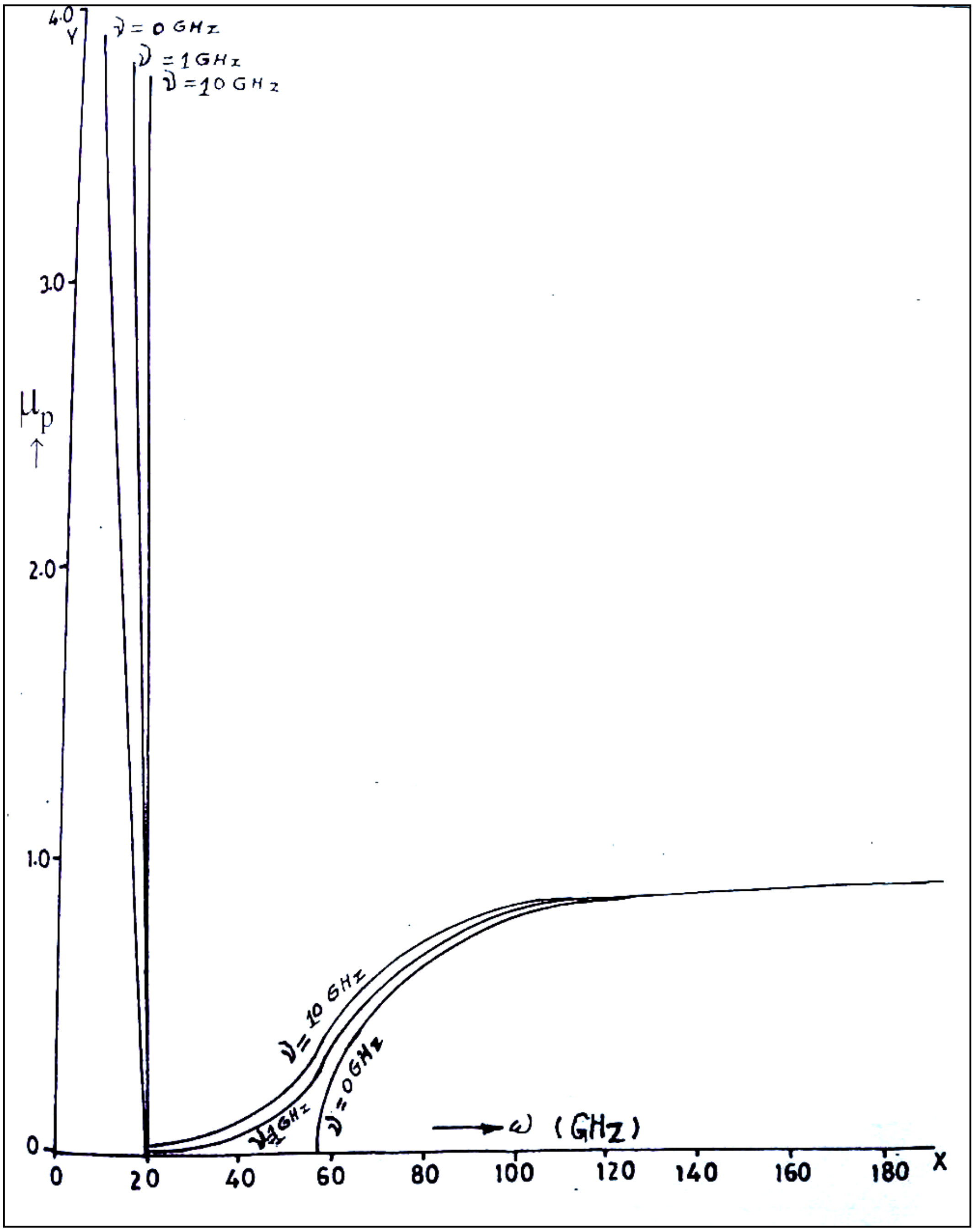

Figure 2.3: Variation of phase refractive index with signal frequency for dominant $TM_{1,1}$ mode in anisotropic TMG parallel plane plasma waveguide.

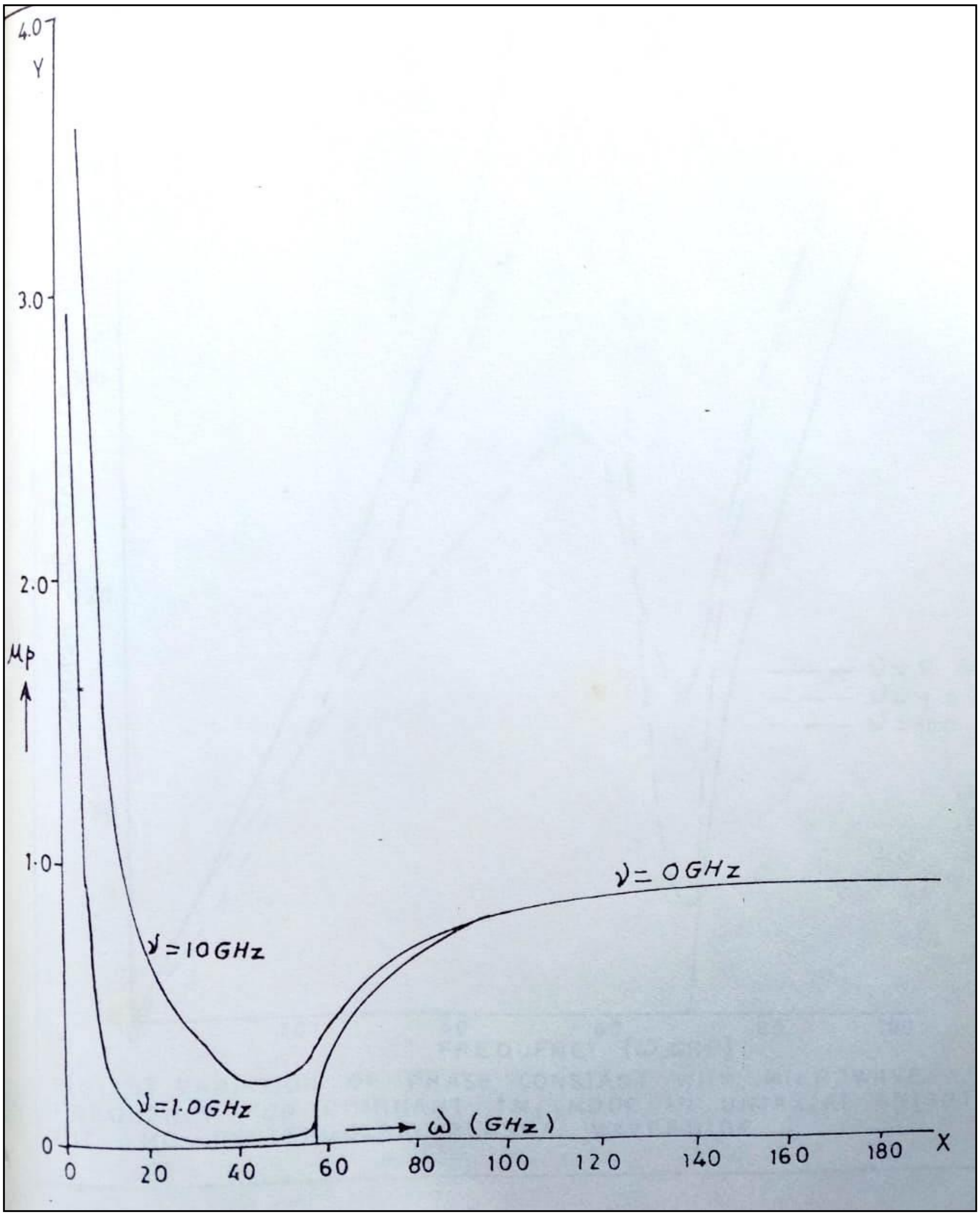

Figure 2.4: Variation of phase refractive index with microwave frequency for dominant TEM mode in TMG parallel plane plasma waveguide.

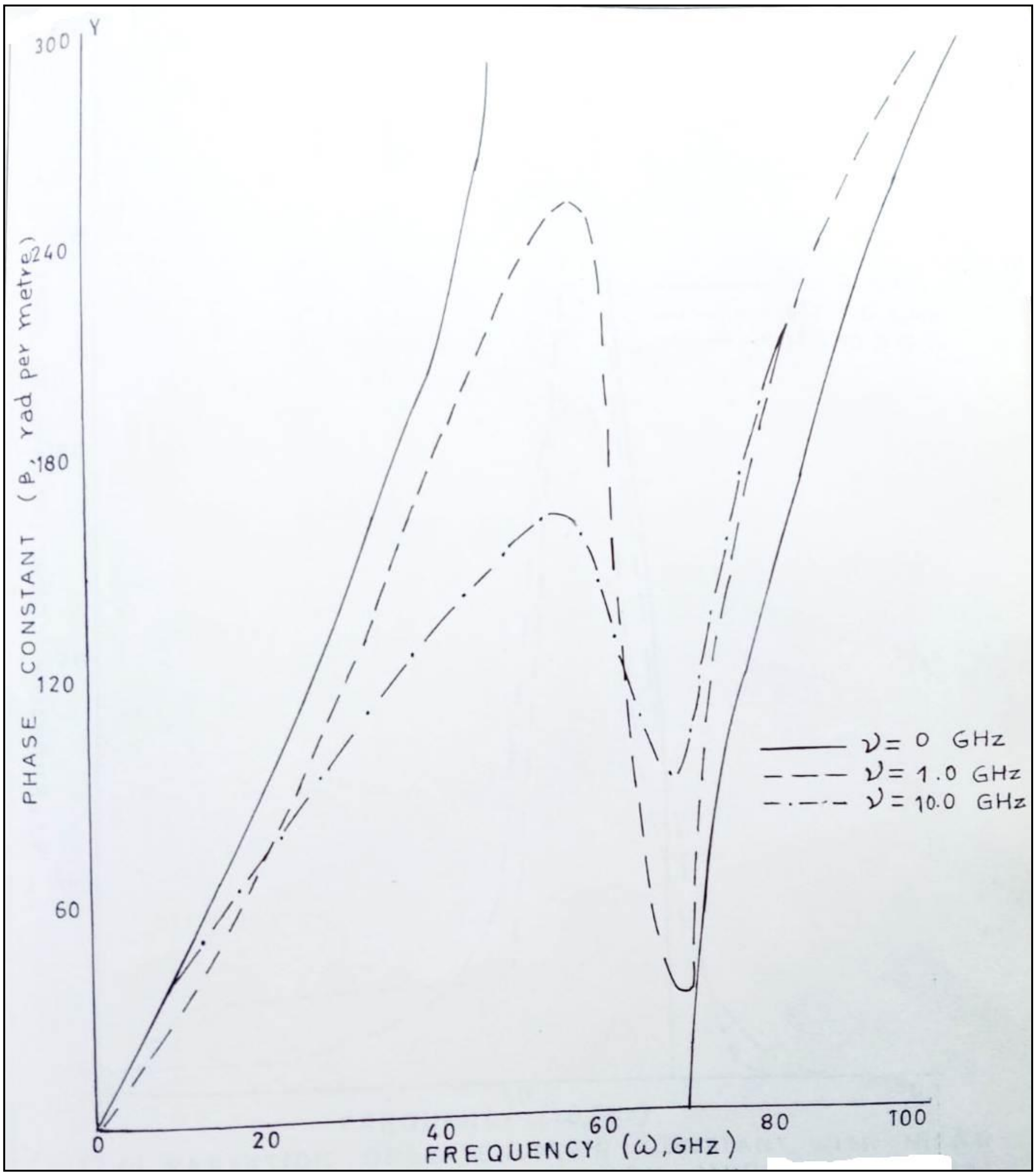

Figure 2.5: Variation of Phase Constant with Microwave Frequency for Dominat $TM_{1,1}$ Mode in Uniaxial Anisotropic LMG Rectangular Plasma Waveduide.

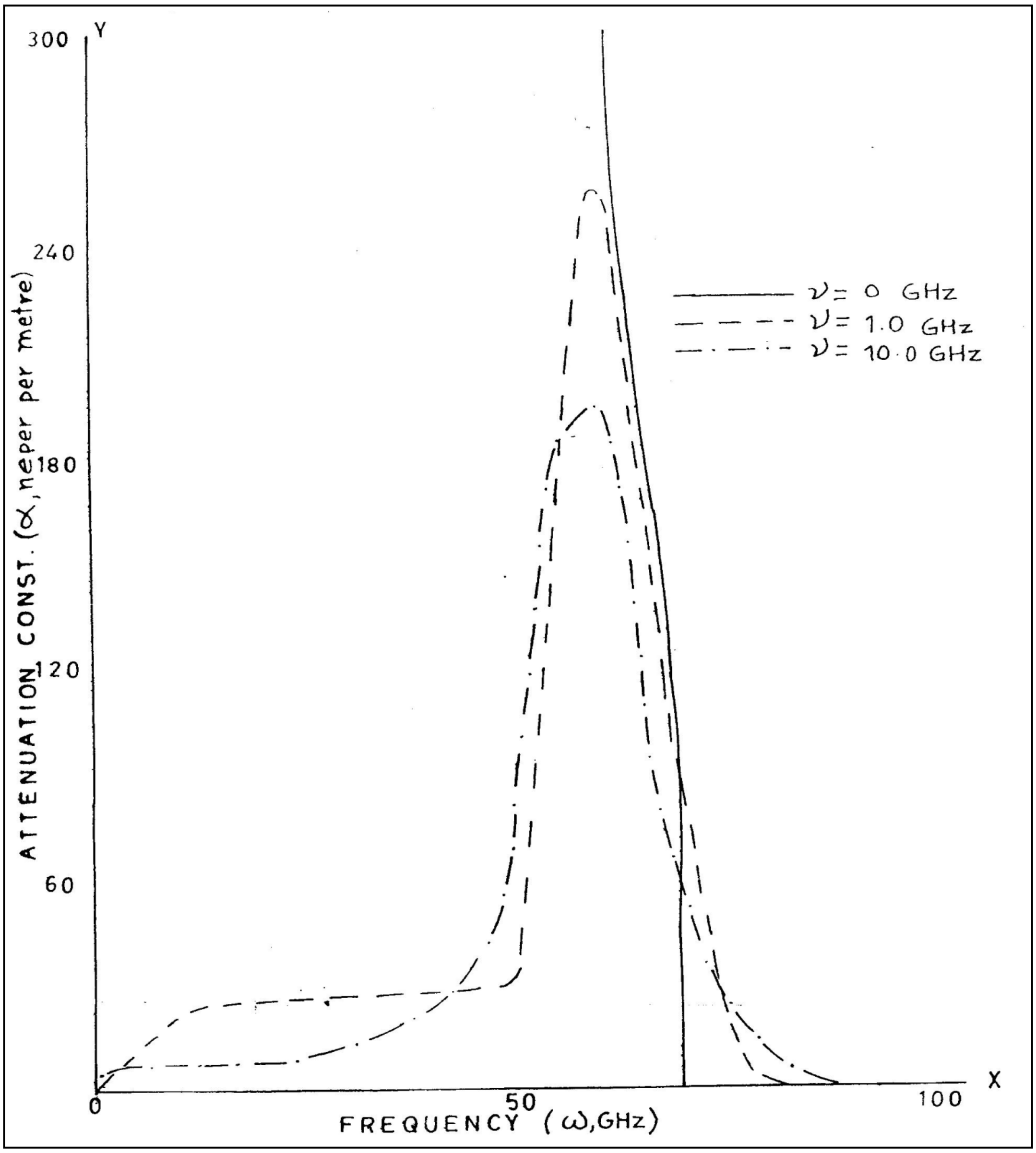

Figure 2.6: Variation of Attenuation Constant with Microwave Frequency for Dominant $TM_{1,1}$ Mode in Unixial Anisotropic LMG Rectangular Plasma Waveguid.

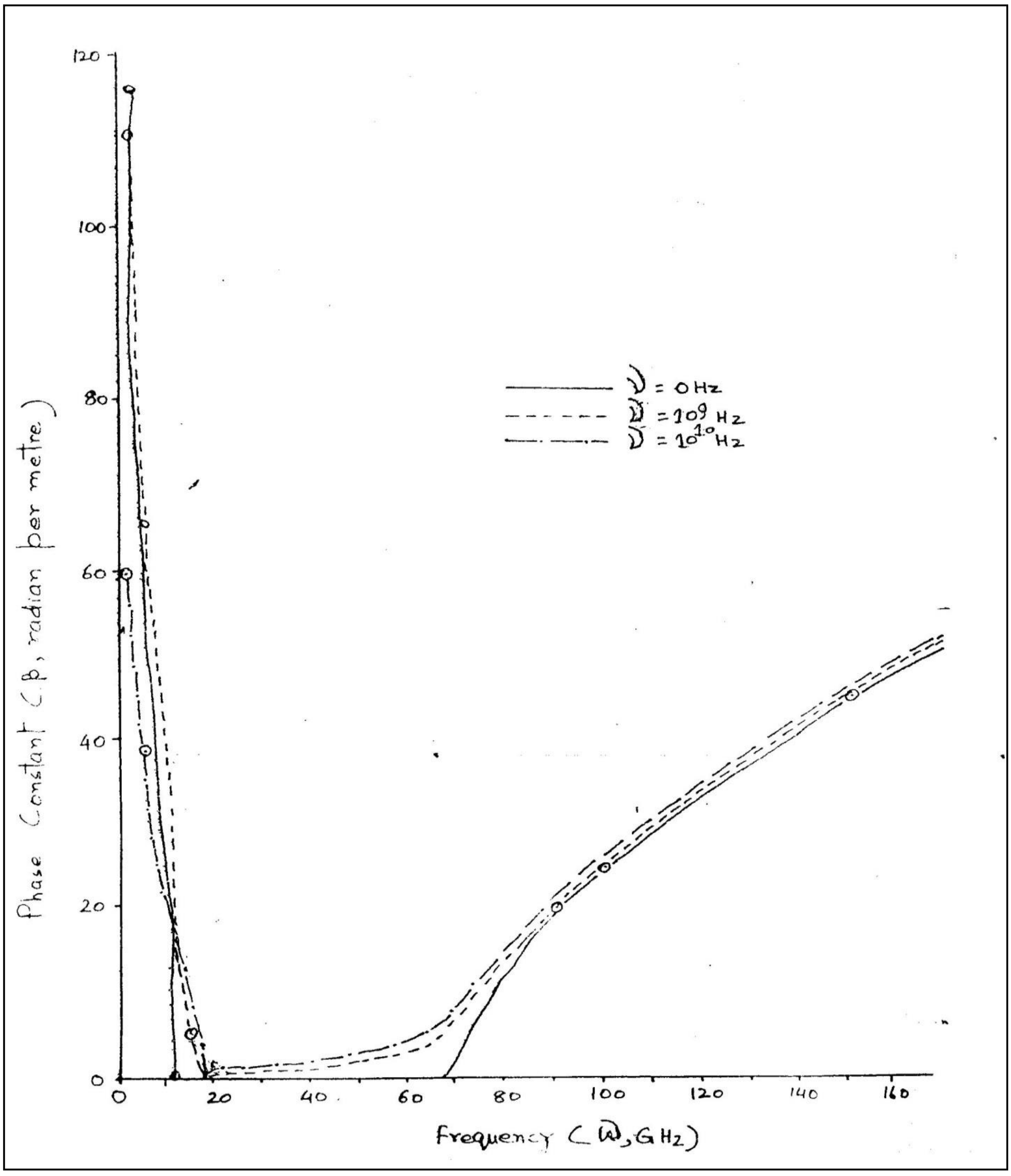

Figure 2.7: Variation of phase constant with microwave frequency for dominant $TM_{1,1}$ mode in anisotropic TMG rectangular plasma waveguide.

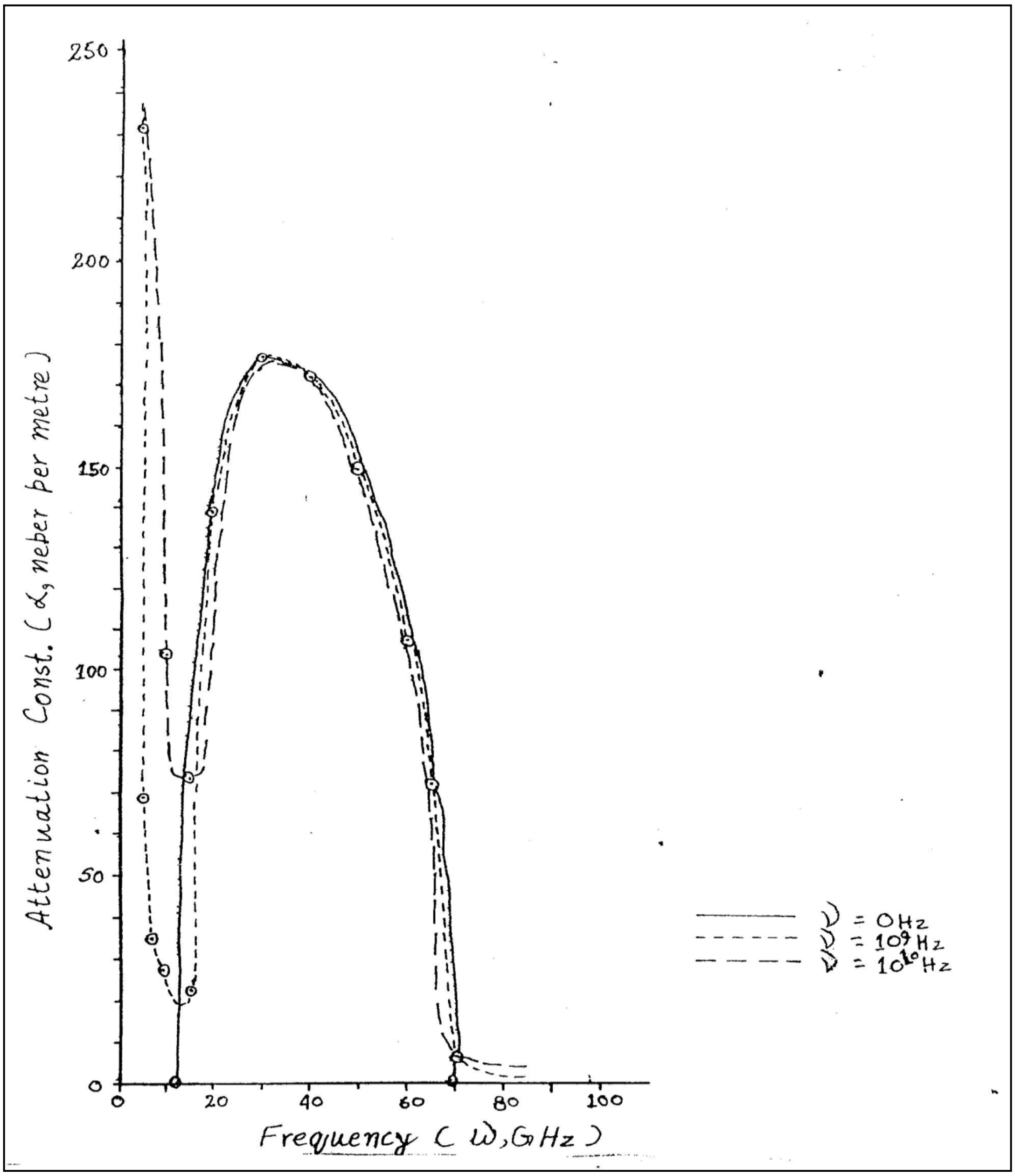

Figure 2.8: Variation of attenuation constant with microwave frequency for dominant $TM_{1,1}$ mode in anisotropic TMG rectangular plasma waveguide.

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Chapter 3

Relativistically Moving Warm Lossy JMG Plasma Waveguide

3.1 Introduction:

The propagation characteristics of waveguide containing warm and stationary plasma in the presence of static magnetic field have been investigated by Samaddar (1964) and Sancer (1965). Tuam (1969) considered the wave propagation in a waveguide filled with warm stationary plasma in the presence of strong transverse magnetic field in the absence of collisions. He showed that the power transfer in the waveguide can take place not only by electromagnetic fields but also by velocity fields. The problem of guided waves in moving media was first discussed by Collier and Tai (1966). Their results showed that the expression for propagation constant in the stationary media was modified by the term depending upon the velocity as well as the constitutive constants of the stationary media but independent of the guide geometry when the media were moving. The results of Minkowski's theory were the foundations of their analysis. Du and Compton (1966) investigated the cutoff phenomena of guided waves in the moving media and showed that for a slowly moving medium there were two critical frequencies, separating three frequency ranges in each of which there was a different type of propagation whereas for a high speed medium cutoff phenomena was found to be absent. Later on another simple method was suggested by Shiozawa (1966), which was based upon the covariance of Maxwell's equations and the principle of invariance of phase [Sommerfield (1952)]. Gruenberg and Daly (1967) derived the perturbation formulae for the changes in the dispersion curves and phase velocity for the modes in an arbitrary composite waveguide structure containing dispersive medium in relative motion. Singh et.al. (1990) studied the cutoff for a rectangular waveguide filled with uniaxial anisotropic stationary warm lossy plasma. The externally applied strong magnetic field has been taken in direction perpendicular to the propagation direction. Jiangiang (1997) analysed an excitation of Cherenkov radiation by a thin annular relativistic electron beam in a plasma-filled, dielectric lined waveguide by use of the selfconsistent linear theory. The effect off the thin annular electron beam on the beam wave interaction is completely described by a jump condition. The dispersion equation and the simultaneous condition of the beam-wave interaction are derived.

Finally, the growth rate of the wave is obtained and the effect of the background plasma density and the electron beam radius on the growth rate of the wave are presented. Jain et.al. (1974) gave an account of the theoretical study of the propagation of electromagnetic waves through a parallel plane waveguide containing homogeneous, lossless, temperate relativistically moving plasma in the presence of strong transverse magnetic field.

The normal propagating modes were classified into TE, TM and TEM waves with respect to the direction of propagation. The nonlinear propagation of relativistically, intense electromagnetic waves into collisionless plasmas with special emphasis on the dimensional plane wave solutions of the propagating, standing and modulated types was discussed by Kaw and Sen (1997). These solutions exhibit a rich variety of phenomena associated with relativistic electron mass variation and coupling between transverse electromagnetic and longitudinal fields. They have important applications to problems of laser propagation, self-focusing in overdense plasmas, particle and photon acceleration and to electromagnetic radiation around pulsars.

The present work in this chapter is an extension of that of Singh et. al. (1990) to include the effect of relativistic movement of plasma on propagation characteristics such as phase constant, cutoff frequency, power transfer down the guide. Our analysis is based on the covariance of Maxwell's equations, invariance of phase and boundary conditions under Lorentz transformations. Basic equations representing the fields and normal modes are written in the rest system and represented by primed frame of reference. Then by making use of principles of special theory of relativity and by applying Lorentz transformations the general equations are formulated in the unprimed system representing the moving frame of reference. Expressions for phase constant, power flow and cutoff frequency in case of a rectangular waveguide filled with relativistically moving warm lossy TMG plasma have been derived. Variation of cutoff frequencies with appropriate laboratory plasma parameters for X-band of rectangular waveguide for some simpler cases have been analysed.

3.2 Plasma Parameters Under Relativistic Effect:

Special theory of relativity was developed in order to prove the invariance of Maxwell's field equations and to unify the electrical and magnetic phenomena. The relativistic transformations and their implications were extensively used to study the electrodynamics of charged particles and of the generation and propagation of electromagnetic waves. The electromagnetic wave theory of Maxwell was formulated during the period 1855 t 1865 where Maxwell studied the consistency of existing experimental laws and unified these laws by adding a new term in Ampere's law. The special theory of relativity was propounded by Einstein in 1884-1905 with a view to reconcile with certain results of Maxwell's field equations.

In this chapter since author is concerned with the properties of moving plasma medium it is necessary to understand the relativistic transformation laws for the electromagnetic field vectors, the wave vector, frequency and the plasma parameters. The result that the Maxwell's equations are covariant under Lorentz transformation is well known [Somme field (1952), Moller (1952) and Papas 1965)].

Electric and magnetic fields transform into one another in different moving inertial frames. This is achieved with the help of Lorentz transformation equation and the invariance of total charge in the moving inertial frame. According to special theory of relativity the field vectors $\left(\overline{B}', \overline{D}', \overline{E}', \overline{H}'\right)$ in the printed system (moving system) s' by the Lorentz transformations:

$$\overline{E}' = r\left(\overline{E} + \overline{v_0} \times \overline{B}\right) + \left(1 + r\right)\frac{\overline{V_0}.\overline{E}}{v_0^2}\bar{v}_0$$

$$\overline{B}' = r\left(\overline{B} - \frac{1}{c^2}\bar{v}_0 \times \overline{E}\right) + \left(1 - r\right)\frac{\overline{V_0}.\overline{B}}{v_0^2}\bar{v}_0$$

$$\overline{D}' = r\left(\overline{D} - \frac{1}{c^2}\bar{v}_0 \times \overline{H}\right) + \left(1 - r\right)\frac{\overline{V_0}.\overline{D}}{v_0^2}\bar{v} \tag{3.1}$$

$$\overline{H'} = r\left(\overline{H} + \overline{v_0} \times \overline{D}\right) + (1 + r)\frac{\overline{V_0}.\overline{E}}{v_0^2}\bar{v}_0$$

Where $r = \left(1 - u^2\right)^{-1/2}, u = \dfrac{v_0}{c}$

c is velocity of light.

Further the Lorentz transformation of time and space co-ordinates is

x' = x

y' = y

z' = r (z –v_0t)

$$t' = r\left(t - \frac{v_0}{c^2}z\right) \quad (3.2)$$

Now the principle of phase invariance [Sommerfeld (1952)] suggests that

$$(\omega't' - \gamma'z') = (\omega t - \gamma z) \quad (3.3)$$

Where γ and γ' are propagation constants in two frames of reference S and S'. By making use of equations (3.2) and (3.3) one obtains the following relations

$$\gamma' = r\left(\gamma - \frac{u}{c}\omega\right) \quad (3.4)$$

$$\gamma' = r\,(\omega - cu\gamma) \quad (3.5)$$

The transformation laws for plasma parameters have been considered by Getmantsev and Report (1960), Scarf (1961) and Unz (1966a). Scarf's statement that the plasma frequency, the cyclotron frequency, the collision frequency and resonant frequency all undergo Doppler shift under transformation from S' and S is not correct, since all these frequencies are the natural frequencies of plasma. Unlike the electromagnetic wave frequencies, the natural frequencies of the plasma do not have a wave vector associated with them, and thus do not have the property of phase invariance which results in a Doppler shift.

It is assumed that the rest system of co-ordinance S' is moving with a moving plasma at velocity V_o with respect to the laboratory system of co-ordinates S. The corresponding plasma frequency ω'_p, ω_p of the same moving plasma in the respective frames is given [Budden, (1961)] as

$$\omega'^2_p = \frac{n'e'^2}{m'\in_0}, \qquad \omega_0^2 = \frac{ne^2}{m\in_0} \tag{3.6}$$

Where (–e', m') and (–e,m) are the electron charge and mass n' and n are the electron number density in the respective frames and $\in_0$ is the free space permittivity. From the special theory of relativity mass of a moving particle increases while its charge remains invariant i.e.

$$e' = e, \ m' = \frac{m}{r} \tag{3.7}$$

From the Lorentz- Fitzgerald contraction one has the following transformation for the volume (Pauli, 1958) in the two respective frames

$$V' = rv \tag{3.8}$$

Since the total number of electrons in V' as counted in S' is the same as the total number of electrons in V a counted in the S frame of reference, the n' and n the corresponding electron number volume densities (number/volume) one has

$$n' = \frac{n}{r} \tag{3.9}$$

Thus one obtains relation for plasma frequency in S' and S

$$\omega'_p = \omega_p \tag{3.10}$$

Invariant under Lorentz transformations. Taking the average counted number of collisions of an electron with heavier particles in the rest frame of reference S' during an interval of time 't one obtains for collision frequency relation

$$\upsilon' = r\,\upsilon \qquad (3.11)$$

The corresponding gyromagnetic (cyclotron) frequencies in the frame of reference S' and S is given (Budden, 1961) as

$$\omega_p^{'} = \frac{e'B'_0}{m'}, \qquad \omega_b = \frac{eB_0}{m} \qquad (3.12)$$

For a neutral moving plasma with no net electrostatic field in the laboratory co-ordinate system, taking the static magnetic field along the velocity direction the relation between ω'_b and ω_b is

$$\omega'_b = r\omega_b \qquad (3.13)$$

under the Lorentz transformation, the scalar pressure is invariant (Pauli, 1958).

$$p' = p \qquad (3.14)$$

and for the temperature (Chawla and Unz, 1971),

$$T' = rT \qquad (3.15)$$

The scalar pressures in two frames of references in terms of electron number densities and temperatures are given by:

$$p' = Kn'T' \qquad \text{and } p = KnT \qquad (3.16)$$

Thus one obtains for acoustic velocity in the electron gas a', a in the frames of references S' and S (unz, 1966)

$$a' = ra \qquad (3.17)$$

Above equations represent complete transformations of parameters relating to plasma in two frames so as to study the propagation of electromagnetic waves in relativistically moving plasma.

3.3 Cutoff Frequency of a Waveguide Containing Warm Lossy Relativistically Moving TMG Plasma:

3.3.1 Development of Theory:

We consider a rectangular waveguide of dimensions’ l and b of perfectly conducting wells containing lossy warm plasma, which is supposed to be moving with respect to the guide walls with a constant velocity V_o in the Z- direction. A strong static magnetic field is applied in the X-direction. The plasma is taken to be weakly ionized in which electrons are warm with an average temperature higher than temperature of neutral particles. We consider two frames of reference, the primed system in the plasma and the unprimed system which is attached to the guide walls.

With the help of linearized hydrodynamic equation of motion, equation of continuity and the equation of state Maxwell's equations can be put into the following form [Tiwari et.al. (1975)], as

$$\nabla'\times H' = j\omega'\in_0 \overline{\overline{\in}}' + \hat{x}\left[\frac{-je}{\omega' m'\left(1-\frac{j\upsilon'}{\omega'}\right)}\right]\frac{\partial p'}{\partial x'} \tag{3.18}$$

$$\nabla'\times\overline{E}' = -j\omega'\mu_0\overline{H}' \tag{3.19}$$

The collisional drag term is due to collisions between electrons and neutrals.

Permittivity tensor in primed system can be written as

$$\overline{\overline{\in}}' = \in_1^{'}\ \hat{x}'.\hat{x}' + \hat{y}'\ \hat{y}' + \hat{z}'\ \hat{z}' \tag{3.20}$$

where

$$\in_1^{'} = 1 - \frac{\omega'^2_p}{\omega'(\omega' - j\upsilon')}$$

$$\omega'^2_p = \frac{n' e'^2}{m' \in_0}$$

and $\mu_0, \in_0$ are permeability and permittivity of free space. Equations (3.18) and (3.19) are basic equations which characterize the uniaxial aand compressible property of plasma inside the guiding structure in the primed system. Using equations (3.18) and (3.19) and assuming z' dependence of the type $e^{-\gamma' z'}$ in primed system one can express transverse fields with respect to the x' direction as,

$$\left(\frac{\partial^2}{\partial x'^2} + k'^2_0\right)\begin{pmatrix} E_y^{'} \\ E_z^{'} \end{pmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial x' \partial y'} & j\omega\mu_0\gamma' \\ -\frac{\gamma' \partial}{\partial x'} & j\omega' \mu_0 \frac{\partial}{\partial y'} \end{bmatrix}\begin{pmatrix} E_x^{'} \\ H_x^{'} \end{pmatrix} \tag{3.21}$$

$$\left(\frac{\partial^2}{\partial x'^2} + k'^2_0\right)\begin{pmatrix} H_y^{'} \\ H_z^{'} \end{pmatrix} = \begin{bmatrix} j\omega \in_0 \gamma' & \frac{\partial^2}{\partial x' \partial y'} \\ -j\omega' \in_0 \frac{\partial}{\partial y'} & -\frac{\gamma' \partial}{\partial x'} \end{bmatrix}\begin{pmatrix} E_x^{'} \\ H_x^{'} \end{pmatrix} \tag{3.22}$$

Where $k'^2_0 = \omega'^2 \mu_0 \in_0$

and $\gamma' = \alpha' + j\beta'$; α', β' being attenuation and phase constants in the primed system E'_x, H'_x and p satisfy the following equations

$$\left[\frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \gamma'^2 + k'^2_0 \in_1^{'}\right] E_x^{'} = \frac{e'}{\omega'^2 m' \in_0}\left[\frac{k'^2_0}{\left(1 - \frac{j\upsilon'}{\omega'}\right)} - \frac{\omega'^2}{a'^2}\right]\frac{\partial p'}{\partial x'} \tag{3.23}$$

$$\left[\frac{\partial^2}{\partial x'^2}+\frac{\partial^2}{\partial y'^2}+\gamma'^2+k'^2_0\right]H'_x=0 \tag{3.24}$$

$$\left[\frac{\partial^2}{\partial x'^2}+\frac{\omega'^2}{a'^2}\left(1-\frac{j\upsilon'}{\omega'}\right)\right]p'=-\frac{n'e'\partial E'_x}{\partial x'} \tag{3.25}$$

Propagation characteristics of warm lossy TMG plasma waveguide will be aanalysed in terms of two normal modes: transverse electric (TE) modes and transverse magnetic ™ modes.

3.3.2 Transverse Electric (TE) Modes:

For TE modes, no longitudinal electric field exists, set $E'_x = 0$, then p' is found identically zero from equations (3.23) and (3.25). The only potential function H'_x, which must satisfy equation (3.24) gives the dispersion relation for TE modes. The solution of equation (3.24) for H_z satisfying boundary conditions is written in the form as:

$$H'_x=H'_0\cos\frac{m_1 x}{l'}x'\cos\frac{m_2 x}{l'}y' \tag{3.26}$$

Where H'_0 is constant value of amplitude and γ' stands for propagation constant in the primed system.

Using the appropriate boundary conditions one obtains for TE modes

$$\gamma'^2_{TE}=k'^2_c-k'^2_0 \tag{3.27}$$

Where $k'^2_0=\omega'^2\,\mu_0\in_0, k'^2_c=\left(\frac{m_1\pi}{l'}\right)^2+\left(\frac{m_2\pi}{b'}\right)^2=k'^2_c$

Where m1 and m2 are number of half sine variations in × and y directions. The phase constant in primed notation obtainable from equation (3.27) is appropriate only for TE modes as is evident from equation (3.24). Phase constant β_{TE} can be obtained in unprimed system by using relativistic transformation as

$$\beta_{TE} = \left(\frac{\omega^2}{c^2} - k_c^2 \right)^{1/2} \tag{3.28}$$

The cutoff for TE modes are found by setting β_{TE} from equation (3.28) equal to zero hence equation (3.28) yields

$$\omega_{c0} = k_c c \tag{3.29}$$

It is evident from equations (33.28) aand (3.29) that the propagation characteristics are unaffected by plasma parameters and relativistic movement.

It is same as that obtained in case of free space rectangular waveguide. The cutoff frequency depends only one dimensions of waveguide.

3.3.3 Transverse Magnetic (TM) Modes:

It is interesting to study the dispersion relation and propagation property of TM modes. For TM modes set $H_x = 0$.

The potential functions E'_x and p' satisfy the coupled wave equations ((3.23) and (3.25). Let E'_x and p' have solutions in the following form:

$$E'_x = c_1 \quad \cos \frac{m_1 \pi}{l'} x' \qquad \text{Sin } \frac{m_2 \pi}{b'} y' \tag{3.30}$$

$$p' = c_2 \text{ Sin } \frac{m_1 \pi}{l'} x' \qquad \text{Sin } \frac{m_2 \pi}{b'} y' \tag{3.31}$$

Where C_1 and C_2 are constants

The values of $k'_1 = \frac{m_1\pi}{l'}$ and $k'_2 = \frac{m_2\pi}{b'}$ are to be determined by applying boundary conditions.

Substituting equation (3.25) in to equation (3.23) and applying appropriate boundary conditions that tangential component of electric field and normal component of magnetic field are zero at the conducting walls of waveguide, the dispersion relation for TM modes for warm lossy relativistically moving TMG plasma waveguide in primed system is obtained as:

$$\beta'^{4+}_{TM}\left[k'^2_c - \left(1 - \frac{\omega'^2_p}{\omega'^2 + \upsilon'^2}\right) + \frac{\omega'^2_p k'^2_1 \left(k'^2_1 a'^2 - \omega'^2\right)}{\left(k'^2_1 a'^2 - \omega'^2\right)^2 + \left(\omega' + \upsilon'\right)^2}\right.$$

$$\left. - \frac{k'^2_0 \omega'^2_p k'^2_1 a'^2 \left(k'^2_1 a'^2 - \omega'^2 + \upsilon'^2\right)}{\omega'^2 \left(k'^2_1 a'^2 - \omega'^2 + \upsilon'^2\right)^2 + \upsilon'^2 \left(2\omega'^2 - k'^2_1 a'^2\right)}\right]\beta'^2_{TM}$$

$$- \frac{\upsilon'^2 \omega'^2}{4}\left[\frac{1}{c^2}\frac{\omega'^2_p}{\upsilon'^2 \omega'^2} + \frac{1}{c^2}\frac{\omega'^2_p k'^2_1 a'^2 \left(2\omega'^2 - k'^2_1 a'^2\right)}{\left[\omega'^2 \left(k'^2_1 a'^2 - \omega'^2 \upsilon'^2\right)^2 + \upsilon'^2 \left(2\omega'^2 - k'^2_1 a'^2\right)^2\right]}\right.$$

$$\left. - \frac{k'^2_1 \omega'^2_p}{\left(k'^2_1 a'^2 - \omega'^2\right) + \left(\omega' \upsilon'\right)^2}\right]^2 = 0 \tag{3.32}$$

Where

$$k_1' = \frac{m_1\pi}{l'} = \frac{m_1\pi}{l} = k_1, k_2' = \frac{m_2\pi}{b'} = \frac{m_2\pi}{b} = k_2$$

and

$$k'^2_c = \left(\frac{m_1\pi}{l'}\right)^2 + \left(\frac{m_2\pi}{b'}\right)^2 = k_c^2$$

Now we use the transformation relations for plasma parameters, waveguide dimensions used in equation (3.32) from primed system to those in unprimed system. Again using normalised parameters for phase constant, wave frequency, collision frequency in terms of plasma frequency, the following dispersion relations corresponding to various modes in the unprimed system is obtained as

$$B^4 + 2u\Omega B^3 + \left[6u^2\Omega^2 + M\right]B^2 + 2u\Omega\left[2u^2\Omega^2 + M\right]B + \left[u^4\Omega^4 + u^2 M\Omega^2 - N\right] = 0$$

Where

$$M = \frac{\Omega_c^2}{r^2} - \left[1 - \frac{1}{r^2\left(\Omega'^2 + Z^2\right)}\right]\Omega'^2 - \frac{1}{r^2}\left[\frac{\Omega'^2\,\Omega_1^2\delta^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)}{\Omega'^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)^2}\right]$$

$$+\frac{1}{r^2}\left[\frac{\Omega_1^2\left(\Omega_1^2\delta^2 - \Omega'^2\right)}{\left(\Omega_1^2\delta^2 - \Omega'^2\right)^2 + \left(\Omega' Z\right)^2}\right] \qquad (3.34)$$

$$N = \frac{\Omega'^2 Z^2}{4}\left[\frac{1}{r^2\left(\Omega'^2 Z^2\right)} - \frac{\Omega_1^2 / r^4}{\left(\Omega_1^2\delta^2 - \Omega'^2\right)^2 + \left(\Omega' Z\right)^2}\right.$$

$$\left. + \frac{\Omega_1^2\delta^2\left(2\Omega'^2 - \Omega_1^2\delta^2\right)}{r^4\left[\Omega'^2\left(\Omega_1^2\delta^2 - \Omega'^2 + Z^2\right)^2 + Z^2\left(2\Omega'^2 - \Omega_1^2\delta\right)^2\right]}\right]^2 \qquad (3.35)$$

$\Omega' = \Omega - uB$

$r^2 + (1 - u^2) - 1$

$$B = \frac{\beta c}{\omega_p}, \Omega = \frac{\omega}{\omega_p}, Z = \frac{\upsilon}{\omega_p} u = \frac{V_0}{c}, \delta = \frac{a}{c}$$

$$\Omega_0 = \frac{k_c}{\omega_p} \text{ and } \Omega_1 = \frac{k_1 c}{\omega_p}$$

It is evident from equation (3.31) that the terms dependent on collision frequency, drift velocity and compressible property of plasma modify the characteristic equation for TM modes.

The cutoff frequency for TM modes in case of relativistically moving warm lossy plasma waveguide can be obtained by setting B = 0 in equation (3.33) equal to zero.

$$u^4\Omega_{c0}^4 + u^2 M'\Omega_{co}^2 - N' = 0 \tag{3.36}$$

where M' and N' are obtained by putting B = 0 in expressions of M and N given by equations (3.34) and (3.35) and $\Omega = \Omega_{c0}$. It is evident from equation (3.33) and (3.36) that the normalized phase constant and cutoff frequency depends upon relativistics velocity, plasma parameters (such as plasma frequency, compressible property and collisional property) and waveguide dimensions. In next sections variation of cutoff frequencies with appropriate laboratory plasma parameters in some simpler cases have been computed.

It is also desirable to see how the power flow changes. Since the velocity vector only has an X-component, pu.z is always zero. The pressure and velocity fields do not contribute to the power flow in the waveguide.

With the help of equations (3.21) and (3.22), the potential function and transformation relation, it is easy to establish the orthogonal relation for TM modes with a different mode index. Thus, for a TM mode the power flow is given by

$$W = \frac{1}{2}\mathrm{Re}\int_0^1\int_0^b E_x H_y^* dxdy = \frac{1}{2}\frac{\omega \in_0 \beta}{k_0^2 - k_1^2}\left|E_0^2\right|\frac{1b}{4} \tag{3.37}$$

It is clear that expression in equation (3.37) does not contain the acoustic speed 'a' explicitly. However, the temperature affects the power flow through the propagation constant β in equation (3.36). Also we observe that the power flow which depends solely on the field components have the same form as those obtained for stationary media because the expression for the electromagnetic field components do not change in form when the plasma is moving, the only essential difference being in the value of β of TM mode.

3.4 Special Cases:

3.4.1 Waveguide Containing Warm Lossy Stationary TMG Plasma:

In this case the rectangular waveguide is filled with transversely magnetized warm lossy plasma subjected to strong magnetic field. The plasma medium is assumed at rest with respect to conducting waveguide. The dispersion relation and cutoff for waveguide filled with warm stationary lossy transversely magnetized plasma can be obtained by puttin u=0 i.e. r =1 in equations (3.33) and (3.36) respectively. Thus the expression for cut-off frequency obtained from equation (3.36) is given as

$$\frac{1}{\Omega_{c0}^2+Z^2}+\frac{\Omega_1^2\delta^2\left(2\Omega_{c0}^2-\Omega_1^2\delta^2\right)}{\Omega_{c0}^2\left(\Omega_1^2\delta^2-\Omega_{c0}^2+Z^2\right)+Z^2\left(2\Omega_{c0}^2-\Omega_1^2\delta^2\right)^2}=\frac{\Omega_1^2}{\left(\Omega_1^2\delta^2-\Omega_{c0}^2\right)^2+\Omega_{c0}^2Z^2} \tag{3.38}$$

Equation (3.38) gives the relation for cutoff frequency which is independent of free space waveguide cutoff but depends upon plasma frequency, acoustic speed of electrons, collision frequency and transverse wave number along X-direction.

3.4.2 Waveguide Containing Warm Lossless Stationary TMG Plasma

Here inside the conducting rectangular waveguide is a fully ionized warm stationary lossless plasma. A strong static magnetic field is applied in the X-direction and the guide axis is along the Z direction. Thus by putting *r* =1 and Z = 0 in equation (3.33) and (3.36) the dispersion relation and cutoff can be obtained in case of TM modes. The expression for cutoff frequency is obtained from dispersion relation by putting phase constants term equal to zero as

$$\Omega_{c0}=\left[\frac{1}{2}\left\{\left(1+\Omega_c^2+\Omega_1^2\delta^2\right)\pm\sqrt{\left(1+\Omega_c^2+\Omega_1^2\delta^2\right)^2-4\Omega_1^2\left(1+\Omega_1^2\delta^2\right)}\right\}\right]^{1/2} \tag{3.39}$$

This equation is similar to that obtained by Singh (1973).

3.5 Result and Discussion:

It is found for TM mode from equation (3.33) that for lossy compressible plasma, propagation is possible overall frequency ranges with a cutoff frequency dependent on plasma parameters. Finally, at higher microwave frequencies, the value of B approaches that of free space waveguide showing that there is no interaction between electrons and electromagnetic waves. It is concluded that lossy characteristics of plasma makes propagation possible over all frequency ranges.

In case TE mode the wave characteristics and dispersion relation remain the same as that of free space rectangular waveguide due to sufficiently strong external magnetic field. Propagation of waves is possible only in the ranges of parameter for which B^2 is positive and real. The curve for phase constant in TM mode has got two branches, one extending over each frequency making propagation possible and other branch makes propagation possible as high pass filter. The second branch having cutoff is perturbed waveguide mode.

Equations (3.38) and (3.39) have been computed taking normalized appropriate plasma parameters and X-band waveguide dimensions the variation of cut off frequency has been studied. Equation (3.39) in case of uniaxial anisotropic warm stationary lossless TMG plasma has been computed taking appropriate and the variation of cutoff Ω_{co} is plotted in Figure (3.1). It is observed from the graph that due to increase in compressible property of plasma, the cutoff frequency for both propagation modes increases as shown in the figure for Ω_0 = 0.2 and Ω_1= 0.1. The normalized cutoff frequencies Ω_{c0} for appropriate plasma parameters in lossless (Z=0) and temperate (δ = 0) plasma are 0.09519 and 1.015034. The warmer is the plasma the higher is the cutoff frequency. At cutoff the wavelength of electromagnetic wave is infinite and at resonance it is zero and the energy of the wave is disputed heating the medium. When collisions between electrons and neutrals are taken into account again (Ω_{c0} vs δ) curve as obtained from equation (3.38) for fixed value of Z= 0.5, Ω_c = 0.2, Ω_1 = 0.1 has been shown in figure(3.2). It is observed from the graph for perturbed waveguide mode that with the increase in compressible effect for a fixed value of collision frequency the cut off increases slightly whereas for temperate (δ = 0) plasma δ_{c0} = 0.98. Introduction of even small collision events predominates over compressible property.

Figure (3.3) depicts the Ω_{co} versus Z curve for perturbed waveguide mode for normalized parameters $\delta = 0.01$, $\Omega_1 = 0.1$. It is observed that cutoff frequency for TM modes in this case decreases for low collision frequencies and increases slightly for higher collision frequencies.

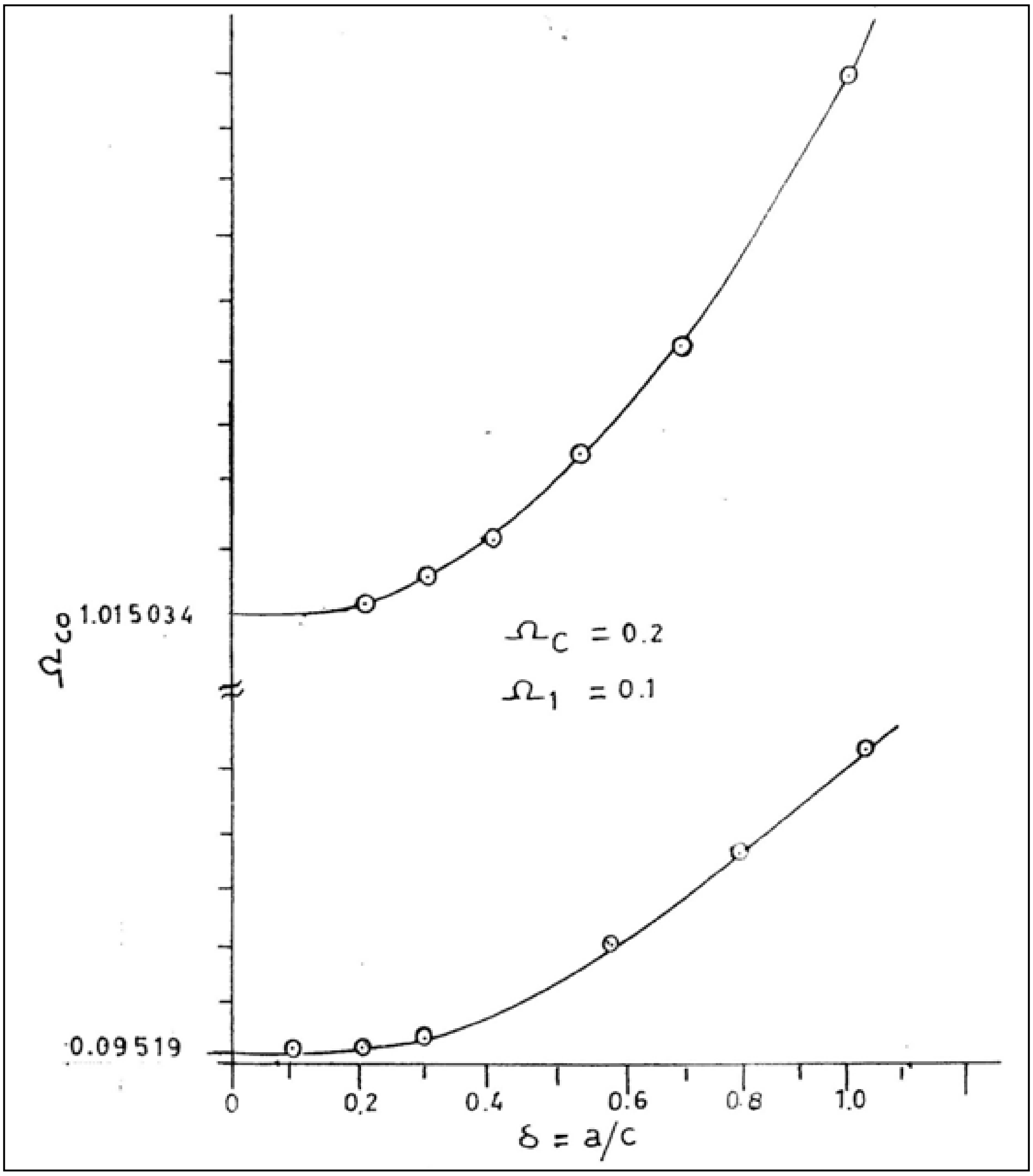

Figure 3.1: Variation of Cutoff Frequency with Normalized Acoustic Speed of Electron for Lossless Case.

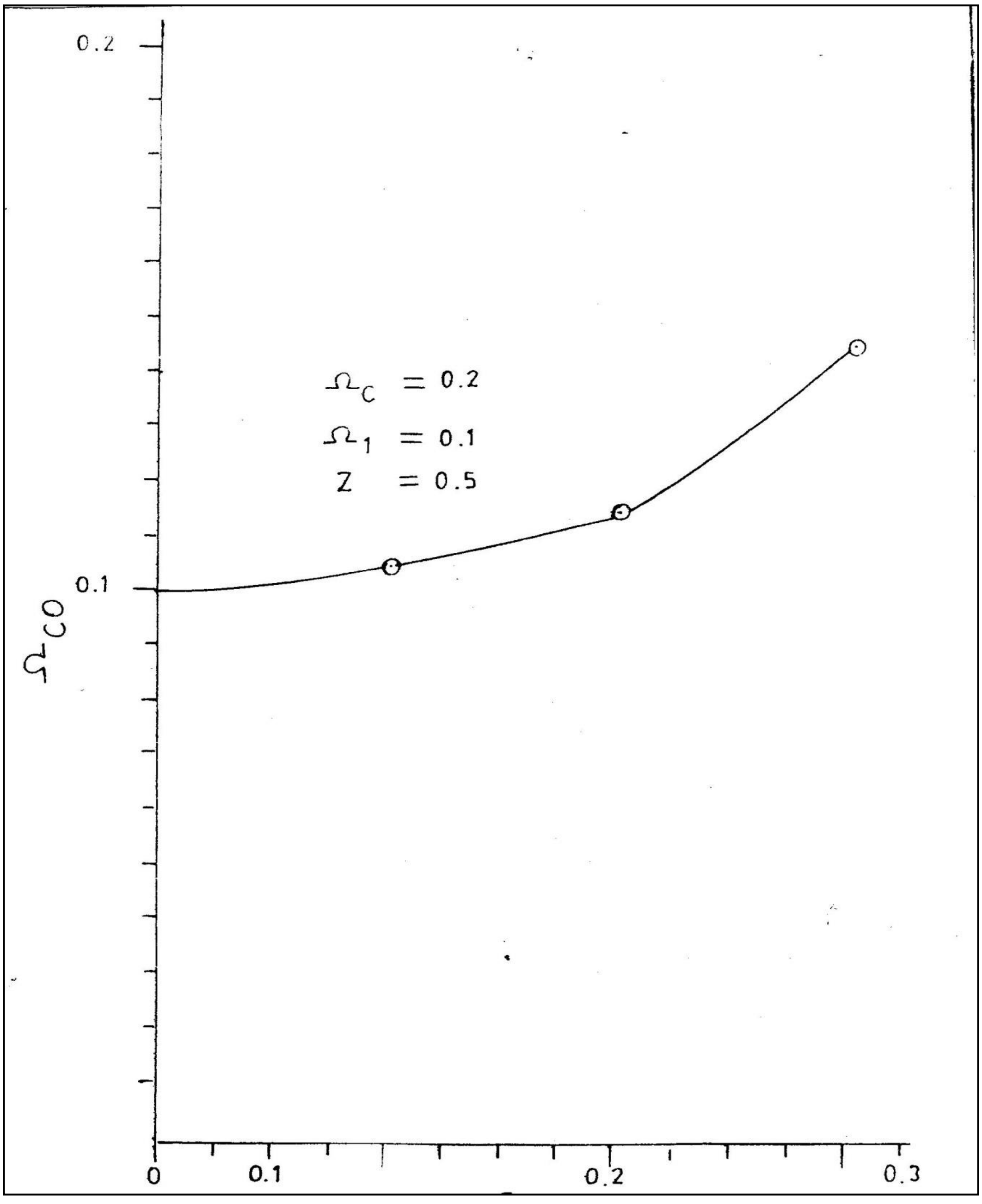

Figure 3.2: Variation of Cutoff Frequency with Acoustic Speed for Perturbed Waveguide Mode for Lossy Case.

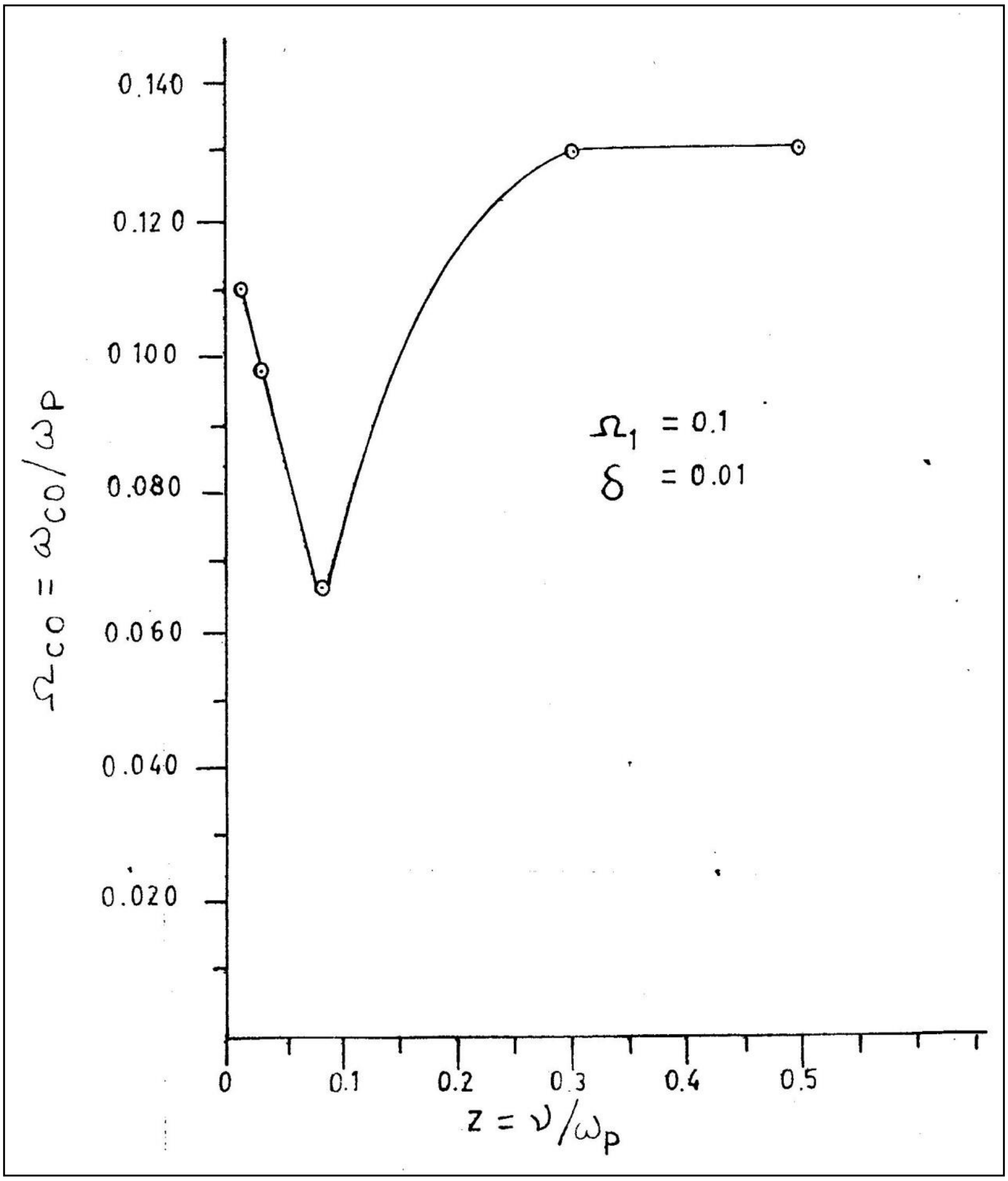

Figure 3.3: Variation of Cutoff Frequency with Normalized Collision Frequency for Lossy Case.

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Chapter 4

Microwave Propagation in LMG Solid State Plasma Waveguide

4.1 Introduction:

Many devices have been developed recently in which the magnetic field is applied in the direction of the propagation of the electromagnetic wave. In this chapter a detailed discussion of the waveguide characteristics filled with a semiconductor in the presence of a longitudinal magnetic field is presented. The presence of longitudinal magnetic field makes the medium anisotropic and the dielectric constant becomes a tensor, because, no one has to consider the electron motion perpendicular to both magnetic and electric fields due to Lorentz force.

The chapter begins with a brief historical introduction which is given in section (4.2). The dispersion characteristics of the waveguide in the presence of finite and strong magnetic field are discussed in section (4.3). The results obtained by the author are presented in section (4.4).

4.2 Historical:

One of the earliest experiments which demonstrated the nonreciprocal transmission characteristics of a waveguide filled with a solid state plasma in the presence of longitudinal magnetic field was described by Kuno and Hershberger (1967). They used n-type Indium antimonide (InSb) at the frequency of 35.95 GHz and the whole experiment was performed at liquid nitrogen temperature (77^0K). They have measured the attenuation experimentally and found that the attenuation in the forward direction is more than the attenuation in the reverse direction. They attributed the attenuation to the Faraday rotation caused by the longitudinal magnetic field. They have further shown that a device employing Faraday rotation can be used as isolators, attenuators and switches in the microwave region.

Steele and Vural (1969) discussed the dispersion characteristics of a cylindrical waveguide filled with InSb containing an electron-hole plasma in the presence of an external magnetic field along the guide axis. They have discussed in general various modes which can exist inside a waveguide containing mobile charges in an applied magnetic field under a variety of conditions. May and McLeod (1968) observed experimentally the nonreciprocal characteristics in a waveguide partially filled with n-InSb under a longitudinal magnetic field operating at liquid nitrogen temperature using Faraday rotation. The theoretical and experimental study of a circular waveguide in the presence of longitudinal static magnetic field containing a coaxial InSb rod was done by McLeod and May (1971), who described the use of such a system as an isolator at 35 GHz. All the experimental work was carried out at liquid nitrogen temperature to achieve nonreciprocal operation in the presence of magneto-static field. Two types of 35 GHs semiconductor isolators namely field displacement isolator and mode coupling isolator have been demonstrated at liquid nitrogen temperature for coaxial solid state plasma waveguide in the presence of longitudinal magnetic field. In the case of field displacement isolator described by effective dielectric constant of the solid state plasma has a larger real part for one particular direction of applied magnetic field and hence the wave will be excluded from the plasma medium. For the reverse direction of applied magnetic field plasma has a small effective dielectric constant which is entirely imaginary and wave will not be strongly excited in the medium. This leads to property of field displacement isolator. Whereas, in the case of mode coupling isolator described by McLeod and May (1971) the variation of coupling between two possible modes at various strength of applied magnetic field has been considered. The coupling gradually changes from the low loss to the high- loss mode as the static magnetic field increases. This will give a technique of obtaining a new type of isolator know as mode coupling isolator. Design of mode coupling isolator can yield a device with low forward loss and very high isolation. The experimental studies have been made under the following conditions.

- The longitudinal applied magneto-static field is strong enough to make cyclotron frequency greater than microwave signal frequency.
- The product of conductivity mobility and the static magnetic field (i.e. $\mu_n B_o$) must be large in comparison with unity.

The experimental and theoretical results of propagation characteristics were found to be in good agreement, so far as the general shape and amount of isolation is concerned. The propagation characteristics of electromagnetic waves in a circular waveguide containing a coaxial annular column of n-type InSb at liquid nitrogen temperature have been discussed by Kanda and May (1974).

Such a system can be utilized for the development of millimeter waveguide isolators. It has been observed experimentally by Ishizuka and Obunai that the 70 GHz millimeter wave propagation characteristics in a transversely magnetized waveguide loaded with a p-type InSb slab are markedly varied by applying a current considerably smaller than that used in the previous studies employing n-InSb.

It is suggested that the shift of the slow- surface-wave resonant magnetic field in this waveguide is responsible for the observed variation. High resistivity silicon substrates demonstrated strong potential for applications as a microwave and millimeter wave substrate. A method to simulate a coplanar waveguide (CPW) on a doped semiconductor substrate is presented by Larocca et.al. (1996). Its salient point is the inclusion of a voltage dependent depletion width and built-in voltage due to the metal- semiconductor (Schottky) contact. The attenuation, effective dielectric constant and characteristic impedance are determined for different modes and applied biases.

4.3 Propagation Characteristics in Rectangular Waveguide Containing Semiconductor Plasma in The Presence of Longitudinal Magnetic Field:

The propagation characteristics of electromagnetic wave investigated by earlier authors through a waveguide containing a semiconductor plasma in the presence of a longitudinal magnetic field, have been discussed in section (4.2).

In this section the author has developed the dispersion relation and discussed the attenuation and phase characteristics for the propagation of electromagnetic wave in a rectangular waveguide containing solid state plasma. A general dispersion relation for a bounded system in the presence of finite magnetic field parallel to the direction of propagation is developed and this is applied to the special case of a strong longitudinal magnetic field.

4.3.1 General Consideration:

Consider a rectangular waveguide of uniform cross- section and of perfectly conducting walls completely filled with a semiconducting material. The external magnetic field B_0 is along the Z-direction which is also the direction of propagation off the wave.

The response of the semiconducting material to the electromagnetic fields in the presence of an external finite longitudinal magnetic field with drift and diffusion neglected can be described by a dielectric tensor and is given by

$$\|\epsilon\| = \epsilon_0 \cdot \epsilon_L \begin{Vmatrix} \epsilon_1 & -\epsilon_3 & 0 \\ \epsilon_3 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_2 \end{Vmatrix} \tag{4.1}$$

where

$$\epsilon_1 = 1 - \frac{\omega_p^2(\omega - j\upsilon)}{\omega\left[(\omega - j\upsilon)^2 - \omega_c^2\right]}$$

$$\epsilon_2 = 1 - \frac{\omega_p^2}{\omega(\omega - j\upsilon)}$$

$$\epsilon_3 = \frac{j\omega_p^2\omega_c}{\omega\left[(\omega - j\upsilon)^2 - \omega_c^2\right]}$$

It has been assumed that the field vectors very as exp $(j\omega t - \gamma z)$ where γ is the propagation constant which is assumed independent of position and is an arbitrary complex number $\alpha + j\beta$. In equation (4.1), υ is the collision frequency, ω_p and ω_c are the plasma and cyclotron frequencies respectively for electrons given by

$$\omega_p = \left(\frac{ne^2}{m^* \in_0 \in_L} \right)^{1/2}$$

$$\omega_C = \frac{eB_0}{m^*}$$

Where $\in_L$ is the relative static dielectric constant of solid state plasma, which is much greater than unity for the semiconducting materials, m* is the effective mass for electron single free carrier, $\in_0$ is the scalar free space permittivity.

From Maxwell's equations the differential equations for E_z and H_z components can after some manipulations, be written in the following coupled form:

$$\nabla_T^2 E_z + \Psi_1 E_z = \Psi_2 H_z \tag{4.2}$$

$$\nabla_T^2 H_z + \Psi_3 H_z = \Psi_4 E_z \tag{4.2}$$

where

$$\Psi_1 = \frac{\in_2}{\in_1}\left(\gamma^2 + k_0^2 \in_1 \in_1\right)$$

$$\Psi_2 = j\omega\mu_0\gamma \frac{\in_3}{\in_1}$$

$$\Psi_3 = \gamma^2 + k_0^2 \frac{\in_1}{\in_1}\left(\in_1^2 + \in_3^2\right)$$

$$\Psi_4 = -j\omega\gamma \in_0 \in_1 \in_2 \frac{\in_3}{\in_1}$$

and

$$k_0^2 = \omega^2 \mu_0 \in_0$$

K_0 is free space propagation constant and μ_0 is the scalar permeability assumed to be that of vacuum (assuming a nonmagnetic conducting medium). The presence of charged particles does not affect the permeability of the medium. ∇_T includes only differentiations with respect to the directions transverse to the direction of wave propagation.

Equations (4.2) and (4.3) show that, in general, medium gives rise to a coupling between. TM and TE modes.

Also one can get a solution for the transverse fields in terms of the longitudinal fields [Allis et.al. (1963)]

$$\begin{bmatrix} P' & R' & Q' & S' \\ T' & P' & U' & Q' \\ -Q' & -S' & P' & R' \\ -U' & -Q' & T' & P' \end{bmatrix} \begin{bmatrix} \nabla_T \quad E_Z \\ \nabla_T \quad H_z \\ \hat{k} \times \nabla_T E_z \\ \hat{k} \times \nabla_T H_z \\ \end{bmatrix} \begin{bmatrix} E_T \\ H_T \\ \hat{k} \times E_z \\ \hat{k} \times H_z \end{bmatrix} \tag{4.3}$$

Where,

$$P' = \frac{\gamma\left(\gamma^2 + k_0^2 \in_1 \in_1\right)}{D'}$$

$$R' = \frac{j\omega\mu_0 k_0^2 \in_1 \in_3}{D'}$$

$$Q' = \frac{\gamma k_0^2 \in_1 \in_3}{D'}$$

$$S' = \frac{j\omega\mu_0\left(\gamma^2 + k_0^2 \in_1\in_1\right)}{D'}$$

$$T' = \frac{j\omega \in_0\in_L \gamma^2 \in_3}{D'}$$

$$U' = \frac{-j\omega \in_0\in_L \left(\gamma^2 \in_1 + k_0^2 \in_L\in_1^2 + k_0^2 \in_L\in_3^2\right)}{D'}$$

and

$$D' = \left[\left(\gamma^2 + k_0^2 \in_L\in_1\right)^2 + \left(k_0^2 \in_L\in_3\right)^2\right]$$

$\hat{k}$ is the unit vector along the Z-direction?

From the above two coupled second order differential equations (4.2) and (4.3), one can derive the following fourth order differential uncoupled wave equations for E_z and H_z

$$\left[\nabla_T^4 + \left(\psi_1 + \psi_3\right)\nabla_T^2 + \left(\psi_1\psi_3 - \psi_2\psi_4\right)\right]E_z = 0$$

(4.5)

$$\left[\nabla_T^4 + \left(\psi_1 + \psi_3\right)\nabla_T^2 + \left(\psi_1\psi_3 - \psi_2\psi_4\right)\right]H_z = 0$$

(4.6)

above equations admit solution of the from (–jΓx), hence

$$\Gamma^4 + \left(\psi_1 + \psi_3\right)\Gamma^2 + \left(\psi_1\psi_3 - \psi_2\psi_4\right) = 0$$

(4.7)

and the four roots are given as

$$\Gamma_1^2 = \frac{1}{2}\left[(\psi_1 + \psi_3) + (\psi_1 - \psi_3)\left\{1 + \frac{4\psi_2\psi_4}{(\psi_1 - \psi_3)^2}\right\}^{1/2}\right]$$

(4.8)

$$\Gamma_2^2 = \frac{1}{2}\left[(\psi_1 + \psi_3) - (\psi_1 - \psi_3)\left\{1 + \frac{4\psi_2\psi_4}{(\psi_1 - \psi_3)^2}\right\}^{1/2}\right]$$

(4.9)

By using binomial expansion, the four roots can be evaluated as

$$\Gamma_1^2 = \left[\psi_1 + \frac{\psi_2\psi_4}{(\psi_1 - \psi_3)}\right]$$

$$\Gamma_2^2 = \left[\psi_3 - \frac{\psi_2\psi_4}{(\psi_1 - \psi_3)}\right]$$

It is often more convenient to give solutions for each value of Γ_i^2 , of the form

$$\left[\nabla_T^2 + \Gamma_i^2\right]E_{zi} = 0 \qquad (4.10)$$

Where I = 1, 2

Since equation (4.7) is of fourth order, two independent solutions of the form of equation (4.10) will in general be required.

Thus

$$E_z = E_{z1} + E_{z2} \qquad (4.11)$$

$$H_z = \left[\left(\frac{\psi_1 - \Gamma_1^2}{\psi_2} \right) E_{z1} + \left(\frac{\psi_1 - \Gamma_2^2}{\psi_2} \right) E_{z2} \right] \tag{4.12}$$

4.3.2 Dispersion Relation for Parallel Plane Waveguide When the Longitudinal Magnetic Field is Finite:

It is very difficult to solve the boundary value problem for the case of rectangular waveguide filled with semiconductor plasma in the presence of finite magnetic field, for which the variables cannot be separated and the expression for the dispersion relation cannot be obtained. For the sake of simplicity author has considered the case of a parallel plane waveguide for which the variation of field vectors along the Y-direction is assumed to be zero i.e. = $\partial/\partial y = 0$. The solution for E_z is of the form

$$E_z = \left[\left(A_3 e^{j\Gamma_1 x} + B_3 e^{j\Gamma_1 x} \right) + \left(C_3 e^{j\Gamma_2 x} + D_3 e^{j\Gamma_2 x} \right) \right] \exp(j\omega t - \gamma z)$$

(4.13)

Where A_3, B_3, C_3 and D_3 are four arbitrary constants which can be evaluated by applying appropriate boundary conditions. The general solution for magnetic field component independent of Y-direction can be given as

$$H_z = \left[\left(\frac{\psi_1 - \Gamma_1^2}{\psi_2} \right) \left(A_3 e^{j\Gamma_1 x} + B_3 e^{j\Gamma_1 x} \right) + \left(\frac{\psi_1 - \Gamma_2^2}{\psi_2} \right) \left(C_3 e^{j\Gamma_2 x} + D_3 e^{j\Gamma_2 x} \right) \right] \exp(j\omega t - \gamma z)$$

(4.14)

The transverse components (i.e. E_x and E_y) can be determined with the help of equations (4.4), (4.13) and (4.14)

$$E_x = R_1 \frac{\partial E_{Z1}}{\partial x} + R_2 \frac{\partial E_{Z2}}{\partial x} \tag{4.15}$$

$$E_y = P_1 \frac{\partial E_{Z1}}{\partial x} + P_2 \frac{\partial E_{Z2}}{\partial x} \tag{4.16}$$

Where

$$R_1 = P' + R'\left(\frac{\psi_1 - \Gamma_1^2}{\psi_2}\right)$$

$$R_2 = P' + R'\left(\frac{\psi_1 - \Gamma_2^2}{\psi_2}\right)$$

$$P_1 = Q' + S'\left(\frac{\psi_1 - \Gamma_1^2}{\psi_2}\right)$$

$$P_2 = Q' + S'\left(\frac{\psi_1 - \Gamma_2^2}{\psi_2}\right)$$

Applying the boundary conditions

E_z	=	0,	at x = 0	amd x = b
E_y	=	0,	at x = 0	amd x = b

One obtains a system of four linear homogeneous equations in arbitrary constants A_3, B_3, C_3, and D_3 as

$$A_3 + B_3 + C_3 + D_3 = 0 \tag{4.17}$$

$$A_3 e^{j\Gamma_1 b} + B_3 e^{-j\Gamma_1 b} + C_3 e^{j\Gamma_{21} b} + D_3 e^{-j\Gamma_2 b} = 0 \tag{4.18}$$

$$A_3\Gamma_1P_1 - B_3\Gamma_1P_1 + C_3\Gamma_2P_2 + D_3\Gamma_2P_2 \qquad =0 \qquad (4.19)$$

$$A_3\Gamma_1P_1e^{j\Gamma_1 b} - B_3\Gamma_1P_1e^{j\Gamma_1 b} + C_3\Gamma_2P_2e^{j\Gamma_2 b} - D_3\Gamma_2P_2e^{j\Gamma 2b} = 0 \qquad (4.20)$$

he equations (4.17) to (4.20) can be put conveniently in the following matrix form:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{j\Gamma_1 b} & e^{-j\Gamma_1 b} & e^{j\Gamma_2 b} & e^{-j\Gamma_2 b} \\ \Gamma_1P_1 & -\Gamma_1P_1 & \Gamma_2P_2 & -\Gamma_2P_2 \\ \Gamma_1P_1e^{j\Gamma_1 b} & -\Gamma_1P_1e^{j\Gamma_1 b} & \Gamma_2P_2e^{j\Gamma_2 b} & -\Gamma_2P_2e^{j\Gamma_2 b} \end{vmatrix} = 0 \qquad (4.21)$$

From the above determinantal equation one obtains the following transcendental equation equation determining the propagation constant γ, the transverse wave number Γ_1, Γ_2, the angular frequency ω, the plasma parameters ω_p and ω_c and the dimensions and the dimensions of the waveguide.

$$-2\Gamma_1\Gamma_2P_1P_2\left[1+\cos\Gamma_1 b Cos\Gamma_2 b\right] = \left[\Gamma_1^2 p_1^2 + \Gamma_2^2 p_2^2\right] Sin\Gamma_1 b Sin\Gamma_2 b \qquad (4.22)$$

If it is assumed that $(\Gamma_1 b)$ and $(\Gamma_2 b) \ll 1$, one can approximately

Write

$Sin\Gamma_1 b = \Gamma_1 b$

$Sin\Gamma_2 b = \Gamma_2 b$

$$Cos\Gamma_1 b = 1 + \frac{\Gamma_1^2 b^2}{2} \qquad (4.23)$$

$$Cos\Gamma_2 b = 1 + \frac{\Gamma_2^2 b^2}{2}$$

Hence equation (4.22) can be written as

$$-\left[\frac{4}{b^2}+\left\{\Gamma_1^2+\Gamma_2^2\right\}\right]=\left[\Gamma_1^2\frac{P_1}{P_2}+\Gamma_2^2\frac{P_2}{P_1}\right] \tag{4.24}$$

Which is the dispersion relation for longitudinally magnetized parallel plane waveguide having plate separation of width b. From the dispersion equation, the propagation constant γ is determined by examining both attenuation constant α and phase constant β.

4.3.3 Dispersion Relation for Rectangular Waveguide in The Presence of Infinitely Strong Longitudinal Magnetic Field:

Due to mathematical difficulties encountered in the analysis of the dispersion relation (4.24) for α and β at various frequencies another simpler case of a rectangular magnetic field is considered.

Consider a rectangular waveguide filled with semiconducting material with an infinitely strong magnetic field along the direction of propagation of the wave, so that all electron motions perpendicular to the applied static magnetic field can be neglected. With this approximation all the non-diagonal elements of the dielectric tensor vanish and all the diagonal elements perpendicular to the magnetic field reduce to unity. Under the influence of strong external magnetic field, the elements of dielectric tensor of equations (4.1) can give as

$$\in_1 = 1$$

$$\in_3 = 0 \tag{4.25}$$

$$\in_2 = 1-\frac{\omega_p^2}{\omega(\omega-j\upsilon)}$$

One can get the following dispersion relation for TM modes (i.e. $H_z = 0$)

$$\left(\gamma^2+k_0^2\in_L\right)\left[1-\frac{\omega_p^2}{\omega(\omega-j\upsilon)}\right]=k_c^2 \tag{4.26}$$

Where

$$k_c^2 = \left(\frac{m_1\pi}{1}\right)^2 + \left(\frac{m_2\pi}{b}\right)^2$$

l and b are waveguide dimensions in the X and Y directions and m1 and m_2 are integers showing modes.

In order to get above dispersion relation, the solution of the following form for E_z have been used with appropriate boundary conditions as

$$E_z = E_0 \text{ Sin } K_1x \text{ Sin} k_2y \exp(j\omega t - \gamma z) \quad (4.27)$$

where

$$k_1 = \frac{m_1\pi}{1}$$

$$k_2 = \frac{m_1\pi}{b}$$

Separating real and imaginary parts of equations (4.26) one can obtain the expression for phase and attenuation constants after normalization in ω_p as

$$B^2 = \frac{1}{2}\left[-X \pm \sqrt{X^2 + Y}\right] \quad (4.28)$$

$$A^2 = \frac{1}{2}\left[-X \pm \sqrt{X^2 + Y}\right] \quad (4.29)$$

where

$$X = \Omega^2 \left[\frac{\Omega_c^2(\Omega^2 - Z^2 - 1)}{\{(\Omega^2 - 1)^2 + \Omega^2 Z^2\}} - \in_L\right]$$

$$Y = \Omega^2 \left[\frac{\Omega_c^4 Z^2}{\left\{ \left(\Omega^2 - 1 \right)^2 + \Omega^2 Z^2 \right\}} \right]$$

with

$$\Omega = \frac{\omega}{\omega_p}; B = \frac{\beta c}{\omega_p}; B = \frac{\alpha c}{\omega_p}; Z = \frac{\upsilon}{\omega_p} and \Omega_c = \frac{k_0 c}{\omega_p}$$

The electric and magnetic field components transverse to the direction of propagation can be obtained from equation (4.4) taking H_z =0 for TM modes

$$E_x = \frac{\gamma \in_2 k_1}{k_c^2} E_0 \cos k_1 x \sin k_2 y \exp(-\gamma z)$$

$$E_y = \frac{\gamma \in_2 k_2}{k_c^2} E_0 \sin k_1 x \cos k_2 y \exp(-\gamma z)$$

$$H_x = \frac{j\omega \in_0 \in_1 \in_2 k_2}{k_c^2} E_0 \sin k_1 x \cos k_2 y \exp(-\gamma z)$$

$$H_y = \frac{j\omega \in_0 \in_1 \in_2 k_1}{k_c^2} E_0 Cos k_1 x Sin k_2 y \exp(-\gamma z) \qquad (4.30)$$

For the of TM modes (E_z=0) usual set of equations are obtained which are similar to free space filled waveguide under the influence of strong longitudinal magnetic field.

4.4 Result and Discussion:

The phase and attenuation constants have been computed using equations (4.28) and (4.29) for a rectangular waveguide filled with n-type germanium for different values of normalized frequency in the microwave range.

The parameters that are used at liquid nitrogen and room temperatures are given below.

At liquid nitrogen temperature (i.e. 77k) a n-type germanium sample having a carrier concentration of $n = 4 \times 10^{19}$ electrons/m^3. electrons mobility $\mu_n = 3.5 m^2$/v.sec., and conductivity $\sigma = 22.0$/ohm.m is used. At room temperature the following conditions for n-type germanium are applied. Carrier concentration $n = 10^{20}$ electrons/m^3, electron mobility $\mu_n = 0.3600 m^2$/v.sec., conductivity $\sigma = 5.76$/ohm.m., the effective mass for electrons $m^* = 0.3 m_0$, which is considered to be constant at both the temperature. The values of collision frequency at these two temperatures are taken 1.67×10^{11}/sec. and 1.63×10^{12}/sec. respectively.

Figures (4.1) and (4.2) show the results for phase and attenuation constants as a function of frequency with the given parameters and the sample of n-type germanium for the rectangular waveguide in the presence of strong longitudinal magnetic field. It can be seen that the phase constant increases as the frequency increases, whereas the attenuation constant increases as the signal frequency increases and attains a maximum value at a frequency depending upon the temperature. As the frequency is further increased the attenuation constant decreases. The peaks occur at 17.99 GHZ and 4.1 GHz for the temperature of 77K and 300K respectively. From the graphs it is concluded that there does not exist any characteristic cutt-off and propagation as well as attenuation is possible over all frequency ranges.

At liquid nitrogen temperature the value of the phase constant is large while the value of the attenuation is small in comparison with values at 300K. This shows that the amount of attenuation can be reduced by reducing the concentration. This is to be expected because the collision frequency decreases as the carrier concentration decreases. The graphs also indicate that as the carrier concentration decreases of increases, there is large change in attenuation. For high concentration the attenuation constant is quite large implying that the electromagnetic wave is largely attenuated as it propagates through the waveguide.

4.5 Conclusion:

The dispersion relation for a bounded solid state plasma system in the presence of finite magnetic field in the direction of guide axis has been derived. In a special case the propagation characteristics of a rectangular under the influence of strong longitudinal magnetic field have been discussed at liquid nitrogen and room temperatures. Solutions for propagation constant and field components are obtained for TM modes. The dispersion diagrames show that the value of phase constant increases with increase of single frequency for the propagation of TM Mode, whereas the attenuation constant increases with the increases of frequency and attains a maximum for a particular frequency depending upon the temperature. With further increase of frequency, the attenuation constant decreases.

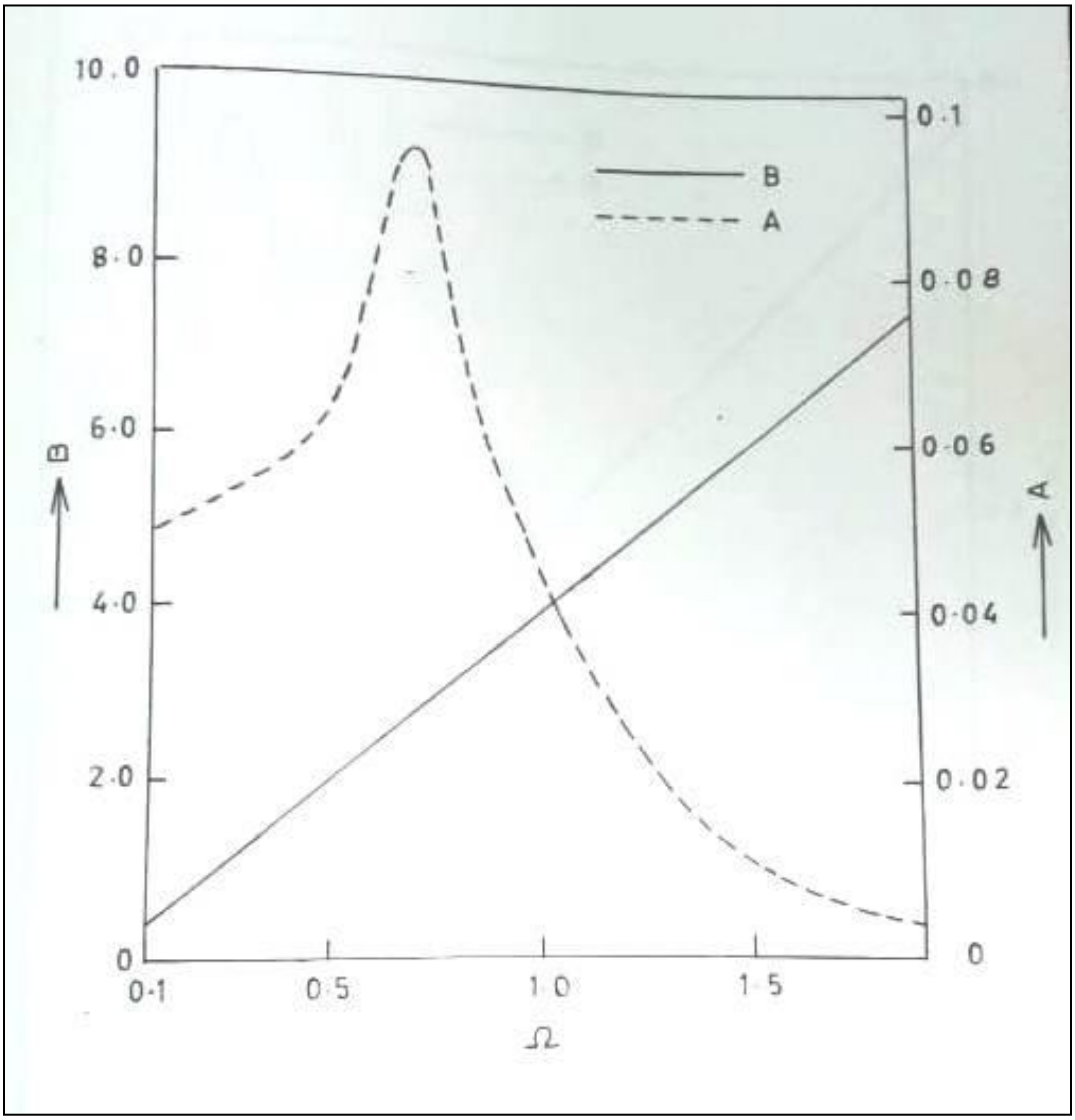

Figure 4.1: Dispersion Diagram for Z= 1.03 and Ω_C =0.61 at Liquid Nitrogen Temperature I.E. 77k

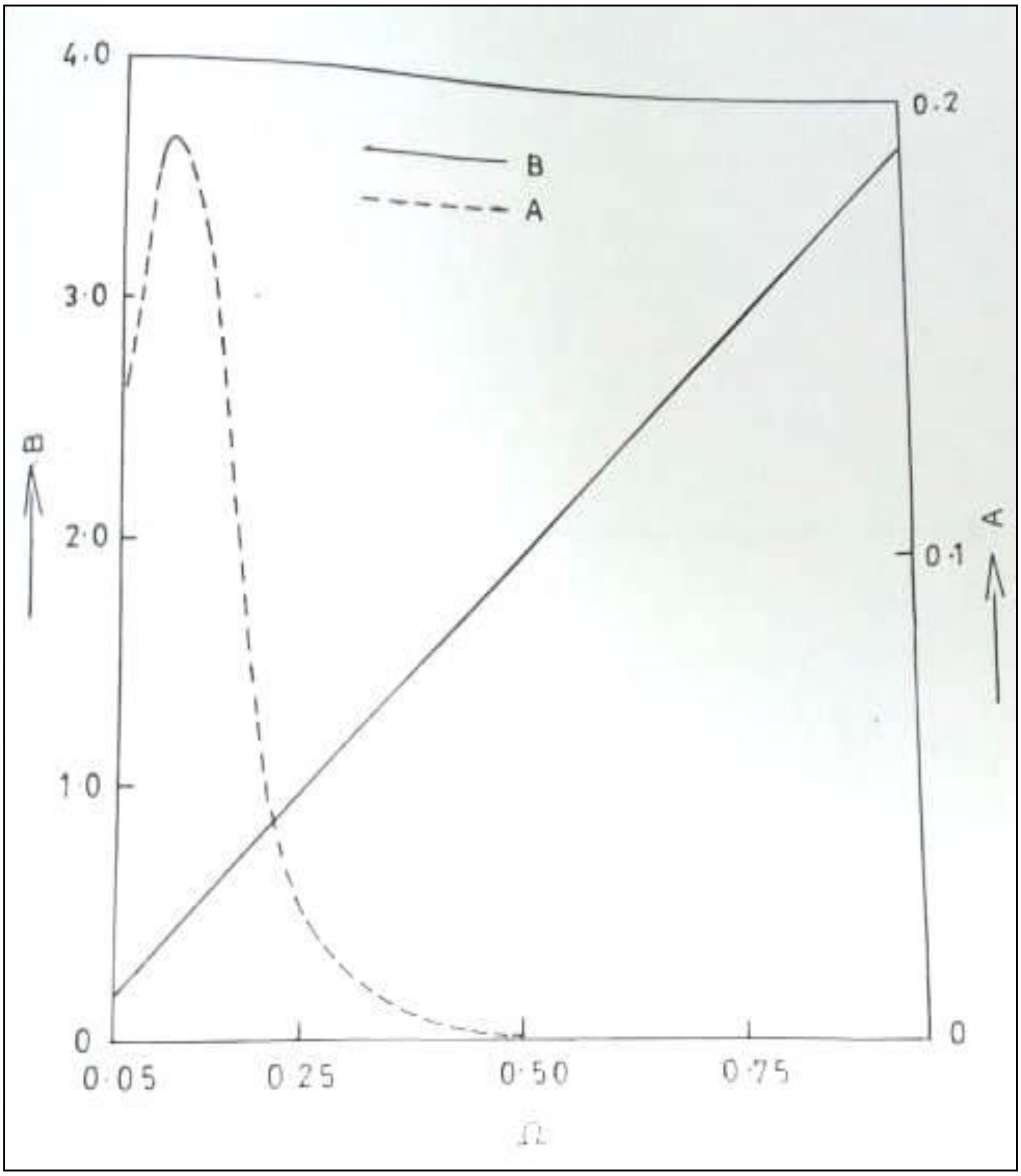

Figure 4.2: Dispersion Diagram For Z = 6.32 and Ω_C =0.33 at Room Temperature (I.E. 300k)

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Chapter 5

Growing Surface Waves in a Semiconductor (InSb) in the Presence of a Transverse Magnetic Field

5.1 Introduction:

Various types of wave propagations in gas plasma and in solid state plasma have undergone considerable study with regard to their potential applications in microwaves and millimeter wave devices [Crawford (1971), Allis et.al. (1963) Obunai (1993)]. Many types of wave propagation modes in plasma find two important classes of device applications: in nonreciprocal devices [Hirota and Suzuki (1966), Hirota and Suzuki (1970), Arnold and Resonbaum (1971), Sorrentino (1976), McLeodd and May (1971), Kanda and May (1974), Champman and Glover (1970)] and in active microwave devices [Verma and Gandhi (1973)]. The influence of transversal density non uniformity on the propagation of electromagnetic surface waves along plasma slabs and columns in the presence of collisions is analysed [Grosse et.al. (1997)] by numerical evaluations for phase diagrams of axially homogeneous plasmas.

Moreover, the changes in the dispersion and damping caused by the occurrence of plasma resonances in the transversal/radian density profile are studied. In addition, complex modes and backward waves in the collisionless dispersion curves of isotropic plasmas are treated. The surface modes in the plasma cylindrical waveguide in a lossy material are analyzed by Hu and Ruan (1997), particularly for the variations of their propagations properties with plasma parameters and surrounding material.

The characteristic equations of surface modes are derived and their relevant approximate solutions are given. The analysis shows that it can be used to improve the properties of mode suppressors and fabricate cut-off attenuators. Surface carrier wave amplification is InSb at X-band has been discussed in detail by Verma and Gandhi (1973).

Slow wave propagations in a particular type of transversly magnetized, partially filled, millimeter wave, parallel plate. Solide- plasma waveguide has been studied by Obunai and Sekiguchi (9174) with a view towards application in a slow wave circuit in a travelling wave amplifier where two different slow surface-wave modes propagate.

The propagation characteristics in gas plasma waveguides can be controlled very rapidly allowing its application to many types of microwave and millimeter wave devices such as rapidly tunable attenuators and phaser (Crawford (1971)]. Obunai (1993) investigated experimentally the propagation characteristics of a 70GHz slow surface wave in a transversely magnetized, partially filled, parallel. plate, solid- plasma waveguide using n-InSb as the plasma material to which a pulsed current was applied.

The surface wave resonant magnetic field increased substantially with an increase in the applied current. Subsequently, a variation of the electron density of the plasma material with the current was determined by measuring the reflecting from the plasma slab. The results indicate that the variation of the propagation characteristics is primarily explained by the variation of the electron density in the plasma material.

The possibility of externally controlling the propagation characteristics of a slow surface wave in a solid plasma waveguide has been shown experimentally. Slow surface wave resonance in a partially filled transversely magnetized 70GHz parallel plate solid plasma waveguide has been studied [Obunai and Sekiguchi (1974)]. It has been confirmed experimentally that the propagation characteristics of the slow wave can be rapidly controlled by varying the plasma density [Obunai and Yoshida (1985)]. Obunai and Takeda (1995) showed that the transmission characteristics of a 70GHz transversely magnetized partially filled solid-plasma waveguide can be varied by irradiating the plasma material with a semiconductor laser. The experimental results are explained by the increase in the slow surface wave resonant magnetic field in the waveguide. These findings indicate the possibility of controlling the propagation characteristics of the slow surface wave in the solid plasma waveguide by means of optical injection. Ishizuka and Obunai (1995) observed experimentally that the 70GHz millimeter-wave propagation characteristics in a transversely magnetized waveguide loaded with a p-type InSb slab are markedly varied by applying a current considerably smaller than that used in employing n-InSb.

It is suggested that the shift of the slow-surface wave resonant magnetic field in this waveguide is responsible for that observed variation. Kimura et.al. (1995) studied 70 GHz slow wave propagation characteristics in a transversely magnetized partially filled ring-form solid plasma waveguide experimentally, employing n-type InSb ring at liquid nitrogen temperature as the plasma material. Obtained results were compared with those for the previously studied straight parallel-plate configuration.

It is clearly shown that the surface wave resonance corresponding to the one observed in the straight plasma waveguide takes place also in the curved structure, and the propagation characteristics are quite similar to those of the straight configuration. Reduction of the resonant magnetic field is found with increasing the curvature of the waveguide. Effects of plasma density non-uniformity on the surface wave propagation along plasma- plasma and plasma-dielectric interfaces are investigated by Aliev et.al. (1995). Plasma in-homogeneities in longitudinal and transverse directions with respect to the direction of wave propagation are considered.

The influence of the in homogeneity on the propagation properties (dispersion, absorption) of the surface waves and the space distribution of the wave fields is analysed. The wave behavior in the region of the quasi-static surface wave resonance is also studied. Girka (1994) studied dispersion properties of electromagnetic extraordinary surface waves propagating across external magnetic field in uniform cold collisinless plasma, fully filling rectangular cross-sectional (with sides a and b) metallic waveguide by using perturbation theory. In semiconductor physics such waves are called surface magneto-plasma polaritons. Comparison of these SW properties with those of azimuthal SW propagating in ring waveguide and SW in planer layers was made.

Yadokawa et.al. (1997) studied slow surface wave propagation characteristics in a 70 GHs waveguide containing transversely magnetized p-type and n-type InSb slabs. Surface wave resonance occurs in this configuration. It is also shown that the magnetic field required for slow surface wave resonance is greatly reduced by the addition of the p-layer. The propagation characteristics of the mangetostic surface wave in a composite waveguide with a YIG film and YBCO were investigated theoretically and experimentally by Fukusako and Tsutsumi (1995). First, using the equivalent complex permittivity of YBCO obtained by the two fluid model and the permeability tensor of a YIG film, the dispersion relationship of the waveguide was derived. It was proved theoretically that a strong non-reciprocity within the 2.4GHz bandwidth with low loss could be achieved by placing superconducting material in the waveguide. The influence of magnetic loss of YIG on the superconducting material also was discussed.

The possibility of observing a low loss or amplifying carrier wave in a semiconductor has been the subject of recent theoretical investigations [Kino (1965), Gueret (1968), Robinson (1967) and Kino (1968)]. A carrier wave in an infinite semiconductor is normally regarded as a longitudinal space charge wave with a phase velocity close to the drift velocity of one type of carrier. It has also been shown by Kino (1965) that another type of carrier wave could possibly exist, a surface wave with a phase velocity close to the drift velocity of one type of carrier. Growing waves have been observed experimentally in a semiconductor by Burke and Kino (1968) in the situation where the electrons are drifting through holes in the presence of a magnetic field perpendicular to the direction of drift. The growth rates are extremely large depending on magnetic field and electron velocity. The velocity of the waves can be predicted accurately from considerations of a simple longitudinal carrier wave in the presence of holes in an infinite semiconductor. However, considerations of electron and hole interactions involving. However, considerations of electron and hole interactions involving the presence of magnetic fields [Gueret 91968)] or the inertial terms in the equations of motion [Kino (19650], yield rates of growth several order of magnitude less than the observed effect. In this chapter we have computed phase constant and growth rate of a surface carrier wave in a near intrinsic semiconductor and compared the result with that of infinite semiconductors [Singh et.al. (1965)]. Such waves have been observed by Burke and Kino 91968) in a near-intrinsic InSb in a transverse magnetic field in the frequency range from 10 to 40 MHz which grows spatially above a threshold drift velocity.

5.2 Theory:

A near intrinsic semi-infinite semiconductor (InSb bar) with its bounding surface along the plane $y = 0$ is considered. The d.c. magnetic field B_0 is taken to be in X-direction and all *rf* and dc motions to be only in the Y and Z directions. All the quantities within the semiconductor will be denoted by the subscript A, and its permittivity by $\in$.

All quantities outside the semiconductor will be denoted by the subscripts B, dc and *rf* quantities by the subscripts 0 and 1, respectively, and the electrons and holes by the subscripts e and h respectively.

The *rf* magnetic field will be neglected because of the very low carrier wave velocity. It will therefore be assumed that the *rf* electric field can be derived from a scalar potential ϕ_1 such that

$$E_1 = \nabla\phi_1 \tag{5.1}$$

The equation of motion for the electrons is

$$V_{1ze} = \mu_e\left(E_{1z} - V_{1ye}B_0\right) \tag{5.2}$$

$$V_{1ye} = \mu_e\left(E_{1y} - V_{1ze}B_0\right) \tag{5.3}$$

Components of electron velocity from equations (5.2) and (5.3) are written as

$$V_{1ze} = \frac{\mu_e}{1+\alpha_e^2}\left(E_{1z} - \alpha_e E_{1y}\right) \tag{5.4}$$

$$V_{1ye} = \frac{\mu_e}{1+\alpha_e^2}\left(E_{1y} - \alpha_e E_{1z}\right) \tag{5.5}$$

where mobility μ_e is negative for electrons and

$$\alpha_e = \mu_e B_0 \tag{5.6}$$

Here the relations

$$i_{1e} = \rho_{oe} V_{1e} + \rho_{1e} V_{ie} \tag{5.7}$$

and continuity equation

$$\nabla . i_{1e} + jw\rho_{1e} = 0 \tag{5.8}$$

It is assumed that all quantities vary as exp. $\left[j(\omega t-\beta z)+\gamma y\right]$.

It may then be shown for electron charge density using equations (5.1), (5.4), (5.5), (5.7) and (5.8) that

$$\rho_{1e}=\frac{\omega_{ce}^{*}\in\left(\gamma^{2}-\beta^{2}\right)}{j\left(\omega-\beta_{oe}\right)}\phi_{1} \tag{5.9}$$

Where γ is transverse wave number in Y-direction. A similar relation for hole charge density can be obtained as

$$\rho_{1h}=\frac{\omega_{ch}\in\left(\gamma^{2}-\beta^{2}\right)}{j\omega}\phi_{1} \tag{5.10}$$

where

$$\omega_{ce}=\frac{\mu_{e}\rho_{oe}}{\in},\omega_{ch}=\frac{\mu_{h}\rho_{oh}}{\in}\text{ and }\omega_{ce}^{*}=\frac{\omega_{ce}}{1+\alpha_{e}^{2}}$$

It has been assumed that $\left|\beta V_{oh}\right|<<\omega$ and $\mu_{h}B_{0}=\alpha_{h}<<1$ which is reasonable for InSb. Both ω_{ce} and ω_{ch} are positive whereas μ_e is negative.

Poisson's equation taken the form

$$\left(\gamma^{2}-\beta^{2}\right)\phi_{1}=\frac{-\left(\rho_{1e}+\rho_{1h}\right)}{\in} \tag{5.11}$$

Putting values of ρ_{1e} and ρ_{1h} from equations (5.9) and (5.10) in equation (5.11) one has

$$\left(\gamma^{2}-\beta^{2}\right)\left\{1+\frac{\omega_{eh}}{j\omega}+\frac{\omega_{ce}^{*}}{j\left(\omega-\beta V_{oe}\right)}\right\}\phi_{1}=0 \tag{5.12}$$

5.2.1 Carrier waves in Infinite semiconductor:

From equation (5.12) we find that

$$1+\frac{\omega_{eh}}{j\omega}+\frac{\omega_{ce}^{*}}{j(\omega-\beta V_{oe})}=0$$

a solution which is equivalent to that for waves in an infinite semiconductor.

Thus for carrier wave in an infinite semiconductor from equation (5.13) we have

$$\beta=\frac{\omega}{V_{0e}}\left[\left(1+\frac{\omega_{ce}^{*}}{\omega_{eh}}\right)-j\frac{\omega_{ce}^{*}\omega}{\omega_{eh}^{2}}\right] \tag{5.14}$$

The real and imaginary parts of above equation yield

$$\beta_r=\frac{\omega}{V_{0e}}\left[1+\frac{\omega_{ce}^{*}}{\omega_{eh}}\right] \tag{5.15}$$

and

$$\beta_i=\frac{\omega}{V_{0e}}\frac{\omega_{ce}^{*}}{\omega_{eh}^{2}} \tag{5.16}$$

The equations (5.15) and (5.16) represent phase constant and attenuation constant in case of bulk carrier wave.

5.2.2 Theory of Surface wave:

Other solution of equation (5.12) i.e.

$$\gamma^2-\beta^2=0 \tag{5.17}$$

is therefore of interest. Equation (5.17) implies that

$\rho 1e = \rho 1h = 0$ and $\nabla^2\phi_1 = 0$

From equation (5.11) inside the semiconductor we may write

$$\phi_{1A} = Ae^{-j\beta z}.e^{\beta y}$$

$$E_{1yA} = -\frac{\partial\phi_{1A}}{\partial y} = -A\beta e^{-j\beta z}$$

$$E_{1zA} = -\frac{\partial\phi_{1A}}{\partial z} = -j\beta e^{-j\beta z}.e^{\beta y} \tag{5.18}$$

Here $\gamma = \beta$ from equation (5.17).

We find similarly for outside the conductor as

$$\phi_{1\beta} = Be^{-j\beta z}.e^{-\beta y}$$

$$E_{1yB} = -\frac{\partial\phi_{1B}}{\partial y} = -B\beta e^{-j\beta z}.e^{-\beta y} \tag{5.19}$$

$$E_{1zB} = -\frac{\partial\phi_{1B}}{\partial z} = -jB\beta e^{-j\beta z}.e^{-\beta y}$$

y has been taken –ve inside the semiconductor. The boundary conditions to be used at the surface are those described and discussed by Hahn and Kino for semiconductor. These are

$$\phi_{1A} = \phi 1B \tag{5.20}$$

and

$$\in_0 E_{1yB} - \in E_{1yA} = \rho_{1se} + \rho_{1sh} \tag{5.21}$$

Following Hahn, we write

$$\rho_{1se} = \rho_{oe} y_{1e} = \frac{\rho_{oe} V_{1ye}}{j(\omega - \beta V_{oe})} \tag{5.22}$$

$$\rho_{1sh} = \rho_{oh} y_{1h} = \frac{\rho_{oh} V_{1yh}}{j(\omega - \beta V_{oh})} \tag{5.23}$$

where y_1 is the displacement of electrons or holes in the Y direction near the surface of the semiconductor. For holes equivalent of equations (5.5) is written in similar fashion as

$$V_{1yh} = \frac{\mu_h}{1+\alpha_h^2}\left(E_{1y} + \alpha_h E_{1z}\right) \tag{5.24}$$

Substitution of equations (5.5) and (5.24) in equations (5.22) and (5.23) yield.

$$\rho_{1se} = \frac{\rho_{oe} . \mu_e}{j(\omega - \beta V_{oe})(1+\alpha_e^2)}\left(E_{1y} + \alpha_e E_{1z}\right) \tag{5.25}$$

$$\rho_{1sh} = \frac{\rho_{oh} \mu_h \left(E_{1y} + \alpha_h E_{1z}\right)}{j(\omega - \beta V_{oh})(1+\alpha_h^2)} \tag{5.26}$$

On the surface of semiconductor (i.e. y = 0) equations (5.18) to (5.19) reduce to

$\phi_{1A0} = Ae^{-j\beta z}$

$E_{1yAo} = -A\beta e^{-j\beta z}$

$E_{1zA0} = jA\beta e^{-j\beta z}$ (5.27)

and

$\phi_{1B0} = Be^{-j\beta z}$

$E_{1zB0} = -B\beta e^{-j\beta z}$ (5.28)

$E_{1zB0} = jA\beta e^{-j\beta z}$

Subscript 0 symbolizes quantities on the surface of conductor. From equation (5.2) we have

$$\phi_{1A0} = \phi_{1B0} \tag{5.29}$$

which means

$$A = B \tag{5.30}$$

Using equations (5.27), (52.8) and (5.30) we get relations

$E_{1zA0} = jE_{1yA0}$

$E_{1zB0} = -jE_{1yB0}$

$$E_{1yA0} = E_{1yB0} \tag{5.31}$$

$E_{1zA0} = jE_{1zB0}$

Putting values of ρ_{1se} and ρ_{1sh} from equations (5.25) and (5.26) equation (5.21) and using equation (5.31) we have

$$\in_0 - \in = \frac{\rho_{oe}\mu_e(1 - j\alpha_e)}{j(\omega - \beta V_{oe})(1 + \alpha_e^2)} + \frac{\rho_{oh}\mu_h(1 - j\alpha_h)}{j(\omega - \beta V_{oh})(1 + \alpha_h^2)} \tag{5.32}$$

which after simplification becomes

$$1 - \frac{\in_0}{\in} + \frac{\omega_{eh}^*}{j\omega} + \frac{\omega_{eh}^*(1 - j\alpha_e)}{j(\omega - \beta V_{oe})} = 0 \tag{5.33}$$

If we assume that $\frac{\omega_{eh}}{\omega} >> 1 + \frac{\in_0}{\in}$, the following result is obtained.

$$\beta = \frac{\omega}{V_{oe}}\left[\left(1+\frac{\omega_{eh}^{*}}{j\omega}\right) - j\frac{\alpha_e \omega_{eh}^{*}}{j\omega}\right] \tag{5.34}$$

The real and imaginary parts of above equation yields

$$\beta = \frac{\omega}{V_{oe}}\left[1+\frac{\omega_{eh}^{*}}{\omega_{ch}}\right]$$

(5.35)

and

$$\beta_r = \frac{\omega}{V_{oe}}\frac{\alpha_e \omega_{eh}^{*}}{j\omega} \tag{5.36}$$

Similarly using exact calculation expression for phase constant is

$$\beta_r = \frac{\omega}{V_{oe}}\left\{1+\frac{\alpha_e \omega_{eh}^{*}\left[\in \omega_{eh} - \alpha_e \omega(\in + \in_0)\right]}{\in^2 \omega_{eh}^2 + \omega^2 (\in + \in_0)^2}\right\} \tag{5.37}$$

Exact solution of equation (5.33) yield expression for growth rate as

$$\beta_r = \frac{\omega \in \omega_{eh}^{*}}{V_{oe}}\left[\frac{\omega(\in + \in_0) + \alpha_e \in \omega_{eh}}{\in^2 \omega_{eh}^2 + \omega^2 (\in_0 + \in)^2}\right] \tag{5.38}$$

5.3 Results and Discussion:

The dispersion relation equation (5.34) describes a surface wave in a nominally p-type InSb and expressions for phase constant and wave growth are derived in equations (5.35) and (5.36) for surface waves and in equations (5.15) and (5.16) for carrier waves in infinite semiconductor. It will be seen that both cases predict a carrier wave with a phase velocity $V_{oe} / \left(1+\omega_{ee}^{*} / \omega_{eh}\right)$. the value of phase constant has been computed using equation (5.15) in case of nominally p-type InSb bar for appropriate parameters and the resulting graph for phase constant with frequency for both infinite carrier and surface waves have been predicted in Figure (5.1). The growth rate for surface wave has been computed by using equation (5.36) for appropriate parameters and shown in Figure. (5.2). This wave grows with a growth rate per wavelength equal to $2\pi|\alpha_e|\omega_{ee} / \omega_{sh}(1+\alpha_e^2)$ if α_e is negative due to particular direction of magnetic field. From equation (5.36), the gain is maximum, where $|\alpha_e| = 1$.

The sign of magnetic field required for growing is such that holes and electrons will tend to move away from the top surface. On the other hand, the wave would be strongly attenuated if the magnetic field were reversed.

It has be noted in this theory that even if the first two terms in equation (5.33) are retained, there is always growth. Surface wave theory predicts a growing wave with a large growth rate whereas the infinite semiconductor theory predicts a wave with a small loss shown in Figure. (5.3).

Diffusion effect which are important, have not been taken into account. Diffusion tends to cause high loss at low drift velocities which should be decreased by the effect of a transverse magnetic field.

It is expected that by inclusion of diffusion effect, there would be a threshold velocity for gain and a higher magnetic field for maximum gain. Also it appears from equation (5.18) that carrier wave amplitude decrease away from the surface. This is a new type of interaction between electrons and holes which leads to a growing surface wave in the presence of a transverse magnetic field.

The basic reasons for this interaction are that the velocity of electrons moving towards the surface of a semiconductor depends, on the presence of a transverse magnetic field, on both the normal and parallel components of electric field.

In this situation, the two components are 90^0 out of phase with each other, so the electron current in a particular direction may not necessarily be in phase with the field in the same direction. This can lead to growing waves.

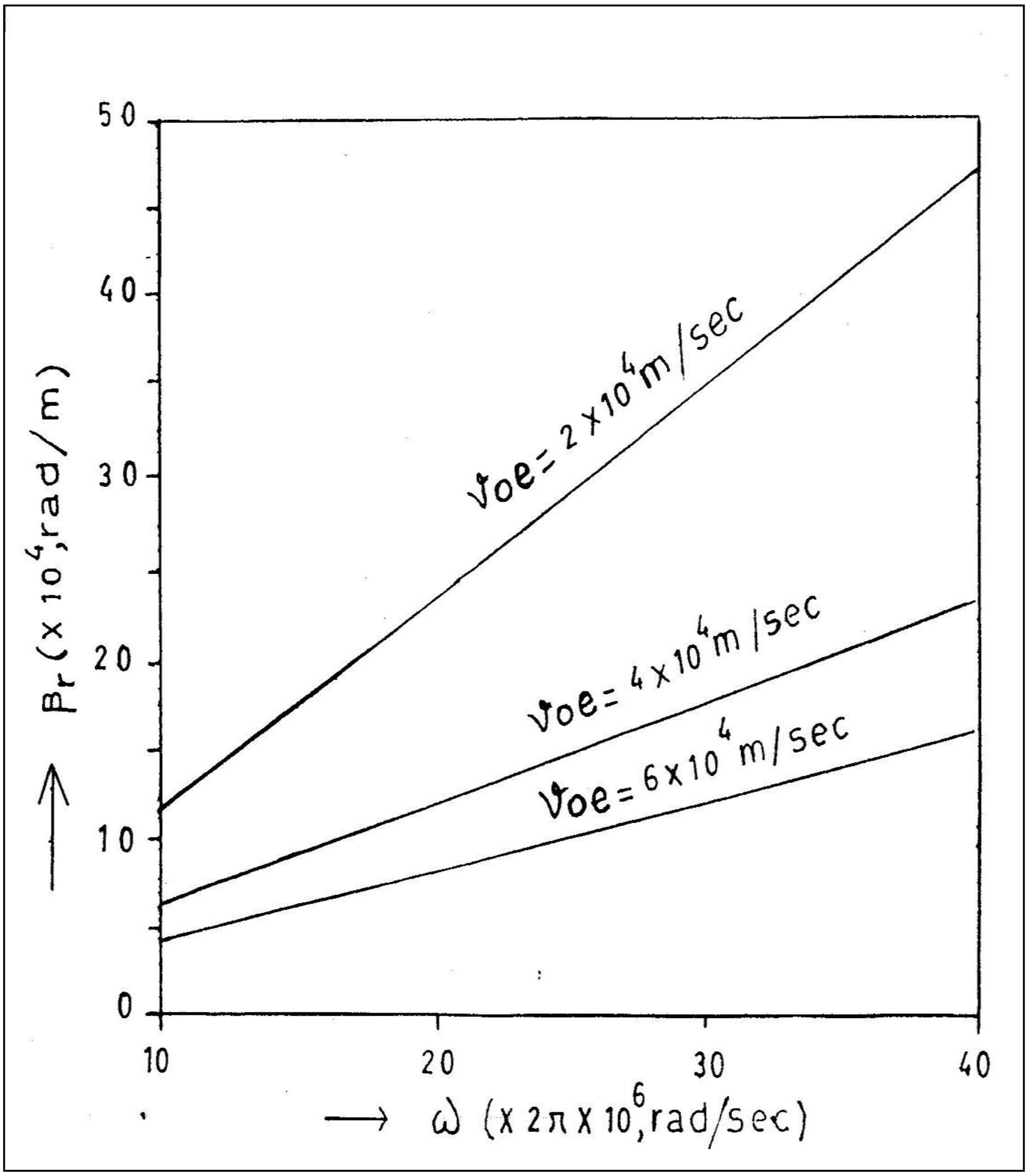

Figure 5.1: Variation of Phase Constant with Frequency of Surface Wave and Carrier Wave.

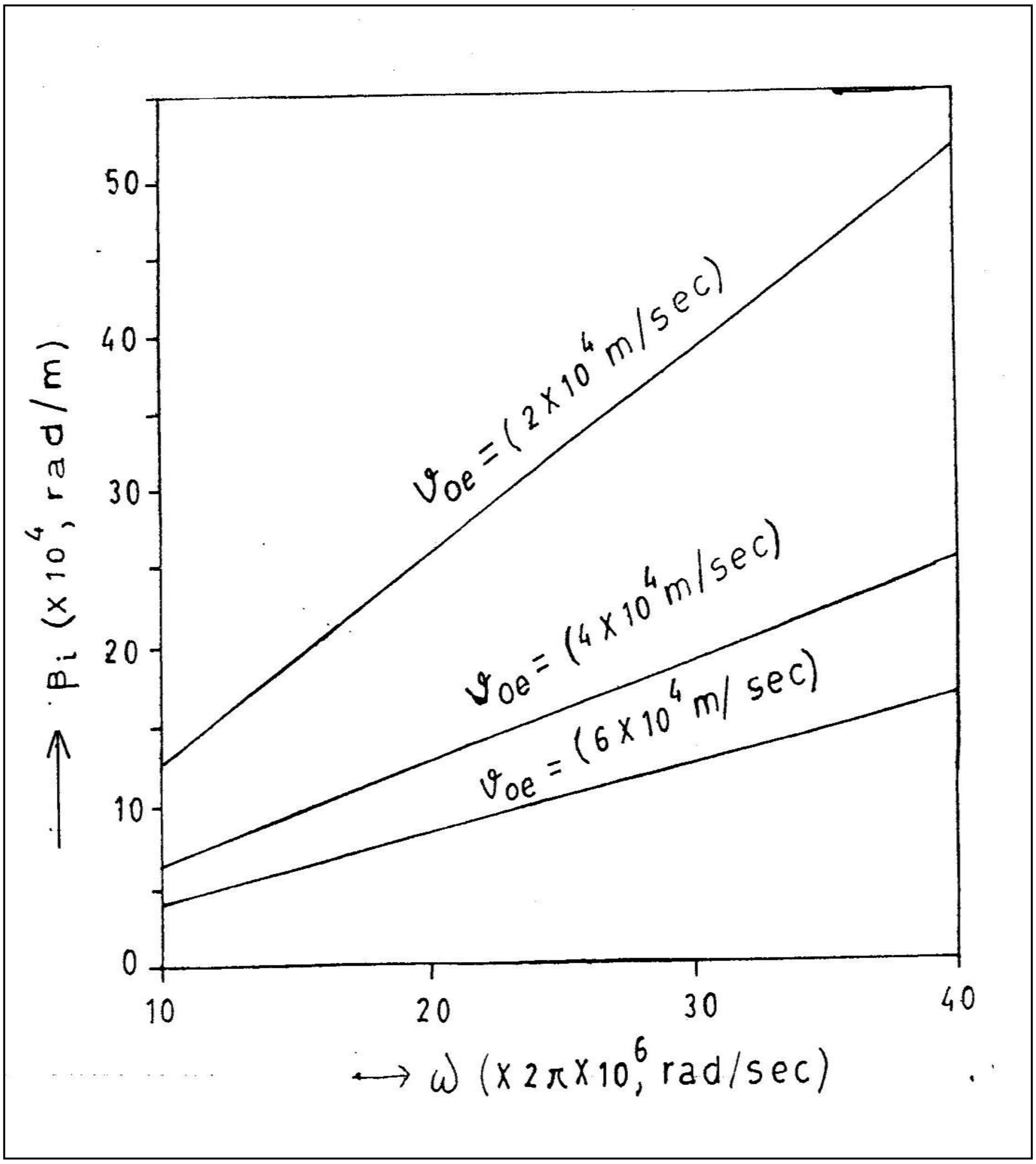

Figure 5.2: Variation of Growth Rate with Frequency of Surface Wave.

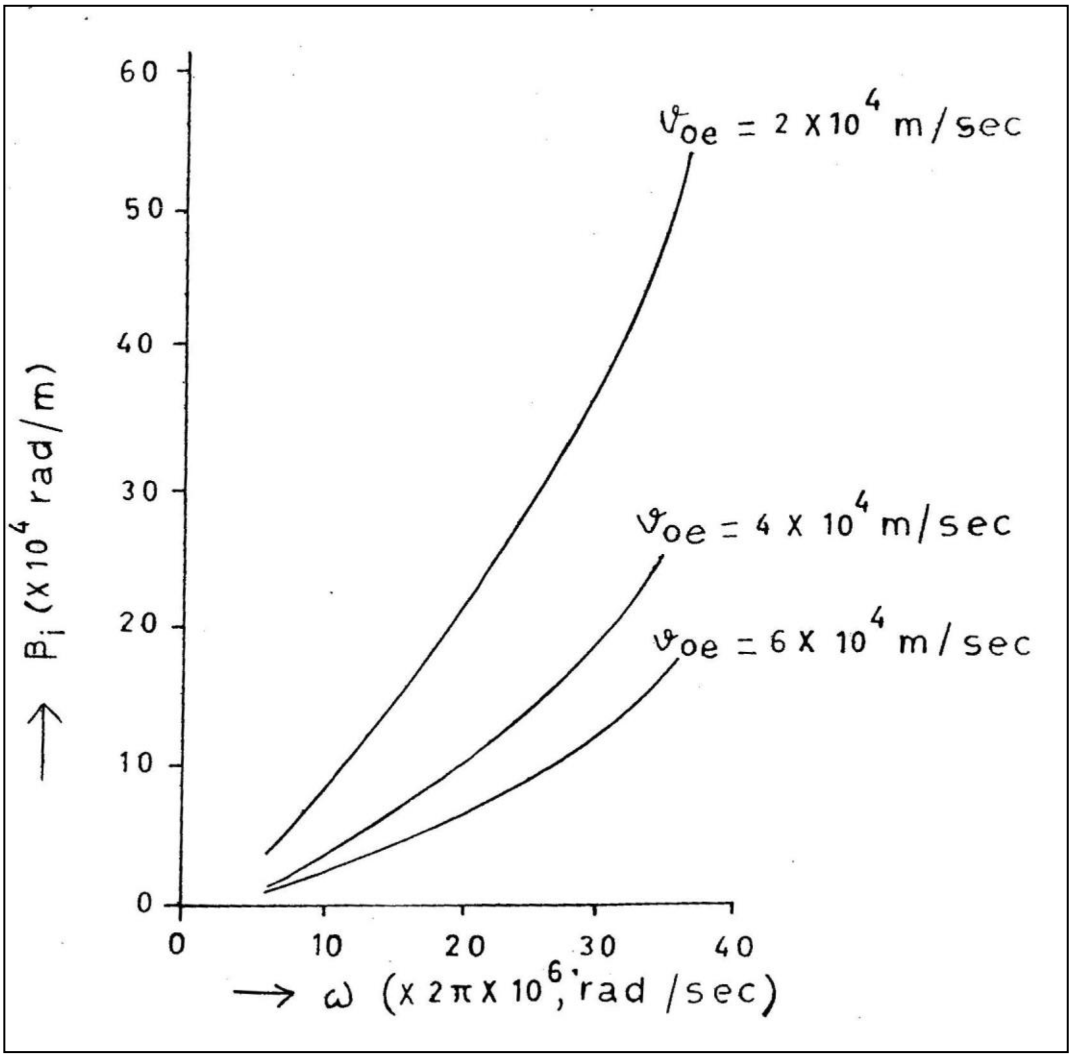

Figure 5.3: Variation of Wave Loss with Frequency of Carrier Wave in Infinite Semiconductor.

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Chapter 6

Two Stream Instability Due to Transferred Electron Effects in N-Type Gaas Semiconducting Plasma Having Negative Differential Conductivity

6.1 Introduction:

During high field experimental work in semiconductor like germanium and silicon [Ryder and Shockely (1951) and Ryder (1953)], no negative differential bulk resistance was observed. The difficulty was principally due to not being able to get a sufficient number of electrons to populate the negative mass states. Optical phonon scattering became the dominant energy-loss mechanism at high fields as it restricted the average energy of electrons to values far below the inflection point in velocity vs electric field curve. Other negative mass proposals in n-type germanium and p-type silicon [Kroemer (1959), Dousmanish (1960)] were also unsuccessful because of the isotropic nature of acoustic scattering. After inventing the transistor, Shockley suggested in 1954 [Shockley (1954)] that two terminal negative resistance devices using semiconductors may have advantage over transistors at high frequencies.

The next step in the search for a negative differential resistance wave given in the search for a negative differential resistance were given in independent theoretical papers by Ridley and Watkins (1961) and Hislum (1962). They described a new method for obtaining negative differential mobility in semiconductors. The principle involved is to heat carriers in a light mass, high mobility sub-band with an electric field so that the carriers can transfer to a heavy mass, low-mobility, higher energy sub-band when they have a high enough temperature. Ridley and Watkins also mentioned structures in the conduction bands. Their theory for achieving negative differential mobility in bulk semiconductors by transferring electrons from high mobility energy bands to low mobility energy bands was taken a step further by Hilsum in 1962 ([Hilsum (1962)].

There are bulk devices where microwave amplification and oscillation are derived from the bulk negative resistance property of uniform semiconductors rather than from the junction negative resistance property between two different semiconductors. J.B. Gunn (1963) discovered a periodic fluctuations of current passing through the n-type gallium arsenide (GaAs) Specimen when the applied voltage exceeded a certain critical value.

Two years later B.C. DeLoach, R.C. Johnston and B.G. Cohen discovered the impact ionization avalanche transit time (IMPATT) mechanism in silicon, which employs the avalanching and transit time properties of the diode to generate microwave frequencies. In later years the limited space charge accumulation diode and the Indium phosphide diode were also successfully developed. We consider a semiconductor having a conduction both with two minima separated by an energy difference.

The lower energy minimum has electrons with a small effective mass and high mobility, whereas the upper energy minimum has electrons with high effective masses and lower mobilities. Initially all the electrons, with density n, are located in the lower minimum. As the applied electric field increases, the current density would increase linearly if all the electrons remained in lower energy minimum. If one imagines all the electrons to have been in the upper minimum, the current density would increase with a smaller slop.

Actually as the electric field in increased, the electrons in lower minimum becomes heated, their temperature thus exceeds the lattice temperature, and at a sufficiently high electric field, some of electrons are transferred from lower minimum to upper minimum. If the transfer is sufficiently rapid with increasing electric field, a negative- differential resistance region in the characteristic curve is realized. Ridley and Watkins (1961) and Hilsum (1962) considered the III-V compound semiconductors to have the proper conduction band configuration for a transferred electron negative differential resistance. In particular, gallium arsenide seemed to have the desired properties. Hilsum Estimated that the threshold field for GaAs would be about 3KV/cm. This was later confirmed by the classic experiments of Gunn (1963). Electron transfer model was ultimately confirmed by Hutson et.al. (1965) using hydrostatic pressure to decrease the intervalley energy separation. In this chapter small signal behavior of waves propagating in a medium having a negative differential resistance arising from the electron-transfer mechanism is studied.

The expression for phase velocity and growth rate have been derived and numerically evaluated in terms of parameters of lower and upper valley conduction bands. The equivalence of the two stream instability concept to that of a negative differential resistance medium is established.

In the strong signal limit, the negative differential resistance resulting from transferred electron behavior [Ridley and Watkins (1961) and Hilsum (1962)] in solid such as gallium arsenide (GaAs) leads to the Gunn effect. Butcher (1967) reviewed the detailed study of negative differential resistance in solids. Hilsum (1962) carefully calculated the transferred electron effect in several III-V compounds and was the first to use the terms transferred electron amplifiers (TEAs) and oscillators (TEOs). He predicted accurately that a TEA bar of semi-insulating GaAs would be operated at 373 K at a field of 3200 V/cm. Hilsum's attempt to verify his theory experimentally failed because the GaAs diode available to him at that time was not of sufficiently high quality. A few years to him at that time was not of sufficiently high quality. A few years before the Gunn effect Kroemer (1958) and (1959) proposed a negative mass microwave amplifier, In 1963 J.B. Gunn discovered the Gunn effect from thin disks of n-type GaAs and ny-type InP specimens while studying the noise properties of semiconductors. Ridley (1963) predicted that the field domain is continually moving down through the crystal, disappearing at the anode and then reappearing at a favoured nucleating centreand starting the whole cycle once more. Kroemer (1964) stated that the origin of the negative differential mobility is Ridley- Watkins- Hilsum's mechanism of electron transfer into the satellite valleys that occur in the conduction band of both the n-type GaAs and the n-type InP and the properties of the Gunn effect are the current oscillations caused by the periodic nucleation and disappearance of travelling space charge instability domains. When positive and negative charges are separated by a small distance, then a dipole domain is formed. The electric field inside the dipole domain would be greater than the fields on either side of the dipole Because of the negative differential resistance, the current in the low field side would be greater than that in the high-field side. The two field values will tend toward equilibrium conditions outside the differential negative resistance region, where the low and high currents are the same. Then the dipole field reaches a stable condition and moves through the specimen towards the anode. When the high-field domain disappears at the anode, a new dipole field starts forming at the cathode and the process is repeated. Since starts forming at the cathode and the process is repeated.

Since Gunn first announced his observation of microwave oscillation the n-type GaAs and n-type InP diodes in 1963, various modes of operation have been developed, depending on the material parameters and operating conditions. The formation of a strong space-charge instability depends on the condition that enough charge in available in the crystal and the specimen is long enough, so that the necessary amount of space charge can be built up within the transit time of the electrons. This requirement sets up a criterion for the various modes of operation of bulk negative differential resistance devices. The pure accumulation layer is the simplest form of space charge instability. In the strong signal limit formation of a high field domain responsible for microwave generation and amplification due to a decrease in drift velocity with increasing electric field is discussed in case of multivalley semiconductor compound, such as the n-type GaAs. In this section time rate of growth of space charge accumulation arising out of space charge instability has been discussed for the sample under consideration.

6.2 Theoretical Considerations:

We consider here the specific case of a transferred electron medium from the view point of a two stream instability. Electron densities in the lower and upper valleys remain the same under an equilibrium conditions. When the applied electric field is lower than the electric field of the lower valley, no electrons will transfer to the upper valley. When the applied electric filed is higher than that of the lower valley and lower than that of the upper valley, electrons will begin to transfer to the upper valley. When the applied electric field is higher than that of the upper valley, all electrons will transfer to the upper valley. If electron densities in the lower and upper valleys are n_1 and n_2, the conductivity of the n-type GaAs is

$$\sigma = e\,(\mu_1 n_1 + \mu_2 n_2) \qquad (6.1)$$

where e is the electron charge, μ_1, μ_2, n_1 and n_2 are mobilities electron densities corresponding to lower valley and upper valley.

When a sufficiently high field E is applied to the specimen, electrons are accelerated and their effective temperature rises above the lattice temperature.

Furthermore, the lattice temperature also increases. The electron density and mobility are both functions of electric field. Differentiating equation (6.1) with respect to E yields

$$\frac{d\sigma}{dE} = e\left(\mu_1 \frac{dn_1}{dE} + \mu_2 \frac{dn_2}{dE}\right) + e\left(n_1 \frac{d\mu_1}{dE} + n_2 \frac{d\mu_2}{dE}\right) \tag{6.2}$$

If the total electron density is given by $n_0 = n_1 + n_2$ and it is assumed that μ_1 and μ_2 are proportional to E^b, where b is a constant, then

$$\frac{d}{dE}(n_1 + n_2) = \frac{dn}{dE} = 0 \tag{6.3}$$

$$\frac{dn_1}{dE} = -\frac{dn_2}{dE} \tag{6.4}$$

and

$$\frac{d\mu}{dE} \alpha \frac{dE^b}{dE} = bE^{b-1} = \frac{bE^b}{E} \alpha \frac{b\mu}{E} \tag{6.5}$$

Substituting equations (6.3) to (6.5) into equation (6.2) results in

$$\frac{d\sigma}{dE} = e(n_1 - n_2)\frac{dn_1}{dE} + e(n_1\mu_1 + m_2\mu_2)\frac{b}{E} \tag{6.6}$$

Differentiating Ohm's law $J = \sigma E$ with respect to E yields

$$\frac{dJ}{dE} = \sigma + \frac{d\sigma}{dE}E \tag{6.7}$$

or

$$\frac{dJ}{\sigma dE} = 1 + \frac{d\sigma / dE}{\sigma / E} \tag{6.8}$$

For negative resistance, the current density J must decrease with increasing field or the ratio dJ/dE must be negative i.e. right hand side of equation (6.8) is less than zero. In other words, the condition for negative resistance is

$$\frac{d\sigma / dE}{\sigma / E} > 1 \tag{6.9}$$

Putting values from equations 6.1) and (6.6), equation (6.9) becomes

$$\left[\left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2 f}\right)\left(-\frac{Edn_1}{n_1 dE}\right) - b\right] > 1 \tag{6.10}$$

The field exponent b is a function of the scattering mechanism and should be negative and large in order to satisfy the unequality. When impurity scattering is dominant, the mobility rises with increasing field making b positive. When lattice scattering is carrier temperature. The first bracket in equation (6-10) must be positive i.e.$\mu_1 > \mu_2$. Electrons must begin in a low mass valley and transfer to a high mass valley when they are heated by the electric field. The maximum value of this term is unity i.e. $\mu_1 \gg \mu_2$. The factor $\frac{dn_1}{dE}$ in the second bracket must be negative. This quantity represents the rate of the carrier density with field at which electrons transfer to the upper valley and depends on differences between electron densities, electron temperature and gap energies in the two valleys.

6.2.1 Small Signal Limit:

The transferred electron mechanism in n-GaAs under consideration creats the two streams. One dimensional model and the response of such medium to a periodic perturbation of the type exp j(ωt –kz) is assumed. The field dependence of n_1 and n_2 is assumed of the following form:

$$n_1 = \frac{n_0}{1 + a\left(\frac{E_0}{E_t}\right)^b} \tag{6.11}$$

$$n_2 = n_0 \frac{\alpha\left(\frac{E_0}{E_t}\right)^b}{1+\alpha\left(\frac{E_0}{E_t}\right)^b} \tag{6.12}$$

Where E_0 is the applied dc electric field, E_t is the threshold electric field and a and b are appropriate constants, with b≫1. Hence, as E_0/E_1 is increased, more and more electrons are transferred from the lower to the higher energy valley of the conduction band.

At a given applied electric filed E_0 such that E_0/E_1 >1, there are significant numbers of electrons in both valleys, drifting with velocities v_1 and v_2 where v_1 ≫v_2. Hence we have two-stream system consisting of electrons in n-GaAs solid state plasma. The response of such a system to a periodic perturbation is given by the dispersion relation for two streams as

$$1 - \frac{\omega_{p1}^2}{(\omega - kv_1 - j\upsilon_{\gamma 1})(\omega - kv_1 - j\upsilon_1 - \upsilon_1 D_1 k^2)}$$

$$- \frac{\omega_{p2}^2}{(\omega - kv_2 - j\upsilon_{\gamma 2})(\omega - kv_2 - j\upsilon_2 - \upsilon_2 D_2 k^2)} = 0 \tag{6.13}$$

Where plasma frequency of electrons in the lower valley of conduction band

$$\omega_{p1} = \left(\frac{n_1 e}{\in_0 \in_L\ m_1}\right)^{1/2} \tag{6.14}$$

and plasma frequency of electrons in the upper valley of conduction band

$$\omega_{p2} = \left(\frac{n_2 e}{\in_0 \in_L\ m_2}\right)^{1/2} \tag{6.15}$$

The subscript 1 refers to quantities in the lower valley and the subscript 2 refers to quantities in the upper valley where n_1 and n_2 are given by equations (6.11) and (6.12).

Symbols v_1, v_2, $\upsilon_{\gamma 1}, \upsilon_{\gamma 2}, \upsilon_1, \upsilon_2, D_1, D_2$ m_1 and m_2 are drift velocities, recombination frequencies, collision frequencies, diffusion constants and mass of electrons in the lower and upper valleys respectively. The equation (6.13) was unstable (or growing) solutions, depending on parameters of the system. To make the equation analytically tractable, we make some simplifying yet realistic assumptions and thus assume

$\upsilon_{\gamma 1} = \upsilon_{\gamma 2} = 0$ $\qquad$ $D_1 = D_2 = 0$

$\upsilon 1 \gg \omega, kv_1$ $\qquad$ $\omega_{p2}^2 > \left(\omega^2 + \upsilon_2^2\right)$

and as a first approximation, we neglect recombination and diffusion effects. We also assume that $v2 \cong 0$, i.e. electrons in the upper valley are treated as a stationary background plasma in which the mobile electrons in the lower valley drift. The equation (6.13) then reduces to

$$1 - \frac{\omega_{p1}^2}{(\omega - kv_1)(-j\upsilon_1)} - \frac{\omega_{p2}^2}{\omega(\omega - j\upsilon_2)} = 0 \qquad (6.16)$$

Diving throughout by

$$\left[1 - \frac{\omega_{p2}^2}{\omega(\omega - j\upsilon_2)}\right]$$

we have

$$1 - \frac{j\omega_{p1}^2}{\upsilon_1(\omega - kv_1)\left[1 - \frac{\omega_{p2}^2}{\omega(\omega - j\upsilon_2)}\right]} = 0 \qquad (6.17)$$

$$1 - \frac{j\omega_{p_{eff}}^2}{\upsilon_1(\omega - kv_1)} = 0 \qquad (6.18)$$

where new effective plasma frequency is written as

$$\omega_{peff} = \frac{\omega_{p1}^2}{1 - \dfrac{\omega_{p2}^2}{\omega(\omega - j\upsilon_2)}} \tag{6.19}$$

$$k = \frac{\omega}{v_1} - \frac{j\omega_{peff}^2}{v_1\upsilon_1} \tag{6.20}$$

The solution describes a wave with a phase velocity equal to the drift velocity. In case of υ_2 = 0, the wave may be either growing or decaying depending on sign of $\omega_{p_{eff}}^2$. If $\omega_{p_{eff}}^2 < 0$, space charge growing and in case of $\omega_{p_{eff}}^2 > 0$ it decays.

However, in general $\upsilon_2 \neq 0$, the solution from equation (6.16) becomes

$$\omega - kv_1 = \frac{j\omega_{p1}^2\omega(\omega - j\upsilon_2)}{\upsilon_1\omega(\omega - j\upsilon_2) - \upsilon_1\omega_{p1}^2} \tag{6.21}$$

$$k = \frac{\omega\upsilon_1(\omega - j\upsilon_2) - \omega\omega_{p2}^2\upsilon_2 - j\omega_{p1}^2\omega(\omega - j\upsilon_2)}{(\omega^2\upsilon_1 v_1 - \omega_{p2}^2 v_1\upsilon_1) - j\omega v_1\upsilon_1\upsilon_2} \tag{6.22}$$

Real and imaginary parts of equation (6.22) have to be separated to analyse the quantities such as phase velocity and growth rate of space charge wave. Thus equation (6.22) becomes

$$\omega^5\upsilon_1^2 v_1 - 2\omega^3\omega_{p2}^2\upsilon_1^2 v_1 + \omega\omega_{p2}^4 v_1\upsilon_1^2 + \omega\omega_{p1}^2\omega_{p1}^2 v_1\upsilon_1\upsilon_2 + \omega^3 v_1\upsilon_1^2\upsilon_2^2$$

$$k = \frac{j\{\omega^2\omega_{p1}^2 V_1\upsilon_1(\omega^2 - \omega_{p2}^2 + \upsilon_2^2)\}}{\omega^4\upsilon_1^2 V_1^2 + \omega_p^4\upsilon_1^2 V_1^2 - 2\omega^2\omega_{p2}^2\upsilon_1^2 V_1^2 + \omega^2 V_1^2\upsilon_1^2\upsilon_2^2} \tag{6.23}$$

Taking real part

$$k_{\mathrm{Re}} = \frac{\omega^5 \upsilon_1^2 v_1 - 2\omega^3 \omega_{p2}^2 \upsilon_1^2 v_1 + \omega \omega_{p2}^4 v_1 \upsilon_1^2 + \omega \omega_{p1}^2 \omega_{p2}^2 v_1 \upsilon_1 \upsilon_2 + \omega^2 v_2 v_1 \upsilon_1^2 \upsilon_2^2}{\omega^4 \upsilon_1^2 v_1 + \omega^2 V_1^2 \upsilon_1^2 \upsilon_2^2 - 2\omega^2 \omega_{p2}^2 V_1^2 \upsilon_1^2 + \omega_{p2}^2 V_1^2 \upsilon_1^2} \tag{6.24}$$

$$k_{\mathrm{Re}} = \frac{\omega V_1 \upsilon_1^2 \left(\omega^4 - 2\omega^2 \omega_{p2}^2 + \omega_{p2}^2 + \omega^2 \upsilon_2^2\right)}{V_1^2 \upsilon_1^2 \left(\omega^4 + \omega^2 \upsilon_2^2 - 2\omega^2 \omega_{p2}^2 + \omega_{p2}^2\right)} + \frac{\omega \omega_{p1}^2 \omega_{p2}^1 \upsilon_2}{v_1 \upsilon_1 \left(\omega^4 2\omega^2 \omega_{p2}^2 + \omega^2 \upsilon_2^2 + \omega_{p2}^2\right)} \tag{6.25}$$

$$k_{\mathrm{Re}} = \frac{\omega}{v_1} + \frac{\dfrac{\omega_{p1}^2 \omega_{p2}^2 \upsilon_2}{\omega^3}}{v_1 \upsilon_1 \left\{ \left(\dfrac{\omega^2 - \omega_{p2}^2}{\omega^2} \right)^2 + \dfrac{\upsilon_2^2}{\omega^2} \right\}} \tag{6.26}$$

Phase velocity of space charge wave is then obtained as

$$V_{ph} = \frac{\omega}{k_{\mathrm{Re}}} = \frac{v_2}{1 + \dfrac{\left(\omega_{p1}^2 \omega_{p2}^2 \upsilon_2 / \omega^3\right)}{\omega \upsilon_1 \left(1 - \dfrac{\omega_{p2}^2}{\omega^2}\right)^2 + \dfrac{\upsilon_2^2}{\omega^2}}} \tag{6.27}$$

Taking imaginary parts of equation (6.23) we have

$$k_{1m} = \frac{\omega^2 \omega_{p1}^2 v_1 \upsilon_1 \left(\omega_{p2}^2 - \omega^2 - \upsilon_2^2\right)}{v_1^2 \upsilon_1^2 \omega^4 - 2 v_1^2 \upsilon_1^2 \omega^2 \omega_{p2}^2 + v_1^2 \upsilon_1^2 \omega_{p2}^4 + v_1^2 \upsilon_1^2 \upsilon_2^2 \omega^2} \tag{6.28}$$

The expression for growth rate is simplified as

$$k_{1m} = \frac{\omega_{p1}^2}{v_1 \upsilon_1} \frac{\left\lfloor \omega_{p2}^2 - \left(\omega^2 + \upsilon_2^2\right) \right\rfloor / \omega^2}{\left[\left(1 - \dfrac{\omega_{p2}^2}{\omega^2}\right)^2 + \dfrac{\upsilon_2^2}{\omega^2} \right]} \tag{6.29}$$

From equation (6.27) we see that phase velocity $v_p = \omega/k_{Re}$ is less than drift velocity v_1 of electrons. Further it is observed from equation (6.29) that growing wave is possible under the condition

$$\omega_{p2}^2 > \omega^2 + \upsilon_2^2$$

According to the energy band theory of the n-type GaAs, data for two valleys in GaAs and data for two valley semiconductors are given in table 6.1 and Table 6.2 respectively.

Table 6.1: Data for two valleys in GaAs

Valley	Effective Mass M_e	Mobility μ	Separation ΔE
Lower	M_{el} = 0.068	μ_1=8000cm^2/V–see	ΔE = 0.36 eV
Upper	M_{eu} = 1.2	μ_u =180cm^2/V–see	ΔE = 0.36 eV

Table 6.2: Data for two-valley semiconductor

Semiconductor	Gap energy (at 300^{0}K) Eg (eV)	Separation energy between two valleys ΔE (eV)	Threshold field Eth (KV/cm)	Peak velocity vp (10^7 cm/s)
Ge	0.80	0.18	2.3	1.4
GaAs InP	1.43 1.33	0.36 0.60 between M & L. 0.80 between U & L	3.2 10.5	2.2 2.5
CdTe	1.44	0.51	13.0	1.5
InAs	0.33	1.28	1.60	3.6
InSb	0.16	0.41	0.6	5.0

6.2.2 Strong Signal Limit:

We consider again a uniform n-type Gallium Arsenide sample with ohmic contacts at both the end surfaces. When a small voltage is applied to the diode, the electric field and conduction current density are uniform through the diode. At low voltage the GaAs is ohmic, since the drift velocity of the electrons is proportional to the electric field. The conduction current density in the specimen is given by

$$J = \sigma E = \sigma V/L = \rho v \quad (6.30)$$

Where

J	=	Conduction current density
σ	=	Conductivity
E	=	Electric field in the X-direction
L	=	Length of the diode
V	=	Applied Voltage
ρ	=	Charge density
v	=	Drift velocity

In the n-type GaAs diode the current is carried by majority free electrons that are drifting through a background of fixed positive charge. The positive charge, which is due to impurity atoms that have donated an electrons (donors), is sometimes reduced by impurity atoms that have accepted an electrons (donors).

As long as the fixed charge is positive, the semiconductor is n-type, since the principal (majority) carriers are the negative charges. The density of donor less the density of acceptors is termed doping. When the space charge is zero, the carrier density is equal to the doping.

When the applied voltage is above the threshold value, which was measured at about 400 V/cm times the thickness of the GaAs diode, a high field domain is formed near the cathode that reduces the electric field in the rest of the material and causes the current to drop to about two-thirds of its maximum value. For a constant voltage V an increase in the electric field within the specimen must be accompanied by decrease in the electric field in the rest of the diode.

The high field domain then drifts with the carrier stream across the electrodes and disappears at the anode contact. When the electric field increases, the electron drift velocity i.e. current density decreases and the GaAs exhibits negative resistance Figure (6.1a).

Specifically, it is assumed that at point A [Figure (6.1b)] there exists an excess or accumulation of negative charge that could be caused by a random noise fluctuation or possibly by a permanent non-uniformity in doping in the n-type GaAs diode. An electric field is then created by the accumulated charges [Figure (6.1d)].

The field to the left of point A is lower than that to the right. If the diode is biased at point E_A [Figure 6.1a)], this situation implies that the carriers (current) flowing into point A are greater than those flowing out of A, thereby increasing the excess negative space charge at A.

Furthermore, when the electric field to the left of point A is lower than it was before, the field to right is then greater than the original one resulting in an even greater space charge accumulation.

This process continues until the low and high fields both reach values outside the differential negative- resistance region and settle at points 1 and 2 in [Figure (6.1a)] where the currents in the two field regions are equal.

As a result of this process, a tavelling space- charge accumulation is formed. This process, of course, depends on the condition that the number of electrons insides the crystal is large enough to allow the necessary amount of space charge to be built up during the transit-time of the space charge layer.

The specimen under consideration is an n-type GaAs, with the concentrations of free electrons ranging from 10^{14} to 10^{17} per cubic centimeter at room temperature. Its typical dimensions are 150×150μm in cross section and 30 μm long.

During the early stages of space-charge accumulation, the time rate of growth of the space charge layers is given by

$$Q(x,t) = Q(x-vt, O) \ \exp \ en_0|\mu_n|t \qquad (6.31)$$

$$Q(x,t) = Q(x-vt, O) \ \exp \left(\frac{t}{\tau_d}\right)$$

where $\tau_d = \frac{\in}{\sigma} = \frac{\in}{en_0|\mu_n|}$ is the magnitude of the negative dielectric relaxation time, n_0 is doping concentration, $\in$ is semiconductor dielectric permittivity, μ_n is negative mobility, e is electronic charge and σ is conductivity.

If equation (6.31) remains valid throughout the entire transit time of the space charge layer, the factor of maximum growth is given by

$$\text{Growth rate} = \frac{Q(L, L/v)}{Q(O,O)} = \exp\left(\frac{Ln_0e|\mu_n|}{\in v}\right) \qquad (6.32)$$

If $x = \frac{Ln_0e|\mu_n|}{\in v}$ then

$$\text{Growth rate} = \exp(\times) = \exp \frac{L}{v\Gamma_d}$$

In equation (6.32) the layer is assumed to start at the cathode at t=0, × = 0, and arrive at the anode at t = L/v and X = L. For a large space- charge growth, this factor must be larger than unity. This means that

$$n_0 L > \frac{\in v}{e|\mu_n|} \tag{6.33}$$

When an electric field of a certain range is applied to a multivalley semiconductor compound such as the n-type GaAs, a decrease in drift velocity with increasing field in certain range can lead to the formation of a high field domain for microwave generation and amplification. It moves from a high field domain the cathode to the anode. The frequency of oscillation is given by the relation

$$f = \frac{V_{dom}}{L_{eff}} \tag{6.34}$$

Where V_{dom} is the domain velocity and L_{eff} is the effective length that domain travels from the time it is formed until the time that a new domain begins to form.

6.3 Result and Discussion:

In case of small signal limit equations (6.27) and (6.29) were computed for two conditions

(i) bias near threshold where

$\omega_{p1} = 86.4\times10^{10}$Hz

and $\omega_{p2} = 20.5\times10^{10}$Hz

(ii) high bias where

$\omega_{p1} = 8.64\times10^{10}$Hz

$\omega_{p2} = 29.1\times10^{10}$Hz

Variation of phase velocity with lower valley electron drift velocities is shown in Figures. (6.2) and (6.3) for near threshold and high bias conditions.

It is found that with increase in drift velocity, phase velocity increases although the two are nearly equal. Phase velocity is always found less than lower valley electron drift velocities.

Figures (6.4) and (6.5) show the variation of wave growth rate with lower valley electron drift velocities for different collision frequencies near threshold and high bias conditions.

It has been observed that growth rate decreases with lower valley electron drift velocities. For lower collision frequencies with curve is more steep whereas for higher collision frequencies the changes in growth rate is small.

Figureure (6.6) and (6.7) show the variation of growth rate with upper valley electron collision frequency near threshold and high bias conditions. Growth rate values near threshold are found much higher than the values computed for high bias condition.

It is observed from the graph that growth rate decreases with upper valley electron collision frequency and becomes zero. On further increasing collision frequency. attenuation of wave takes place.

In case of strong signal limit, the equation (6.32) has been computed for appropriate GaAs parameters to analyse the space charge growth. Parameters taken for calculations are as

$$\in \; = \; \in_0 . \in_L = 8.854 \times 10^{-12} \times 13.1 \qquad \text{Farad/m}$$

$e = 1.6 \times 10^{-19}$ Coulomb

$v_d = 2.5 \times 10^5$m/sec

$|\mu_n| = 0.015 m^2/v.s$

$L = 3 \times 10^{-5}$m

Variation of growth rate with varying doping concentration in the rage $10^{20}/m^3$ to $10^{23}/m^3$ has been depicted in Table 6.3. It is found from the table that the space charge growth increases exponentially with the doping concentration of electrons.

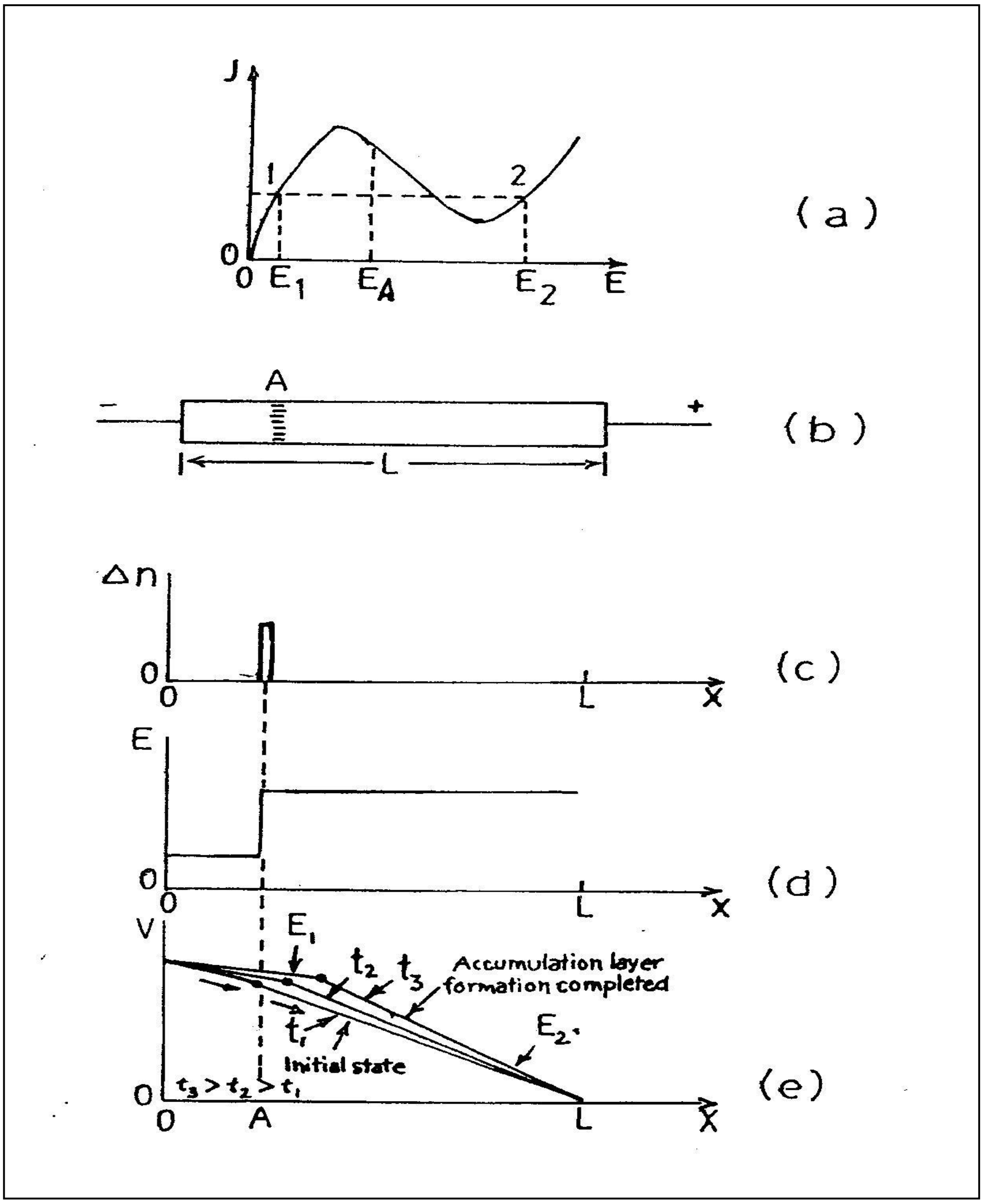

Figure 6.1: Formation of an Electron Accumulation Layer in GaAs.

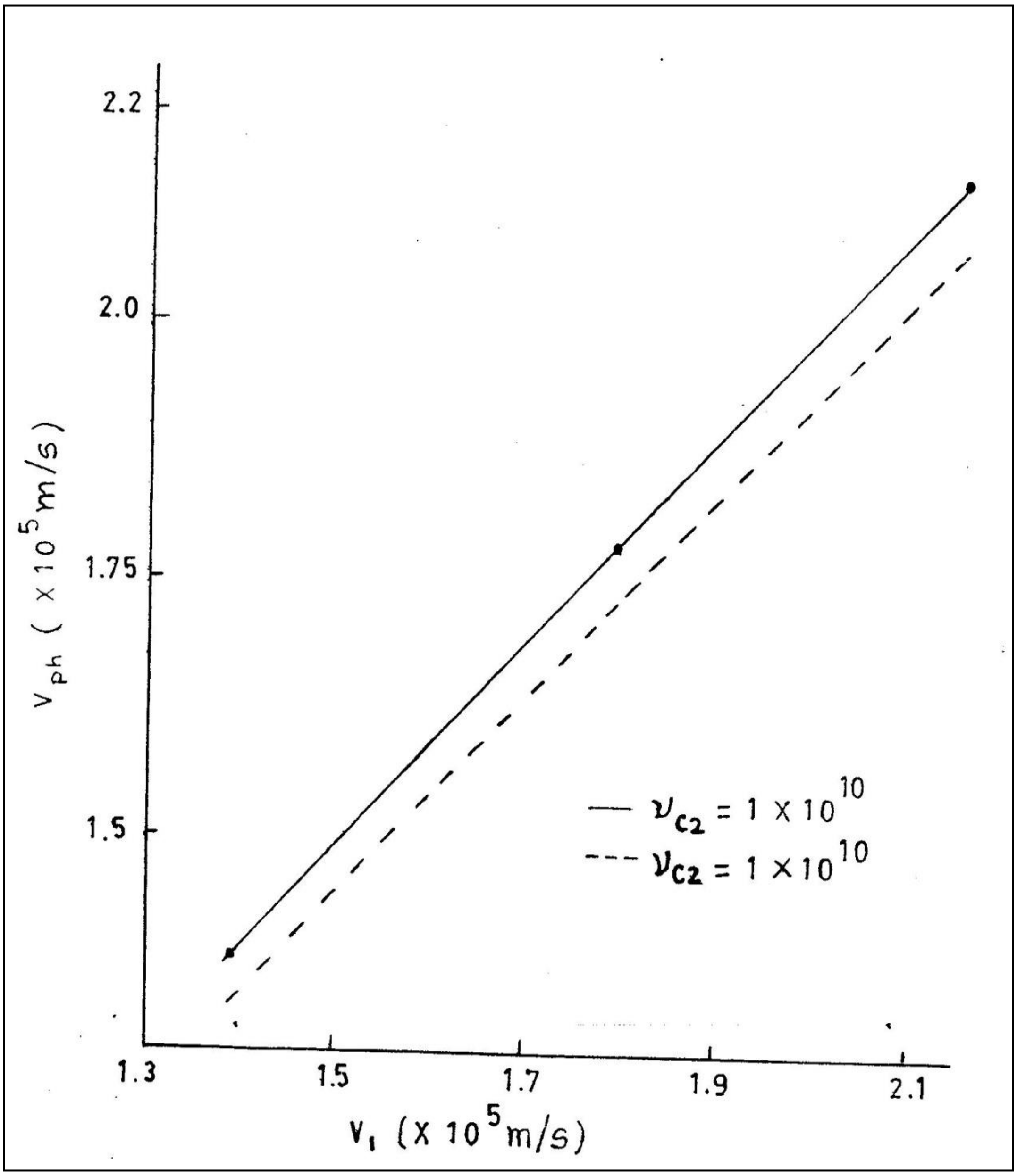

Figure 6.2: Variation of Phase Velocity With Lower Valley Electron Drift Velocity Near Threshold.

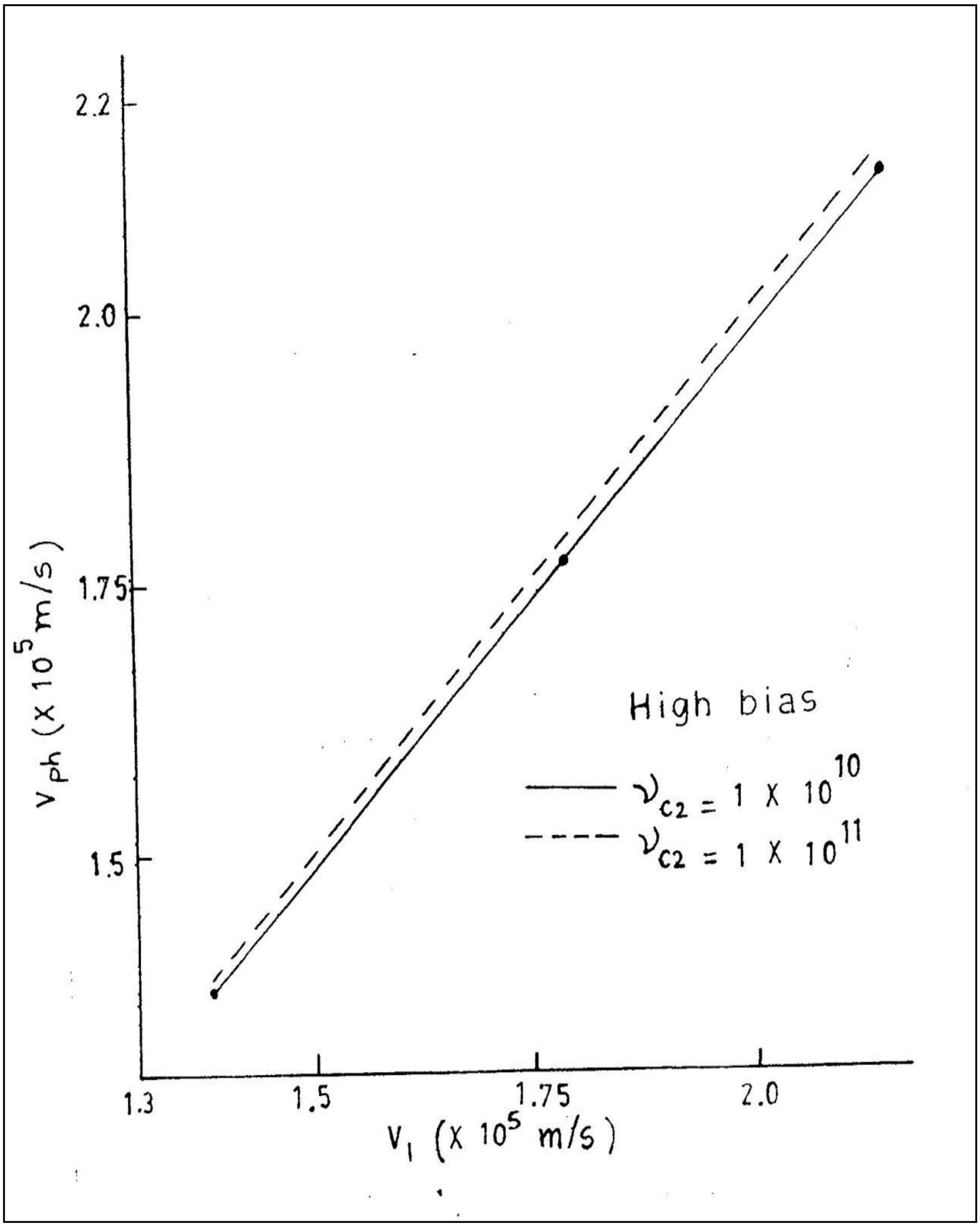

Figure 6.3: Variation of phase velocity with lower valley electron drift velocity for high bias.

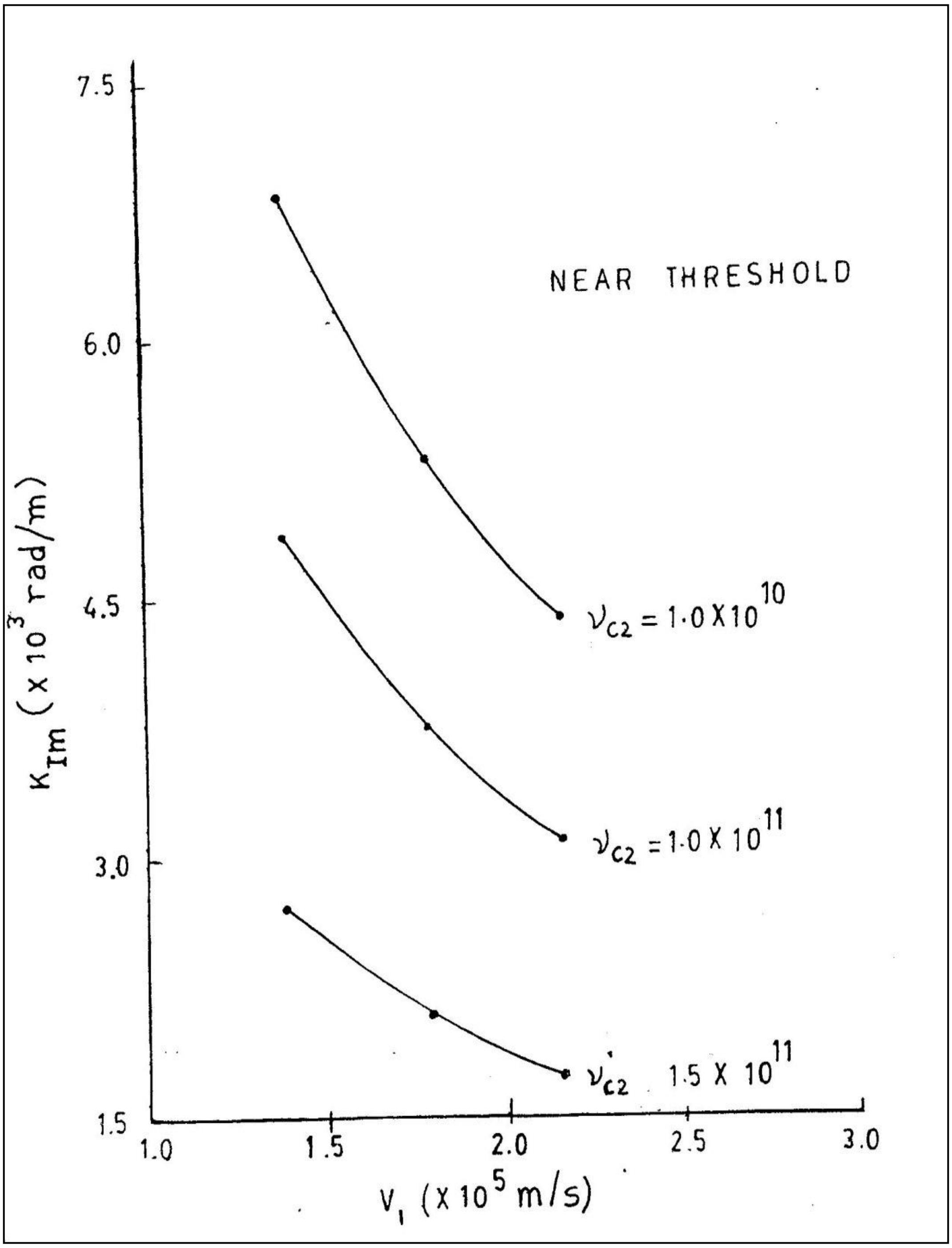

Figure 6.4: Variation of growth rate with lower valley electron drift velocity near threshold.

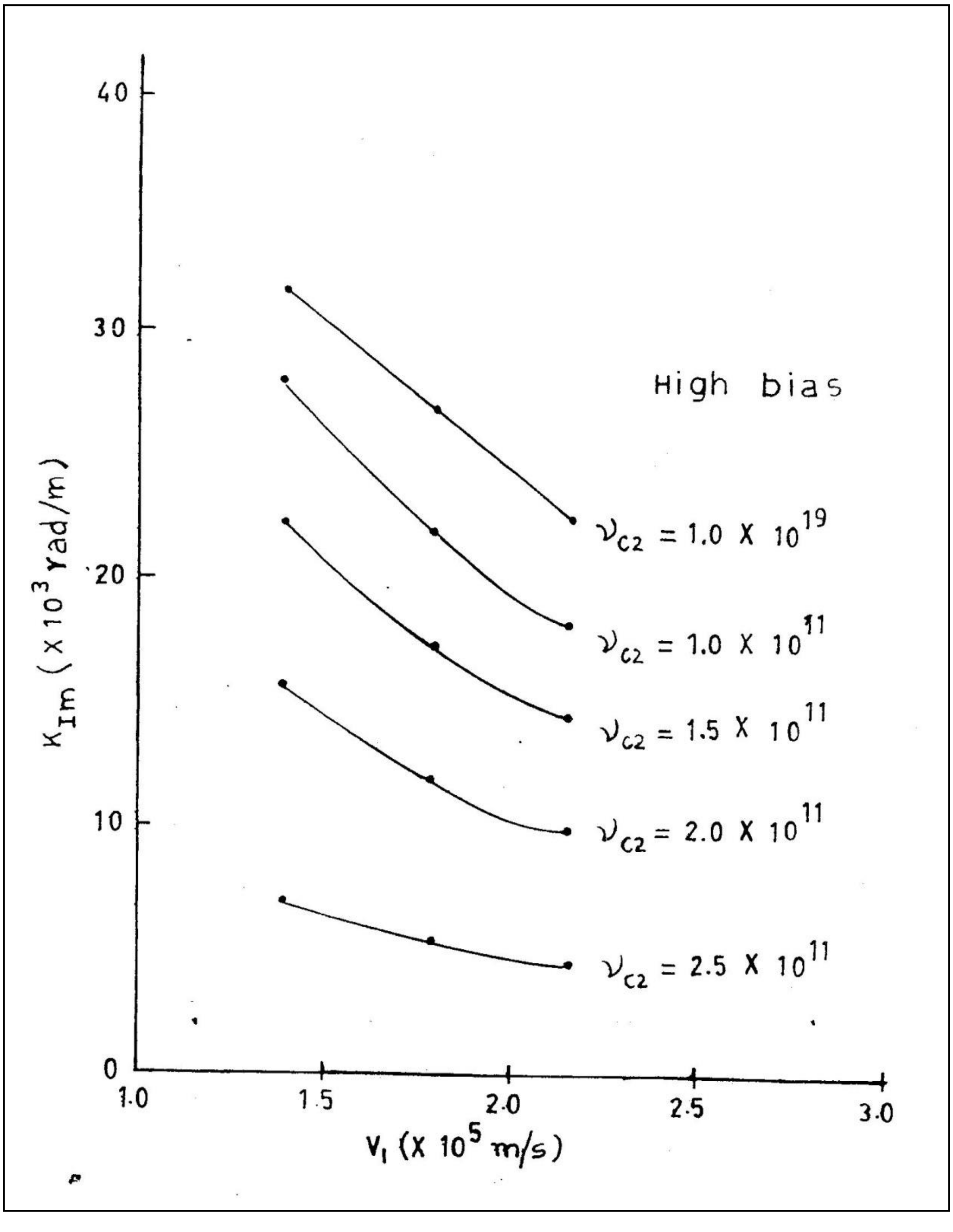

Figure 6.5: Variation of growth rate with lower valley electron drift velocity for high bias.

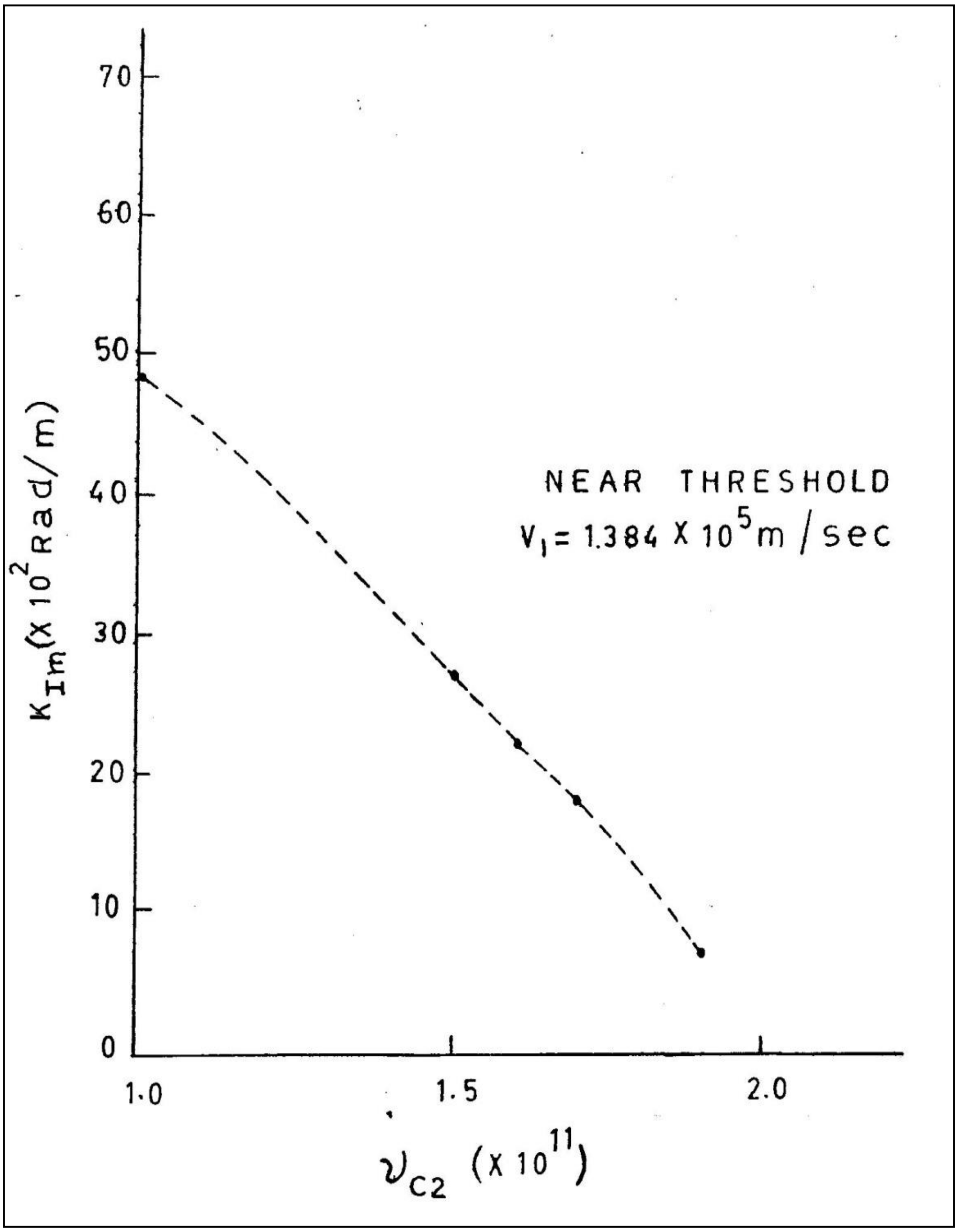

Figure 6.6: Variation of growth rate with upper valley electron collision frequency near threshold.

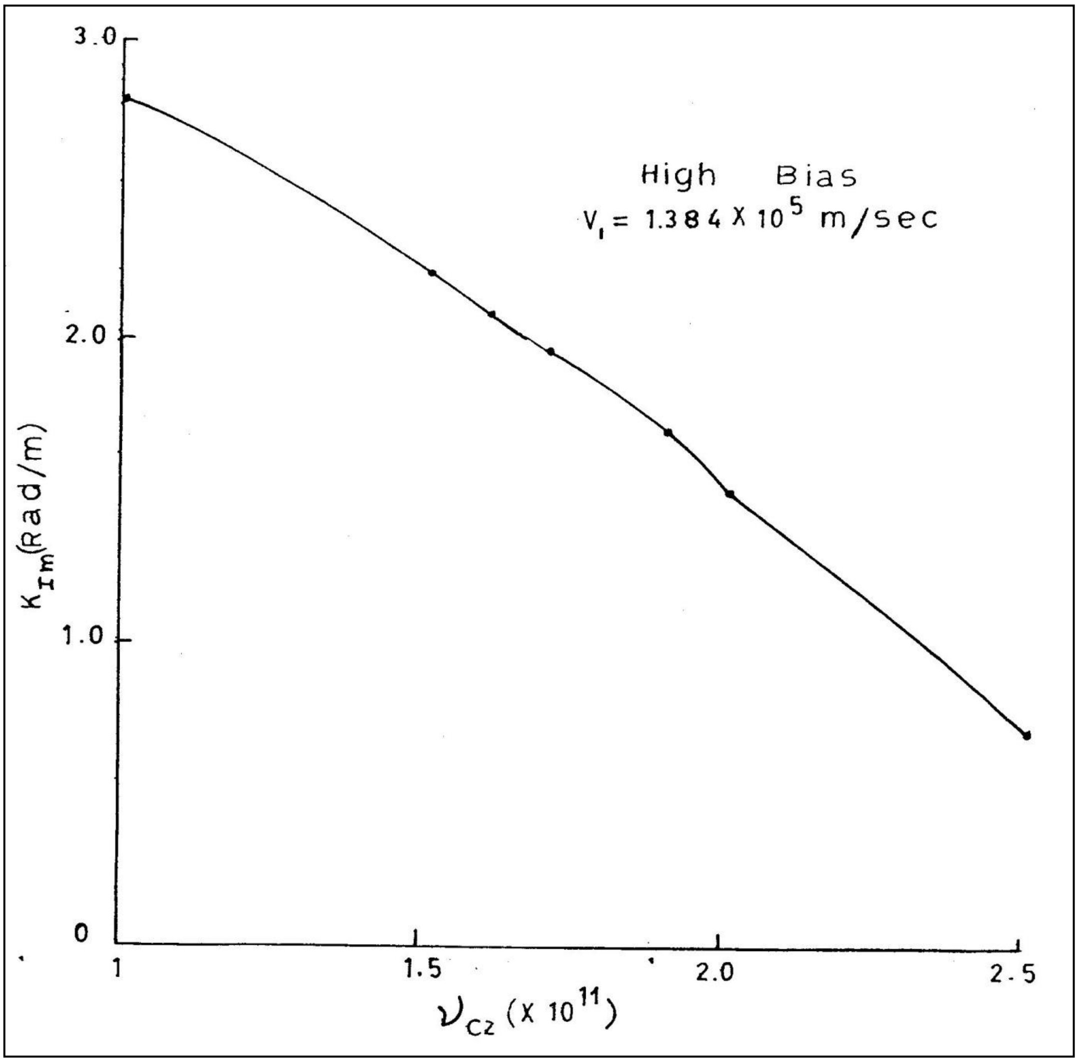

Figure 6.7: Variation of growth rate with upper valley electron collision frequency for high bias.

Table 6.3: Space charge Growth rate for different electron concentration

n_0(per m^3)	x	Growth rate
10^{19}	0.025	1.025
10^{20}	0.248	1.281
10^{21}	2.483	11.980

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Appendix

Einstein (1907) for the first time initiated the study of lattice dynamics of solids. He assumed that each atom or molecule in a crystal behaved as a harmonic oscillator vibrating with a fixed frequency. All oscillators were supposed to vibrate independently without interaction with others. Based on this hypothesis, Einstein formulated his specific heat theory.

This theory could explain the decrease in specific heat with decreasing temperature, but it was found that the decrease predicted by the theory was more rapid than what was observed. Later on many workers studied the lattice waves in metals. Narayan et.al. (1991) and Narayan and Singh (1996) proposed a phenomenological model to study lattice waves in noble metals. We have modified Kreb's model and out proposed model is in equilibrium under zero external stress and gives correct expression for Cauchy discrepancy.

Large scale theoretical and experimental efforts have been focused on the computation of cubic and quartic anharmonic contributions to Debye- Waller factors. Many workers studied the Debye- Waller factors of FCC metals which agreed satisfactorily with experimental observations only upto a certain temperature and thereafter deviation in theoretical and experimental value occurs. Later on it was studied that the deviation is due to the neglect of anharmonic effects. Narayan (1989) studied the anharmonicity for Debye-Waller factors of FCC metals by the inclusion of quartic anharmonic contributions to the Debye- Waller exponent 2W alongwith the harmonic contributions and the thermal expansion correction.

Gautam et.al. (1994) studies the lattice waves in transition metals palladium and platinum on the basis of lattice dynamical model which considers short range pairwise forces effective upto second neighbours, long range screened Coulomb forces on the lines of Krebs and describes the ionic lattice to be in equilibrium in a medium of electrons. The present modified Krebs model gives an adequate explanation of the dispersion of lattice waves and are found to be in a reasonably satisfactory agreement with the experimental values. Narayan et.al. (1995) have followed the Willis approach to study the anharmonic vibrrational effects on the D-W factors for FCC metals, silver and nickel and found that our theoretical values agree well with experimental values.

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ABOUT THE AUTHOR

Dr. A. B. Gautam

Dr. Gautam is currently working as Associate Professor in the Dept. of Physics, S. B. A. N. College Darheta- Lari, Arwal. He has published more than fifteen research articles in national / international journals. He has participated and presented his paper in many national / international seminars/ conferences/workshop. Dr Gautam is life member of ISCA, KOLKATA and several other organizations. He has keen interest in the field of research. Also, he is continuously working in the field of Microwave Plasma for socio economic development of the nation.

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