AN INTRODUCTION TO FUZZY LINEAR PROGRAMMING PROBLEMS

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Kripa Drishti Publications, Pune.

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Book Title: **An Introduction to Fuzzy Linear Programming Problems**

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1st Edition

ISBN: **978-81-19149-24-7**

Published: **April 2023**

Publisher:

Kripa-Drishti Publications

A/ 503, Poorva Height, SNO 148/1A/1/1A,
Sus Road, Pashan- 411021, Pune, Maharashtra, India.
Mob: +91-8007068686
Email: editor@kdpublications.in
Web: https://www.kdpublications.in

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ISBN: 978-81-19149-24-7

Chapter 1

General Introduction

1.1 Introduction:

Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS), which is a generalization of the concept of fuzzy set [1]. Basic arithmetic operations of TIFNs are defined by Deng-Feng Li in [2] using membership and non-membership values. Basic arithmetic operations of TIFNs such as addition, subtraction and multiplication are defined by S.Mahapatra & T.K. Roy in [5], by considering the six tuple number itself. Here we have developed division operation on six tuple TIFN using α, β- cut method. Most of the authors used the membership and non-membership values of TIFNs for ranking. A ratio ranking method of TIFN is developed is defined by L.Shen. et. Al in [8]. Scoring function of a fuzzy number intuitionistic fuzzy value is defined by X.F.Wang in [9]. We have defined ranking of TIFNs using integral value by considering six tuple TIFNs in [6].

The aim of this book is to propose division of TIFN using, α, β- cut, score function and accuracy function of TIFNs. Based on the score functions we compare two TIFNs and it is applied to solve Intuitionistic Fuzzy Variable Linear Programming Problem. An accuracy function is developed to defuzzify TIFN.

1.2 Preliminaries:

1.2.1 Definition [1]:

Given a fixed set X = {x1, x2, x3, xn}, an intuitionistic fuzzy set (IFS) is defined as A = $((x_i, t_A(x_i), f_A(x_i))/x_i \in X)$ Which assigns to each element xi a membership degree $t_A(x_i)$ and a non-membership degree $f_A(x_i)$ under the condition $0 \leq t_A(x_i) + f_A(x_i) \leq 1$, for all $x_i \in X$

1.2.2 Definition: [5]

A fuzzy number Ã (a1, a2, a3) is a Triangular Fuzzy Number if its membership function by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{\;} & , a_1 \leq x \leq a_2 \\ \dfrac{x-a_3}{a_2-a_3} & \leq a_3 \text{ where } a_1, a_2, a_3 \text{ are real numbers} \\ 0 & , \quad \text{otherwise} \end{cases}$$

1.2.3 Definition [5]:

(α, β)-Level intervals or (α, β)-cuts

A set of (α, β) – cut generated by IFS $\tilde{A}^I$, where $\alpha, \beta \in [0, 1]$ are fixed numbers such that $\alpha + \beta$ is defined as $\tilde{A}^I_{\alpha,\beta} = \{(x, \mu_{\tilde{A}I}(x), \upsilon_{\tilde{A}I}(x)): x \in X, \mu_{\tilde{A}I}(x) \leq \alpha, \upsilon_{\tilde{A}I}(x) \leq \beta, \alpha, \beta \in [0, 1]\}$

(α, β)-level interval or (α, β) – cut denoted by $\tilde{A}I\alpha, \beta$ is defined as the crisp set of elements of x which belong to $\tilde{A}^I$ at least to the degree α and which does belong to $\tilde{A}^I$ at most to the degree β.

1.2.4 Definition:

A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}I}(x)$ and non-membership function $\upsilon_{\tilde{A}I}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} x-a_1 & , a_1 \leq x \leq a_2 \\ \dfrac{x-a_3}{a_2-a_3} & \leq a_3 \\ 0 & , \text{otherwise} \end{cases}$$

and

$$\upsilon_{\tilde{A}}(x) = \begin{cases} a_2 - x & , a'_1 \leq x \leq a_2 \\ \dfrac{x-a_2}{a'_3-a_2} & , a_2 \leq x \leq a'_3 \\ 1 & , \text{otherwise} \end{cases}$$

Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3$ and

$\mu_{\tilde{A}I}(x) + \upsilon_{\tilde{A}I}(x) \leq 1$, or $\mu_{\tilde{A}I}(x) = \upsilon_{\tilde{A}I}(x)$, for all $x \in$ R.

This TIFN is denoted by

$\tilde{A}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3) = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$

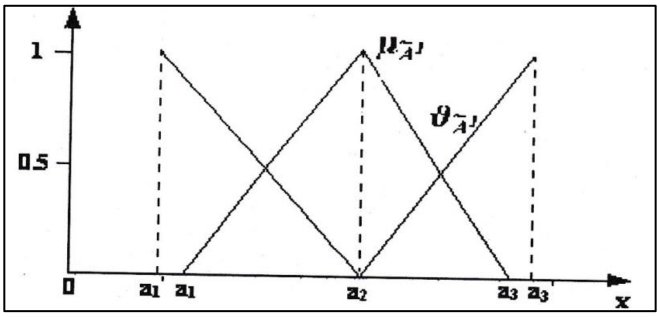

Figure 1.1: Membership and Non-Membership Functions of TIFN

Arithmetic operations of Triangular Intuitionistic Fuzzy Number based on (α, β) – cuts method: [5]

A. If $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B^I = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$ are two TIFNs, then their sum
B. $\tilde{A}^I + B^I = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a'_1+b'_1, a_2 + b_2, a'_3+b'_3)\}$ is also a TIFN.
C. If $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B^I = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$ are two TIFNs, then
D. $\tilde{A}^I - B^I = \{(a_1 - b_1, a_2 - b_2, a_3 - b_3); (a'_1-b'_1, a_2 - b_2, a'_3-b'_3)\}$ is also a TIFN.
E. If $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B^I = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$ are two TIFNs, then their product
F. $\tilde{A}^I \times B^I = \{(a_1b_1, a_2b_2, a_3b_3); (a'_1b'_1, a_2b_2, a'_3b'_3)\}$ is also a TIFN.
G. If TIFN $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and y = ka (with k > 0) then $y^I = k\tilde{A}^I$ is a TIFN $\{(ka_1, ka_2, ka_3); (ka'_1, ka_2, ka'_3)\}$.
H. If TIFN $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and y = ka (with k < 0) then $y^I = k\tilde{A}^I$ is a TIFN $\{(ka_3, ka_2, ka_1); (ka'_3, ka_2, ka'_1)\}$.

1.3 Proposed Division of Two Tifns Based On (A, B) – Cuts Method:

If $\tilde{A}^I = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B^I = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$ are two positive TIFNs, then

$\frac{\tilde{A}^I}{B^I}$ is also a TIFN, $\frac{\tilde{A}^I}{B^I} = \left\{\left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \left(\frac{a'_1}{b'_3}, \frac{a_2}{b_2}, \frac{a'_3}{b'_1}\right)\right\}$

Proof: Let $z = \frac{x}{y}$ be the transformation with the membership functions and non-membership functions of TIFNs.

Then $Z^I = \frac{\tilde{A}^I}{B^I}$ can be found by (α, β) – cuts method:

- α – cut for membership function of $\tilde{A}^I$ is $[a_1 + \alpha (a_2 - a_1), a_3 - \alpha (a_3 - a_2)]$, □□$\alpha$ □ [0, 1] i.e., x□ $[a_1 + \alpha (a_2 - a_1), a_3 - \alpha (a_3 - a_2)]$
- α – cut for membership function of B^I is $[b_1 + \alpha (b_2 - b_1), b_3 - \alpha (b_3 - b_2)]$, □□$\alpha$ □ [0, 1] i.e., y□ $[b_1 + \alpha (b_2 - b_1), b_3 - \alpha (b_3 - b_2)]$

To calculate division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ & B^I, we first divide the α-cuts of $\tilde{A}^I$ & B^I using interval arithmetic

$$\frac{\tilde{A}^{I\alpha}}{B^{I\alpha}} = \frac{[a_1 + \alpha (a_2 - a_1), a_3 - \alpha (a_3 - a_2)]}{[b_1 + \alpha (b_2 - b_1), b_3 - \alpha (b_3 - b_2)]}$$

$$= \frac{[a_1 + \alpha (a_2 - a_1), a_3 - \alpha (a_3 - a_2)]}{[b_3 + \alpha (b_3 - b_2), b_1 - \alpha (b_2 - b_1)]}$$

To find the membership function $\mu_{\tilde{A}I/BI}(x)$, we equate x both the first and second component, which gives

$$x = \frac{a_1 + \alpha (a_2 - a_1)}{b_3 + \alpha (b_3 - b_2)} \quad \text{and } x = \frac{a_3 + \alpha (a_3 - a_2)}{b_1 + \alpha (b_2 - b_1)}$$

Now expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$, we get

$$\mu\tilde{A}^I/B^I(x) = \begin{cases} \dfrac{b_3 x - a_1}{(a_2 - a_1) + (b_3 - b_2)x}, & a_1/b_3 \le x \le a_2/b_2 \\[2ex] \dfrac{a_3 - b_1 x}{(a_3 - a_2) + (b_2 - b_1)\, x}, & a_2/b_2 \le x \le a_3/b_1 \end{cases}$$

- β – cut for non-membership function of $\tilde{A}^I$ is $[a_2 + \beta (a_2 - a'_1), a_2 + \beta (a'_3 - a_2)]$, □□$\beta$ □ [0, 1] i.e., x□ $[a_2 - \beta (a_2 - a'_1), a_2 + \beta (a'_3 - a_2)]$
- β – cut for non-membership function of B^I is $[b_1 - \beta (b_2 - b'_1), b_2 + \beta (b'_3 - b_2)]$, □□$\beta$ □ [0, 1] i.e., y□ $[b_2 - \beta (b_2 - b'_1), b_2 + \beta (b'_3 - b_2)]$

To calculate division of triangular intuitionistic fuzzy numbers $\tilde{A}^I$ and B^I, we now divide the β-cuts of $\tilde{A}^I$ & B^I using interval arithmetic

$$\frac{\tilde{A}^{I\beta}}{B^{I\beta}} = \frac{[a_2 - \beta(a_2 - a'_1),\ a_2 + \beta(a'_3 - a_2)]}{[b_2 - \beta(b_2 - b'_1),\ b_2 + \beta(b'_3 - b_2)]}$$

$$= \frac{[a_1 - \beta(a_2 - a'_1),\ a_2 + \beta(a'_3 - a_2)]}{[b_2 + \beta(b'_3 - b_2),\ b_2 - \beta(b_2 - b'_1)]}$$

To find the membership function $\upsilon_{\tilde{A}I/BI}(x)$, we equate x both the first and second component, which gives

$$x = \frac{a_2 - \beta(a_2 - a'_1)}{b_2 + \beta(b'_3 - b_2)} \quad \text{and } x = \frac{a_2 + \beta(a'_3 - a_2)}{b_2 - \beta(b_2 - b'_1)}$$

Now expressing β in terms of x and setting α = 0 and β = 1, we get

$$\upsilon\tilde{A}^I/B^I(x) = \begin{cases} \dfrac{a_2 - b_2x}{(a_2 - a'_1) + (b'_3 - b_2)x}, & a'_1/b'_3 \le x \le a_2/b_2 \\[2ex] \dfrac{b_2x - a_2}{(a'_3 - a_2) + (b_2 - b'_1)\,x}, & a_2/b_2 \le x \le a'_3/b'_1 \end{cases}$$

Hence the division rule is proved for membership and non-membership functions.

Thus

$\tilde{A}^I/B^I = \{(a_1/b_3, a_2/b_2, a_3/b_1), (a'_1/b'_3, a_2/b_2, a'_3/b'_3)\}$ is also a TIFN.

1.4 Proposed Score Function and Accuracy Function:

Let $\tilde{A} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ be a TIFN, then we define a Score function for membership and non-membership values respectively as

$$S(\tilde{A}^{I\alpha}) = \frac{a_2 + 2a_2 + a_3}{4} \quad \& \; S(\tilde{A}^{I\beta}) = \frac{a'_2 + 2a_2 + a'_3}{4}.$$

Let $\tilde{A} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ be a TIFN, then we define

$$(\tilde{A}^I) = \frac{(a_2 + 2a_2 + a_3) + (a'_2 + 2a_2 + a'_3)}{8},$$

an accuracy function of $\tilde{A}^I$, to defuzzify

the given number.

1.4.1 Ranking Using Score Function:

Let $\tilde{A} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B = \{(b_1, b_2, b_3); (b'_1, b_2, b'_3)\}$ be two TIFNs and $S(\tilde{A}^{I\alpha})$, $S(\tilde{A}^{I\beta})$ & $S(B^{I\alpha})$, $S(B^{I\beta})$ be the scores of $\tilde{A}^I$ & B^I respectively. Then

A. If $S(\tilde{A}^{I\alpha}) \leq S(B^{I\alpha})$ & $S(\tilde{A}^{I\beta}) \leq S(B^{I\beta})$, then $\tilde{A}^I < B^I$
B. If $S(\tilde{A}^{I\alpha}) \geq S(B^{I\alpha})$ & $S(\tilde{A}^{I\beta}) \geq S(B^{I\beta})$, then $\tilde{A}^I > B^I$
C. If $S(\tilde{A}^{I\alpha}) = S(B^{I\alpha})$ & $S(\tilde{A}^{I\beta}) = S(B^{I\beta})$, then $\tilde{A}^I = B^I$

1.4.2 Theorem:

Let $\tilde{A}^I$ & B^I be any two TIFNs.

Then

$S(\tilde{A}^{I\alpha}) \leq S(B^{I\alpha})$ & $S(\tilde{A}^{I\beta}) \leq S(B^{I\beta}) \rightarrow H(\tilde{A}^I) \leq H(B^I)$

Proof: Since $S(\tilde{A}^{I\alpha}) \leq S(B^{I\alpha})$ & $S(\tilde{A}^{I\beta}) \leq S(B^{I\beta})$

We get $S(\tilde{A}^{I\alpha}) + S(\tilde{A}^{I\beta}) \leq S(B^{I\alpha}) + S(B^{I\beta})$

i.e., $$\frac{S(\tilde{A}^{I\alpha}) + S(\tilde{A}^{I\beta})}{2} \leq \frac{S(B^{I\alpha}) + S(B^{I\beta})}{2},$$ i.e., $H(\tilde{A}^I) \leq H(B^I)$.

Hence the proof.

1.5 Intuitionistic Fuzzy Linear Programming:

Linear Programming with Triangular Intuitionistic Fuzzy Variables is defined in [4] as (IFLP)

max $Z^I = \sum_{j=1}^{n} c^I_j\, x^I_j$ Subject to $\sum_{j=1}^{n} a^I_{ij}\, x^I_j \leq b^I_i$

i = 1, 2, …m, j = 1, 2, ….m, where $\tilde{A}^I = (\tilde{a}^I_{ij})$, c^I, b^I, x^I are (m x n), (1 x n), (m x 1), (n x 1) intuitionistic fuzzy matrices consisting of Triangular Intuitionistic Fuzzy Numbers (TIFN).

Standard Form [6] The objective function should be of maximization form (IFLP) max $Z^I = \sum_{j=1}^{n} c^I_j\, x^I_j$ …………………………………. (1.5.1)

Subject to

$$\left.\begin{aligned}
&\tilde{a}^I_{11}x^I_1 + \tilde{a}^I_{12}x^I_2 + \ldots\ldots\ldots\ldots + \tilde{a}^I_{1n}\, x^I_n + x^I_{n+1} = b^I_1\\
&\tilde{a}^I_{21}x^I_1 + \tilde{a}^I_{22} + \ldots\ldots\ldots\ldots + \tilde{a}^I_{2n}\, x^I_n + x^I_{n+2} = b^I_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5.2)\\
&\tilde{a}^I_{m1}\, x^I_1 + \tilde{a}^I_{22}\, x^I_2 + \ldots\ldots\ldots\ldots + \tilde{a}^I_{mn}x^I_n + x^I_{n+m} = b^I_m\\
&x^I_1, x^I_2, \ldots\ldots, xIn, x^I_{n+1}, x^I_{n+m} \geq 0 \ldots\ldots\ldots\ldots (5.3)
\end{aligned}\right\}$$

Intuitionistic Fuzzy Optimum Feasible Solution [6]:

Let X be the set of all intuitionistic fuzzy feasible solutions of (5.1). An intuitionistic fuzzy feasible solution x^I_0 X is said to be an intuitionistic fuzzy optimum solution to (5.1), if $c^I x^I_0 \geq c^I x^I$ for all x^I X, where $c^I = (c^I_1, c^I_2, c^I_3, \ldots\ldots cIn)$, and $c^I x_I = c^I_1 x^I_1 + c^I_2 x^I_2 + \ldots\ldots\ldots + c^I_n x^I_n$.

1.6 Numerical Illustration:

Solve Max $z^I = 5^I x^I_1 + 3^I x^I_2$

Subject to $4x^I_1 + 3^I x^I_2 \leq 12^I$, $1^I x^I_1 + 3^I x^I_2 \leq 6^I$, $x^I_1, x^I_2 \geq 0$ where

$c^I_1 = 5^I = \{(4, 5, 6); (4, 5, 6.1)\}$ $\qquad$ $c^I_2 = 3^I = \{(2.5, 3, 3.5); (2, 3, 3.5)\}$

$\tilde{a}^I_{11} = 4^I = \{(3.5, 4, 4.1); (3, 4, 5)\}$ $\qquad$ $\tilde{a}^I_{12} = 3^I = \{(2.5, 3, 3.5) ;(2.4, 3, 3.6\}$

$\tilde{a}^I_{21} = 1^I = \{(0.8, 1, 2); (0.5, 1, 2.1)\}$ $\qquad$ $\tilde{a}^I_{22} = 3^I = \{(2.8, 3, 3.2); (2.5, 3, 3.2)\}$

$b^I_1 = 12^I = \{(11, 12, 13); (11, 12, 14)\}$ $\qquad$ $bI_2 = 6^I = \{(5.5, 6, 7.5) ;(5,6,8.1)\}$

Solution: Rewriting the problem in standard form:

Solve Max $z^I = 5^I x^I_1 + 3^I x^I_2$

Subject to $4x^I_1 + 3^I x^I_2 + 1^I S^I_1 \le 12^I$, $1^I x^I_1 + 3^I x^I_2 + 1^I S^I_2 \le 6^I$, x^I_1, x^I_2, S^I_1, $S^I_2 \ge 0$. Hence the co-efficient of S^I_1, S^I_2 are given by $1^I = \{(1, 1, 1); (1, 1, 1)\}$ and $0^I = \{(0, 0, 0); (0, 0, 0)\}$.

Initial Iteration: Basic variables are sI1 = 12^I, $s^I_2 = 6^I$

c^I_j 5^I 3^I 0^I 0^I

C_B	BV	x^I_1	x^I_2	s^I_1	s^I_2	b^I	ratio
0^I	s^I_1	4^I	3^I	1^I	0^I	12^I	$12^I/4^I = 3^I$
0^I	s^I_2	1^I	3^I	0^I	1^I	6^I	$6^I/1^I = 6^I$
	z^I_j	0^I	0^I	0^I	0^I	0^I	
	$c^I_j - z^I_j$	5^I	3^I	0^I	0^I	-	

Intuitionistic fuzzy linear programming problem

Since all $c^I_j - z^I_j \ge 0$, the solution is not optimal. x^I_1 is the entering variable, since the most positive value corresponds to the x^I_1 column. Then the ratio is calculated. Using the division procedure and Scoring function as defined in the sections (2 & 3) of this book, we get the following results:

A. $12^I / 4^I = \{(2.68, 3, 3.71); (2.2, 3, 4.67)\} = 3^I$
B. $6^I / 1^I = \{(2.75, 6, 9.375); (2.38, 6, 16.2)\} = 6^I$
C. Score function $S(3^{I\alpha}) = 3.0975$ & $S(3^{I\beta}) = 3.2175$
D. Score function $S(6^{I\alpha}) = 6.03125$ & $S(6^{I\beta}) = 7.645$

Since $S(3^{I\alpha}) < S(6^{I\alpha})$ & $S(3^{I\beta}) < S(6^{I\beta})$, we get $3^I < 6^I$.

So s^I_1 is the leaving variable.

First Iteration: Basic variables are $x^I_1 = 3^I$, $s^I_2 = 3^I$

c^I_j 5^I 3^I 0^I 0^I

C_B	BV	x^I_1	x^I_2	s^I_1	s^I_2	b^I
5^I	x^I_1	1^I	0.75^I	0.25^I	0^I	3^I
0^I	s^I_2	0^I	2.25^I	-0.25^I	1^I	3^I
	z^I_j	5^I	3.75^I	1.25^I	0^I	15^I
	$c^I_j - z^I_j$	0^I	-0.75^I	-1.25^I	0^I	-

Where the Triangular intuitionistic representation for each element based on arithmetic operations is listed below:

$c^I_1 = 5^I = \{(4, 5, 6); (4, 5, 6.1)\}$

$c^I_2 = 3^I = \{(2.5, 3, 3.5); (2, 3, 3.5)\}$

$\tilde{a}^I_{11} = \{(.85, 1, 1.17); (.6, 1, 1.67)\} = 1^I$

$\tilde{a}^I_{12} = \{(.61, .75, 1); (.46, .75, 1.2)\} = 0.75^I$

$\tilde{a}^I_{13} = \{(0.24, 0.25, 0.29); (0.2, 0.25, 0.67)\} = 0.25^I$

$\tilde{a}^I_{14} = \{(0, 0, 0); (0, 0, 0)\} = 0^I$

$b^I_1 = \{(2.68, 3, 3.71); (2.2, 3, 4.67)\} = 3^I$

$\tilde{a}^I_{21} = \{(-0.37, 0, 1.15); (-1.17, 0, 1.5)\} = 0^I$

$\tilde{a}^I_{22} = \{(1.8, 2.25, 2.59); (1.3, 2.25, 2.74)\} = 2.25^I$

$\tilde{a}^I_{23} = - \{(0.24, 0.25, 0.29); (0.2, 0.25, 0.67)\} = -0.25^I$

$\tilde{a}^I_{24} = \{(1, 1, 1); (1, 1, 1)\} = 1^I$

$b^I_2 = \{(1.79, 3, 4.82); (0.33, 3, 5.9)\} = 3^I$

REPRESENTATION OF EACH ELEMENT IN THE ROW z^I_j

$\tilde{a}^I_{33} = \{(3.4, 5, 7.02); (2.4, 5, 10.18)\} = 5^I$

$\tilde{a}^I_{32} = \{(2.44, 3.75, 6); (1.84, 3.75, 7.32)\} = 3.75^I$

$\tilde{a}^I_{33} = \{(0.96, 1.25, 1.74); (0.8, 1.25, 4.08)\} = 1.25^I$

$\tilde{a}^I_{34} = \{(0, 0, 0); (0, 0, 0)\} = 0^I$

$z^I_j = \{(10.72, 15, 22.26) ;(8.8, 15, 28.48)\} = 15^I$

REPRESENTATION OF EACH ELEMENT IN THE ROW $c^I_j - z^I_j$

$\tilde{a}^I_{41} = \{(-3.02, 0, 2.6); (-6.187, 0, 3.7)\} = 0^I$

$\tilde{a}^I_{44} = \{(0, 0, 0); (0, 0, 0)\} = 0^I$

$\tilde{a}^I_{42} = - \{(-0.76, 0.75, 0.76); (-1.66, 0.75, 1.66)\} = -0.75^I$

$\tilde{a}^I_{43} = -\{(0.96, 1.25, 1.74); (0.8, 1.25, 4.08)\} = -1.25^I$

1.7 Conclusion of the Problem:

Since all the elements in the row cIj – zIj are less than or equal to zero, the solution obtained is optimal i.e., Max $z^I_j = 15^I$ when $x^I_1 = 3^I$, $x^I_2 = 0^I$, $s^I_1 = 0^I$, $s^I_2 = 3^I$

1.8 References:

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ISBN: 978-81-19149-24-7

Chapter 2

Generalized Trapezoidal Intuitionistic Fuzzy Number Via New Ranking Method

2.1 Introduction:

In fuzzy environment, ranking of fuzzy numbers play a vital role in decision making problem. In literature, numerous approaches for ranking fuzzy numbers have been extensively studied. Several authors namely Abbasbandy (2009); Chen (1985); Chen and Chen (2009); Wang and Lee (2008) rank fuzzy numbers by different approaches. The concept of fuzzy set theory introduced by Zadeh (1965) was extended to intuitionistic fuzzy sets (IFS) by Atanassov (1986). In IFS, degree of non-membership and non-membership function up to Decision maker's (DMs) satisfacition due to insufficient available information.

As a result, there remains an indeterminsitic part in which reluctance perseveres. Therefore, intuitionistic fuzzy set theory seems to be more consistent to deal with ambiguity and vagueness. In recent past, ranking intuitionistic fuzzy numbers (IFNs) draws the attention of several researches.

Nehi (2010) ranked IFNs based on characteristic values of membership and non-membership functions of IFN. Ranking of trapezoidal IFNs based on value and ambiguity indices were given by De and Das (2012), Rezvani (2012) and many more approaches were subsequently developed.

In 1970, Bellman and Zadeh (1970) introduced the concept of decision making in fuzzy environment. The concept of optimization in intuitionistic fuzzy environment was given by Angelov (1997).

One of the important applications of linear programming is in the area of transportation of goods and services from several supply centres to several demand centres. The simplest transportation model was first presented by Hitchcock (1941) in 1941. Several other extensions were successively developed.

In 1984, Chanas. Et. Al (1984) presented a fuzzy approach to the transportation problem. Fuzzy zero-point method is introduced by Pandian and Natarajan (2010), which was extended to intuitionistic fuzzy zero-point method by Hussain and kumar (2012) to compute optimal solution of transportation problem.

To the best of our knowledge, till now no one has used generalized trapezoidal intuitionistic fuzzy numbers for solving transportation problems.

In this book, new ranking method for ordering generalized trapezoidal intuitionistic fuzzy numbers (GTRIFNs) is introduced.

Intuitionistic max- min method and generalized intuitionistic modified distribution method is introduced for computing the initial basic feasible solution (IBFS) and optimal solution respectively of transportation problem in which the costs are represented by GTRIFNs.

Rest of the book is organized as follows. Section 2 briefly describes some basic concepts. Arithmetic operations over GTRIFNs are defined in section 3.

A new ranking method for GTRINs and significance of the proposed ranking method over existing methods is illustrated in section 4.

In section 5, mathematical model formulation of intuitionistic fuzzy transportation problem and algorithm of proposed methods to solve intuitionistic fuzzy transportation problem is illustrated.

A numerical example is solved in section 6 to demonstrate the efficiency of proposed methods. Finally, the book is concluded in section 7.

2.2 Preliminaries:

In this section, some basic results related to intuitionistic fuzzy set theory are reviewed.

Definition 1 (Atanassov, 1999): Let X be a universal set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form A = {< x, $\mu_A(x)$, $\upsilon_A(x)$ >: x □X} where the functions a: X → [0, 1] define the degree of membership and the degree of non-membership of the element x □X to the set A respectively and for every x □X in A, $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ holds.

Definition 2 (Atanassov, 1999): For every common intuitionistic fuzzy subset A on X, intuitionistic fuzzy index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A.

Obviously, for every x □X, $0 \le \pi_A(x) \le 1$.

Definition 3 (Mahapatra and Mahapatra, 2010): An Intuitionistic Fuzzy Number (IFN) $\tilde{a}^I$ is

a. An intuitionistic fuzzy subset of the real line.
b. Convex for the membership function $\mu_a(x)$, that is, a $(\lambda x_1 + (1-\lambda)\ x_2) \ge \min(\mu_a(x_1), \mu_a(x_2))$□□ x_1, x_2 □□R, λ □□[0, 1].
c. Concave for the non-membership function $\upsilon_a(x)$, that is, a $(\lambda x_1 + (1-\lambda)\ x_2) \le \max(\upsilon_a(x_1), \upsilon_a(x_2))$□□ x_1, x_2 □□R, λ □□[0, 1].
d. Normal, that is, there is some x_0 □□R such that $\mu_a(x_0) = 1$, $\upsilon_a(x_0) = 0$.

Definition 4 (Mahapatra and Mahapatra, 2010): An intuitionistic fuzzy number $\tilde{a}^I = <(a_1, a_2, a_3, a_4)(\bar{a}_1, a_2, a_3, \bar{a}_4)>$ is said to be trapezoidal intuitionistic fuzzy number (TRIFN) if its membership and non-membership functions are respectively given by

$$\mu_a(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\upsilon_a(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - \bar{a}_1} & \text{if } \bar{a}_1 \le x \le a_2 \\ 0 & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{a_3 - \bar{a}_4} & \text{if } a_3 \le x \le \bar{a}_4 \\ 1 & \text{otherwise} \end{cases}$$

Definition 5: An intuitionistic fuzzy number $\tilde{a}^I = <(a_1, a_2, a_3, a_4; \omega_a)(\bar{a}_1, a_2, a_3, \bar{a}_4; \sigma_a)>$ is said to be a generalized intuitionistic fuzzy number (GTRIFN) if its membership and non-membership function are respectively given by

$$\mu_a(x) = \begin{cases} \dfrac{(x - a_1)\,\omega_a}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \omega_a & \text{if } a_2 \le x \le a_3 \\ \dfrac{(a_4 - x)\,\omega_a}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\upsilon_a(x) = \begin{cases} \dfrac{a_2 - x + \sigma_a(x - \bar{a}_1)}{a_2 - \bar{a}_1} & \text{if } \bar{a}_1 \le x \le a_2 \\ \upsilon_a & \text{if } a_2 \le x \le a_3 \\ \dfrac{x - a_3 + a\,(\bar{a}_4 - x)}{a_3 - \bar{a}_4} & \text{if } a_3 \le x \le \bar{a}_4 \\ 1 & \text{otherwise} \end{cases}$$

Where ω_a and σ_a represent the maximum degree of membership and minimum degree of non-membership respectively, satisfying $0 \le \omega_a \le 1$, $0 \le \sigma_a \le 1$, $0 \le \omega_a + \sigma_a \le 1$.

Observation: GTRIFN defined in definition 5 is different from the TRIFNs considered in (De and Das 2012), since in (De and Das, 2012) $\bar{a}_1 = a_1$ and $\bar{a}_4 = a_4$ but in definition 5, $\bar{a}_1$ and $\bar{a}_4$ may not necessarily be equal to a_1 and a_4 respectively. Also, in Wan (2013); Wu and Cao (2013); Shen. et. al (2011), $\upsilon_a(x) = 0$ for $x < \bar{a}_1$ and $x > \bar{a}_4$ but in definition 5, $\upsilon_a(x) = 1$ for $x < \bar{a}_1$ and $x > \bar{a}_4$. Graphical representation of GTRIFN is illustrated in Figure 2.1.

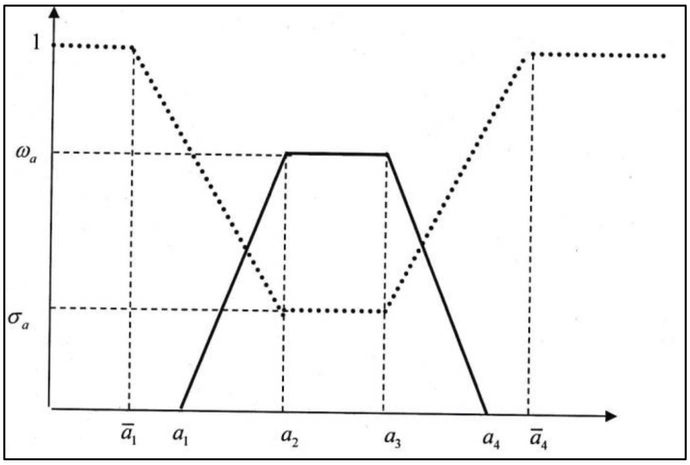

Figure 2.1: Generalized Trapezoidal Intuitionistic Fuzzy Number (GTRIFN)

2.3 Arithmetic Operations:

In a similar way to the arithmetic operations of TRIFNs (De and Das, 2012) and triangular IFNs (Li, 2008), arithmetic operations over GTRIFNs are defined as follows.

Let $\tilde{a}^I = <(a_1, a_2, a_3, a_4; \omega_a)\ (\bar{a}_1, a_2, a_3, \bar{a}_4; \sigma_a)>$ and $b^I = <(b_1, b_2, b_3, b_4; b_y)\ (b_1, b_2, b_3, b_4; \sigma_b)>$ be two GTRIFNs, then

a. $\tilde{a}^I + b^I = <(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(\omega_a, \omega_b))\ (\bar{a}_1 + b_1, a_2 + b_2, a_3 + b_3, \bar{a}_4 + b_4; \max(\sigma_a, \sigma_b))>$
b. $\tilde{a}^I - b^I = <(a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4; \min(\omega_a, \omega_b))\ (\bar{a}_1 - b_1, a_2 - b_2, a_3 - b_3, \bar{a}_4 - b_4; \max(\sigma_a, \sigma_b))>$
c. $\lambda\tilde{a}^I = <(\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \lambda\omega_a)\ (\lambda\bar{a}_1, \lambda a_2, \lambda a_3, \lambda\bar{a}_4; \lambda\sigma_a)>$ if $\lambda > 0$.
d. $\lambda\tilde{a}^I = <(\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; a)\ (\lambda\bar{a}_4, \lambda a_3, \lambda a_2, \lambda\bar{a}_1; \lambda\sigma_a)>$ if $\lambda < 0$.

2.4 Ranking Index of GTRIFN:

In literature there are various algorithms for ranking IFNs, but most of the algorithms are used to rank triangular IFNs or TRIFNs with $\bar{a}_1 = a_1$ and $\bar{a}_4 = a_4$ (De and Das, 2012; Das and Duha, 2013). So, in order to rank GTRIFN, firstly we define a new single function ρ_a involving both membership and non-membership function of GTRIFN $\tilde{a}^I$ as follows:

Define a: $R \rightarrow [0, \omega_a]$ such that

$$\rho_a = \frac{(\mu_a(x) - \upsilon_a(x) + 1)\,\omega_a}{\omega_a - \sigma_a + 1} \qquad □□ x □□R$$

Here, $\mu_a(x)$ and $\upsilon_a(x)$ are membership and non-membership of GTRIFN $\tilde{a}^I$, Lemma: $\rho a = <(x, \rho a(x)): x □□R>$ is trapezoidal non-normal fuzzy number. Proof: Let x □□R be arbitrary. Then,

$$\rho_a(x) = \begin{cases} 0 & \text{if } x \le \bar{a}_1,\ x \ge \bar{a}_4 \\ \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{-a_2 + x - a(x-\bar{a}_1)}{a_2-\bar{a}_1} + 1\right\} & \text{if } \bar{a}_1 \le x \le a_1 \\ \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{x-a_1}{a_2-a_1}\ a - \frac{a_2 - x + \sigma_a(x-\bar{a}_1)}{a_2-\bar{a}_1} + 1\right\} & \text{if } a_1 \le x \le a_2 \\ \omega_a & \text{if } a_2 \le x \le a_3 \\ \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{(a_4-x)\,\omega_a}{a_4-a_3} - \frac{x - a_3 + a(\bar{a}_4 - x)}{a_3-\bar{a}_4} + 1\right\} & \text{if } x_3 \le x \le a_4 \end{cases}$$

$$\frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{-x+a_3+a\,(\bar{a}_4-x)}{\bar{a}_4-a_3}+1\right\} \quad \text{if } a_4 \le x \le \bar{a}_4$$

Therefore, $\rho_a(x)$ can be written as

$$\rho_a(x) = \begin{cases} q(x) & \text{if } \bar{a}1 \le x \le a2 \\ \omega a & \text{if } a_2 \le x \le a_3 \\ r(x) & \text{if } a_3 \le x \le \bar{a}_4 \\ 0 & \text{otherwise} \end{cases}$$

Where q(x) is defined as $q(x): [\bar{a}_1, a_2] \rightarrow [0, \omega_a]$ such that

$$q(x) = \begin{cases} \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{-a_2+x-\sigma_a\,(x-\bar{a}_1)}{a_2-\bar{a}_1}+1\right\} & \text{if } \bar{a}_1 \le x \le a_1 \\ \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{x-a_1}{a_2-a_1}\ \omega_a - \frac{a_2-x+\sigma_a\,(x-\bar{a}_1)}{a_2-\bar{a}_1}+1\right\} & \text{if } a_1 \le x \le a_2 \end{cases}$$

and r(x) is defined as $r(x): [a_3, \bar{a}_4] \rightarrow [0, \omega_a]$ such that

$$r(x) = \begin{cases} \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{(a_4-x)}{a_4-a_3}\,\omega_a - \frac{x-a_3+a\,(\bar{a}_4-x)}{a_3-\bar{a}_4}+1\right\} & \text{if } x_3 \le x \le a_4 \\ \frac{\omega_a}{\omega_a-\sigma_a+1}\left\{\frac{-x+a_3+\sigma_a(\bar{a}_4-x)}{\bar{a}_4-a_3}+1\right\} & \text{if } a_4 \le x \le \bar{a}_4 \end{cases}$$

Here, q(x) is continuous and monotonically increasing function and r(x) is continuous and monotonically decreasing function. Also range of $\rho_a(x)$ lies in $[0, \omega_a]$.

Therefore, $\rho_a = <(x, \rho_a(x);\ x\,\square\square R>$ is non-normal trapezoidal fuzzy number. To rank GTRIFNs, firstly we will find the centroid of fuzzy number ρ_a.

Functions q(x) and r(x) defined in the lemma are both strictly monotone. Let $q^{-1}(y)$: $[0, \omega_a] \rightarrow [\bar{a}_1, a_2]$ and $r^{-1}(y)$: $[0, \omega_a] \rightarrow [a_3, \bar{a}_4]$ be the inverse functions of q(x) and r(x) respectively. Then,

$$q^{-1}(y) = \begin{cases} \dfrac{y(a_2-\bar{a}_1)(\omega_a-\sigma_a+1)+\bar{a}_1(1-\sigma_a)\omega_a}{(1-\sigma_a)\,\omega_a} & \text{if } 0 \le y \le t \\ \dfrac{y(\omega_a-\sigma_a+1)\quad (a_2-a_1)\quad (a_2-\bar{a}_1)-\omega_a(a_4\bar{a}_4\omega_a-a_4a_3\omega_a-a_4\bar{a}_4\sigma_a+a_3\bar{a}_4\sigma_a+a_4\bar{a}_4-a_3\bar{a}_4}{} & \text{if } t \le y \le \omega_a \end{cases}$$

$$\text{where } t = \frac{(a_1-\bar{a}_1)\,(1-\sigma_a)\,\omega_a}{(\omega_a-\sigma_a+1)\,(a_2-\bar{a}_1)} \quad \text{and}$$

$$r^{-1}(y) = \begin{cases} \dfrac{y(\bar{a}_4-a_3)\,(\omega_a-\sigma_a+1)-\bar{a}_4\omega_a(1-\sigma_a)}{(1-\sigma_a)\,\omega_a} & \text{if } 0 \le y \le s \\ \dfrac{y(a_4-a_3)\quad (\bar{a}_4-a_3)\quad (\omega_a-\sigma_a+1)\quad -\omega_a(a_4\bar{a}_4\omega_a-a_4a_3\omega_a-a_4\bar{a}_4\sigma_a+a_3\bar{a}_4\sigma_a+a_4\bar{a}_4-a_3\bar{a}_4}{} & \text{if } s \le y \le \omega_a \end{cases}$$

$$\text{where } s = \frac{(\bar{a}_4-a_4)\,(1-\sigma_a)\,\omega_a}{(\omega_a-\sigma_a+1)\,(\bar{a}_4-a_3)}$$

Since ρ_a is non-normal trapezoidal fuzzy number, so centroid point (x_0, y_0) of a fuzzy number ρa (based on formula of Wang .et .al, 2006) is given by

$$x_0(\rho_a) = \int_{\infty} x\rho_a(x)dx \Big/ \int_{\infty} \rho_a(x)dx$$

$$= \int_{\bar{a}_1}^{a_2} xq(x)dx + \int_{a_2}^{a_3} x\omega a dx + \int_{a_3}^{\bar{a}_4} xr(x)dx \Big/ \int_{\bar{a}_1}^{a_2} q(x)\,dx + \int_{a_2}^{a_3} \omega a dx + \int_{a_3}^{\bar{a}_4} r(x)dx$$

$$= \frac{(1-\sigma_a)(-\bar{a}^2_1-a^2_2-\bar{a}_1a_2+a^2_3+a_3\bar{a}_4+\bar{a}^2_4)+\omega_a(-a^2_1-a^2_2-a_1a_2+a^2_3+a_3\bar{a}_4+\bar{a}^2_4}{3\{(1-\sigma a)(-\bar{a}_1-a_2+a_3+\bar{a}_4)+\omega_a(-\bar{a}_1-a_2+a_3+\bar{a}_4)\}} \quad (1)$$

$$y_0(\rho_a) = \int_0^{\omega_a} y(r^{-I}(y) - q^{-I}(y))\,dy \Big/ \int_0^{\omega_a} (r^{-I}(y)-q^{-I}(y))\,dy \qquad (2)$$

Remark 1: If $\mu_a(x) = 1-\upsilon_a(x)$, then $\bar{a}_1=a_1$, $\bar{a}_4=a_4$, $\omega_a=1-\sigma_a$

Also, ρ_a = <x $\mu_a(x)$: x□□R) >. Thus, ρ_a reduces to a non-normal trapezoidal fuzzy number with membership function $\mu_a(x)$ (as defined in definition 5). By substituting the values in the above centroid formula, we get

$$x_0(\rho_a) = 1/3\left[a_1 + a_2 + a_3 + a_4 - \frac{a_4a_3 - a_1a_2}{(a_4+a_3)-(a_1+a_2)}\right]$$

$$y_0(\rho_a) = \omega_a/3\left[1 + \frac{a_3 - a_2}{(a_4+a_3)-(a_1+a_2)}\right],$$ which is exactly the same centroid formula of a trapezoidal non-normal fuzzy number with membership function $\mu a(x)$, as derived by Wang.et.al (2006).

Remark 2: Let $\mu_a(x) = 1 - \upsilon_a(x)$ and $a_2 = a_3$ then $\bar{a}_1 = a_1$, $\bar{a}_4 = a_4$, $\omega_a = 1 - \sigma_a$.

Also, ρ_a reduces to a non-normal triangular fuzzy number and by substituting the values, we get

$$x_0(\rho_a) = \frac{a_1+a_2+a_4}{3},\quad y_0(\rho_0) = \frac{\omega_a}{3},$$ which is the centroid formula of a triangle.

We employ Wang and Lee, (2008) method for the centroid of ρ_a (defined in 1 and 2) to order GTRIFNs. Then,

a. If $x_0(\rho_a) > x_0(\rho_b)$ then $\tilde{a}^I > b^I$
b. If $x_0(\rho_a) < x_0(\rho_b)$ then $\tilde{a}^I < b^I$
c. If If $x_0(\rho_a) = x_0(\rho_b)$, then $\tilde{a}^I > b^I$

if $x_0(\rho_a) > y_0(\rho_b)$ then $\tilde{a}^I > b^I$;

else if $x_0(\rho_a) < y_0(\rho_b)$ then $\tilde{a}^I < b^I$;

else if $x_0(\rho_a) = y_0(\rho_b)$ then $\tilde{a}^I = b^I$.

Here, we use x_0 value as ranking index. y_0 value is used only to compare GTRIFNs when their x_0 value are same. Significance of the proposed ranking method over existing methods

a. Algorithm given by De and Das (2012); Das and Duha (2013) cannot be used to rank those GTRIFNs where $\bar{a}_1 \neq a_1$ or $\bar{a}_4 \neq a_4$ but the proposed method can be used to rank such GTRIFNs.
b. Algorithm described in (Nayagam et.al. 2008) fails if membership score of $\tilde{a}^I \leq$ membership score of b^I and non-membership score of $\tilde{a}^I \leq$ non-membership score of b^I, where $\tilde{a}^I$ and b^I are IFNs. But in the proposed method, we overcome this situation by defining a single function ρ_a involving both membership and non-membership function of GTRIFN $\tilde{a}^I$.
c. Most of the existing methods discussed in (Dubey and mehra, 2011); (Li, 2010) and many more can be used only for Triangular IFNs. These methods cannot be used to rank GTRIFNs. But our method can be used to rank GTRIFNs as well as triangular IFNs by taking $a_2 = a_3$.

2.5 Mathematical Formulation of Intuitionistic Fuzzy Transportation Problem:

Consider an intuitionistic fuzzy transportation problem (IFTP) with m origins and n destinations.

Let c^I_{ij} be the intuitionistic fuzzy (IF) cost of transporting one unit of the product form i^{th} origin to the j^{th} destination.

Here, the cost c^I_{ij} (i=1, 2, ………, m, j= 1, 2, ………., n) are represented by GTRIFNs. Let a_i be the total availability of the product at the i^{th} origin. Let b_j be the total demand of the product at j^{th} destination.

Let x_{ij} be the quantity transported from i^{th} origin to the j^{th} destination so as to minimize the total IF transportation cost.

Therefore, IFTP in which the DM is uncertain about the precise values of transportation cost from ith origin to the j^{th} destination but sure about the supply and demand of the product can be formulated as

$$\text{Minimize} \quad \sum \sum_{i=1}^{m} \sum_{j=1}^{n} c^I_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad j = 1, 2, \ldots, n$$

$$x_{ij} \ge 0 \qquad \forall\, i, j.$$

if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, then IFTP is said to be balanced, otherwise it is said to be unbalanced IFTP.

The primal of the balanced IFTP can be written as

$$\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} c^I_{ij} x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} \le a_i \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j \qquad j = 1, 2, \ldots, n$$

$$x_{ij} \ge 0 \qquad \forall\, i, j.$$

The dual of the above primal problem can be written as

$$\text{Maximize } \sum_{i=1}^{m} a_i u_i \;\square\; \sum_{j=1}^{n} b_j \upsilon^I_j$$

Subject to u^I_i □ υ^I_j unrestricted

Where u^I_i and υ^I_j are the intuitionistic fuzzy dual variables associated with the i^{th} row and j^{th} column respectively. In IBFS of a primal problem, m + n – 1 variable are basic and the remaining variables are non-basic.

Table no 2.1: Tabular form of IFTP

	1	2	n	Supply
1	c^I_{11}	c^I_{12}	c^I_{1n}	a_1
2	c^I_{21}	c^I_{22}	c^I_{2n}	a_2
m	c^I_{m1}	c^I_{m2}	c^I_{mn}	a_m
Demand	b_1	b_2	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

2.5.1 Proposed IF Max-Min Method for Finding Initial Basic Feasible Solution (IBFS) of IFTP:

In this section, IF Max-Min method is proposed to compute initial basic feasible of IFTP. The steps of the proposed method are as follows.

Step 1: Set up the formulated intuitionistic fuzzy linear programming problem into tabular form known as intuitionistic fuzzy transportation table (IFTP).

Represent the approximate cost by GTRIFNs.

Step 2: Examine whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ or $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_y$.

Case (I): if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. Go to step 3.

Case (II): if $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$, then introduce a dummy column having all its cost as

zero GTRIFNs.

Assume $\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$ as the demand at this dummy destination. Go to step 3.

m n

i=1 j=1

Case (III): if $\sum a_i - \sum b_j$, then introduce dummy row having all its cost zero GTRIFNs.

Assume $\sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$ as availability of the product at the dummy source. Go to step 3.

Step 3: Take the first row and choose its smallest entry (cost) and write it in the front of first row on the right. This is the intuitionistic fuzzy penalty of first row. Similarly, compute the intuitionistic fuzzy penalty of each row and write them in front of each corresponding row.

In the similar way, compute intuitionistic fuzzy penalty computed in step 3 and determine the cost for which this corresponds. Let it be c^I_{ij}. Find $x_{ij} = \min(a_i, b_j)$.

Case (I): If $\min(a_i, b_j) = a_i$, then allocate $x_{ij} = a_i$ in the $(i, j)^{th}$ cell of m x n IFTT. Ignore the i^{th} row to obtain a new IFTT of order $(m – 1)$ x n. Replace b_j by $b_j – a_i$ in obtained IFTT. Go to step 5.

Case (II): If $\min(a_i, b_j) = b_j$, then allocate $x_{ij} = b_j$ in the $(i, j)^{the}$ cell of m x n IFTT. Ignore the j^{th} column to obtain a new IFTT of order m x $(n – 1)$. Replace a_i by $a_i - b_j$ in obtained IFTT. Go to step 5.

Step 5: Calculate the fresh penalties for the reduced IFTT as in step 4. Repeat step 4 until IFTT is reduced into IFTT of order 1 x 1.

Step 6: Allocate all xij in the $(i, j)^{th}$ cell of the given IFTT.

Step 7: The IBFS and initial intuitionistic fuzzy transportation cost are x_{ij} and

$$\sum_{i=1}^{m} \sum_{j=1}^{n} c^I_{ij} x_{ij}$$ respectively.

2.5.2 Generalized Intuitionistic Modified Distribution Method (GIMDM) for Finding Optimal Solution:

In this section, generalized intuitionistic modified distribution method is proposed to find the optimal solution of IFTP. The proposed algorithm is an extension of classical approach. Algorithm of GIMDM is illustrated as follows.

Step 1: Find IBFS by proposed IF Max-Min method.

Step 2: Calculate intuitionistic fuzzy dual variables u^I_i and υ^I_j for each row and column respectively, satisfying u^I_j □ $\upsilon^I_j = c^I_{ij}$ for each occupied cell.

To start with, take any u^I_i or υ^I_j as (-1, 0, 0, 1; 1) (-1, 0, 0, 1; 0)

Step 3: For unoccupied cells, find opportunity cost d^I_{ij} by the relation $d^I_{ij} = c^I_{ij} – (u^I_i$ □ $\upsilon^I_j)$.

Step 4: Calculate the x_0 value of each d^I_{ij}.

Case (I): If $x_0\ (d^I_{ij}) \geq 0$ for all unoccupied cells, then obtained IBFS is intuitionistic fuzzy optimal solution.

Case (II): If at least one $x_0\ (d^I_{ij}) < 0$, then IBFS is not optimal. Go to step 5.

Step 5: Select the unoccupied cell corresponding to which x_0 value of d^I_{ij} is most negative.

Step 6: Construct the closed loop as follows:

Start the closed loop with the selected unoccupied cell (in Step 5) and move horizontally and vertically with corner cells occupied and return to selected unoccupied cell to complete the loop. Assign + and – sign alternatively at the corners loop, by assigning the + sing to the selected unoccupied cell first.

Step 7: Find the minimum allocation value from the cells having – sign.

Step 8: Allocate this value to the selected unoccupied cell and add it to the other occupied cells having + sign and subtract it to the other occupied cell having – sign.

Step 9: Allocation in step 8 will yield an improved basic feasible solution.

Step 10: Test the optimality condition for improved basic feasible solution. The process terminates when $x_0(d^I_{ij}) \geq 0$ for all unoccupied cells.

2.6 Numerical Examples:

Consider the following 3 x 3 IFTT in which the costs are represented by GTRIFNs

Table no. 2.2:

	D1	D2	D3	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15;0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30

	D1	D2	D3	Supply
S3	(2,7,11,15;0.5)(1,7,11, 18;0.3)	(5,9,12,16;0.7)(3,9,12, 19;0.2)	(4,6,8,10;0.6)(3,6,8, 12;0.3)	40
Demand	35	45	15	

Since $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 95$, the problem is balanced.

By first iteration, we have Table no. 2.3:

	D1	D2	D3	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3)	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15;0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
S3	(2,7,11,15;0.5)(1,7,11, 18;0.3)	(5,9,12,16;0.7)(3,9,12, 19;0.2)	(4,6,8,10;0.6)(3,6,8, 12;0.3) **15**	25
Demand	35	45	-	

Therefore, after first iteration, IFTT reduces to Table no. 2.4:

	D1	D2	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2)	(4,8,10,13;0.4) (3,8,10,15;0.3)	30
S3	(2,7,11,15;0.5) (1,7,11,18;0.3)	(5,9,12,16;0.7) (3,9,12,19;0.2)	25
Demand	35	45	

Finally, IBFS is Table no. 2.5:

	D1	D2	D3	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3) **25**	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2 **10**	(4,8,10,13;0.4) (3,8,10,15;0.3) **20**	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
S3	(2,7,11,15;0.[illegible])([illegible]1, 18;0.3) **25**	(5,9,12,16;0.7)([illegible] 19;0.2)	(4,6,8,10;0.6)(3,6,8, 12;0.3) **15**	40
Demand	35	45	15	

Thus, IBFS is x_{12} = 25, x_{21} = 10, x_{22} = 20, x_{31} = 25, x_{33} = 15, and the transportation cost is

25(3, 5, 7, 12; 0.5) (1, 5, 7, 15; 0.3) □10((2, 5, 8, 10; 0.6) (1, 5, 8, 12; 0.2) □20(4, 8, 10, 13; 0.4) (3, 8, 10, 15; 0.3) □25(2, 7, 11, 15; 0.5) (1, 7, 11, 18, 0.3) □15(4, 6, 8, 10; 0.6) (3, 6, 8, 12; 0.3) = (285, 600, 850, 1185; 0.4) (165, 600, 850, 1425; 0.3).

Now we apply GIMDM, to compute optimal solution.

Now, firstly we calculate intuitionistic fuzzy dual variables uIij and υIij for each row and column respectively, satisfying u^I_{ij} □ υ^I_{ij} = c^I_{ij} for each occupied cell.

So, let υ^I_1 = (-1, 0, 0, 1; 1) (-1, 0, 0, 1; 0).

For each occupied cell,

$$u^I_1 + \upsilon^I_2 = (3,5,7,12;0.5)\ (1,5,7,15;0.3)$$

$$u^I_2 + \upsilon^I_1 = (2,5,8,10;0.6)\ (1,5,8,12;0.2)$$

$$u^I_2 + \upsilon^I_2 = (4,8,10,13;0.4)\ (3,8,10,15;0.3)$$

$$u^I_3 + \upsilon^I_1 = (2,7,11,15;0.5)\ (1,7,11,18;0.3)$$

$$u^I_3 + \upsilon^I_3 = (4,6,8,10;0.6)\ (3,6,8,12;0.3)$$

Thus, we get,

$$u^I_3 = (1, 7, 11, 16; 0.5)\ (0, 7, 11, 19; 0.3)$$

$$u^I_2 = (1, 5, 8, 11; 0.6)\ (0, 5, 8, 13; 0.2)$$

$$\upsilon^I_3 = (-12, -5, 1, 9; 0.5)\ (-16, -5, 1, 12; 0.3)$$

$$\upsilon^I_2 = (-7, 0, 5, 12; 0.4)\ (-10, 0, 5, 15; 0.3)$$

$$u^I_1 = (-9, 0, 7, 19; 0.4)\ (-14, 0, 7, 25; 0.3)$$

Therefore,

$$d^I_{11} = (-18, -3, 8, 25; 0.4)\ (-25, -3, 8, 33; 0.3)$$

$$d^I_{13} = (-26, -3, 14, 37; 0.4)\ (-36, -3, 14, 48; 0.3)$$

$$d^I_{23} = (-17, -3, 10, 26; 0.5)\ (-23, -3, 10, 34; 0.3)$$

$$d^I_{32} = (-23, -7, 5, 22; 0.4)\ (-31, -7, 5, 29; 0.3)$$

Since the value of $x_0(d^I_{32})$ is most negative, so IBFS is not intuitionistic fuzzy optimal.

Table no. 2.6: Construction of loop

	D1	D2	D3	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3) **25**	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2) **10**	(4,8,10,13;0.4) (3,8,10,15;0.3) **20**	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
S3	(2,7,11,15;0.5)(1,7,11,18;0.3)(-) **25**	(5,9,12,16;0.7)(3,9,12,19;0.2)(+) **20**	(4,6,8,10;0.6)(3,6,8,12;0.3)	40
Demand	35	45	15	

Since the minimum allocation in the cell marked with (-) sign is 20, so, add 20 to the cell with (+) sign, and subtract 20 from the cell with (-) sign.

Table no. 2.7: Improved basic feasible solution

	D1	D2	D3	Supply
S1	(2,4,8,15;0.6) (1,4,8,18;0.3)	(3,5,7,12;0.5) (1,5,7,15;0.3) **25**	(2,5,9,16;0.7) (1,5,9,18;0.3)	25
S2	(2,5,8,10;0.6) (1,5,8,12;0.2) **30**	(4,8,10,13;0.4) (3,8,10,15;0.3)	(3,6,10,15;0.8) (2,6,10,18;0.2)	30
S3	(2,7,11,15;0.5)(1,7,11,18;0.3) **5**	(5,9,12,16;0.7)(3,9,12,19;0.2) **20**	(4,6,8,10;0.6)(3,6,8,12;0.3) **15**	40
Demand	35	45	15	

Now compute uIij and υIij satisfying u^I_{ij} □ $\upsilon^I_{ij} = c^I_{ij}$ for each occupied cell.

Let $u^I_3 = (-1, 0, 0, 1; 1)\ (-1, 0, 0, 1; 0)$.

For each occupied cell, we have, $uI_1 + \upsilon^I_2 = (3,5,7,12;0.5)\ (1,5,7,15;0.3)$

$$u^I_2 + \upsilon^I_1 = (2,5,8,10;0.6)\ (1,5,8,12;0.2)$$

$$u^I_3 + \upsilon^I_1 = (2,7,11,15;0.5)\ (1,7,11,18;0.3)$$

$$u^I_3 + \upsilon^I_2 = (5,9,12,16;0.7)\ (3,9,12,19;0.2)$$

$$u^I_3 + \upsilon^I_3 = (4,6,8,10;0.6)\ (3,6,8,12;0.3)$$

after solving above equations, we get,

$$\upsilon^I_3 = (3, 6, 8, 11; 0.6)\ (2, 6, 8, 13; 0.3)$$

$$\upsilon^I_2 = (4, 9, 12, 17; 0.7)\ (2, 9, 12, 20; 0.2)$$

$$\upsilon^I_1 = (1, 7, 11, 16; 0.5)\ (0, 7, 11, 19; 0.3)$$

$$u^I = (-14, -6, 1, 9; 0.5)\ (-18, -6, 1, 12; 0.3)$$

$$u^I_1 = (-14, -7, -2, -8; 0.5)\ (-19, -7, -2, 13; 0.3)$$

Thus, for each unoccupied cell,

$$d^I_{11} = (-22, -5, 8, 28; 0.5)\ (-31, -5, 8, 37; 0.3)$$

$$d^I_{13} = (-17, -1, 10, 27; 0.5)\ (-25, -1, 10, 35; 0.3)$$

$$d^I_{23} = (-17, -3, 10, 26; 0.5)\ (-23, -3, 10, 34; 0.3)$$

Since $x_0(d^I_{ij}) \geq 0$ for all unoccupied cells, so optimal solution is $x_{12} = 25$, $x_{21} = 30$, $x_{31} = 5$, $x_{32} = 20$, $x_{33} = 15$, and the minimum transportation intuitionistic fuzzycost is 25(3, 5, 7, 12; 0.5) (1, 5, 7, 15; 0.3) □30(2, 5, 8, 10; 0.6) (1, 5, 8, 12; 0.2) □5(2, 7, 11, 15; 0.5) (1, 7, 11, 18, 0.3) □20(5, 9, 12, 16; 0.7) (3, 9, 12, 19; 0.2) □ 15(4, 6, 8, 10; 0.6)(3, 6, 8, 12; 0.3) = (305, 580, 830, 1145; 0.5) (165, 580, 830, 1385; 0.3).

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ISBN: 978-81-19149-24-7

Chapter 3

An Integrated Method Using Intuitionistic Fuzzy Set and Linear Programming for Supplier Selection Problem

3.1 Introduction:

In the recent years, competitive pressures are forcing enterprises to re-attend on theirs supply chain management (SCM) and use new strategies to design and develop engineering products to quickly and exactly responding the customers' demand. In these new strategies, to ensure the quality and performance of products, suppliers work closely with in-house designers to design some of sub-assemblies and components. Therefore, companies pay particular attention to the indentification and selection appropriate suppliers. Supplier selection involves several conflicting criteria, where decision maker’s knowledge is usually vague and imprecise. Thus, it is a multi-criteria group decision-making problem.

Sonmez (2006) defined supplier selection as “the process of the suppliers where able to provide the buyer with the right quality products and/or services at the right price, at the right quantities and at the right time”. Selection of suppliers has a direct impact on the financial, technical and operational performance of an organization.

It influences products cost and quality. Therefore, it affects directly competitiveness of the organization in the market and end customer satisfaction. Supplier selection is a multiple criteria decision-making (MCDM) problem affected by qualitative and quantitative criteria (Chan et al. 2007). These factors are defined to measure important aspects of the supplier’s business as financial and technical ability, support resources, quality systems and so on.

The overall objective of supplier selection process is to maximize overall value to the buyer, reduce by risk, and build long term relationships between buyers and suppliers (Chena et.al, 2006).

This article develops a new method for supplier selection problem. Analytical hierarchy process (AHP) is used to evaluate the supplier selection factors.

3.2 Background:

Researchers have been widely studied multi-criteria techniques to select the best suppliers. They are used many methods such as cluster analysis, case-based reasoning systems, statistical models’ decision support systems, data envelopment analysis, multi criteria decision making, analytical hierarchy process (AHP), analytical network process (ANP), TOPSIS and SMART.

Total cost of ownership models, activity-based costing, artificial intelligence (neural network, fuzzy set theory), mathematical programming and some of hybrid model such as AHP-LP, ANP-GP, FAHP, F-TOPSIS.

A good review of the methods for supporting supplier selection is represented by Aissaoui et.at (2007) and Ho et. Al (2010). Akarte et. Al (2001) created an AHP system based upon web to evaluate the suppliers. It uses 18 criteria to evaluate the suppliers and related importance weightings are determined by using a pairwise comparison. Gencer et al. (2007) considered supplier selection as a multi-criteria decision problem. They developed a model usage of analytic network process (ANP) in supplier selection criteria in feedback systematic.

A hierarchy model based upon fuzzy set theory is presented by Chen et al. (2006) to select best supplier. They used linguistic variables to assess supplier factors. This model considered both quantitative and qualitative criteria.

Many works integrated several approaches to evaluate the performance of suppliers and to select the best suppliers.

Mendoza et al. (2008) used goal programming to develop an integrated AHP-GP approach to sort best suppliers while determine the optimal order quantity.

For first time, amid et al. (2006) presented a fuzzy multi-objective linear model to overcome the vagueness of the information and used different weights for various objectives.

Ount et al. (2009) developed a supplier evaluation approach based on the ANP and the TOPSIS methods, under the fuzzy environment, to help a telecommunication company. They used Fuzzy ANP to calculate criteria weights and Fuzzy TOPSIS to select a supplier.

Lee (2009) developed a fuzzy analytic hierarchy process (FAHP) model to evaluate suppliers, which incorporates the benefits, opportunities, costs and risks (BOCR) concept and a performance ranking of the suppliers is obtained.

The rest of this book is structured as follows: section 3 discusses the proposed method for supplier selection. This section is included the overview of Intuitionistic fuzzy set, and developing model. Solution methodology present in section 4.

Section 5 represents a numerical example to select the best suppliers by suggested method and in final section; conclusion and future research is presented.

3.3 Background:

This book develops a hybrid method by using intuitionistic fuzzy sets (IFS) and linear programming to select suppliers for manufacturing firms. Use of IFSs provides a formal language for explaining lack of information in the human reasoning, to generate decisions. The following sections describe intuitionistic fuzzy sets and applied method for solving multi-objective model.

3.3.1 Intuitionistic Fuzzy Sets:

In some real-life situations, a decision maker (DM) may not be able to accurately express his/her preferences for alternative due to that DM may not possess a precise level of knowledge and the DM is unable to express the degree to which one alternative are better than others. In such cases, the DM may provide his/her preferences with a degree of doubt. IFSs are suitable for these situations. Intuitionistic fuzzy set (IFS) is a generalization of fuzzy set theory, introduced by Atanassov (1986). It characterized by a membership function and a non-membership function.

Intuitionistic fuzzy set A is introduced by:

$$A = \{\mu_A(X), \upsilon_A(X) | x \in X\}$$

$$0 \leq \mu_A(X) + \upsilon_A(X) \leq 1$$

$\mu_A(x)$ and $\upsilon_A(x)$ are membership and non-membership functions, respectively. IFSs have a third parameter that usually known as the DM's hesitation degree. This index expresses lack of knowledge whether x belongs to A or not.

$$\pi A = 1 - \mu_A(X) - \upsilon_A(X) \quad (1)$$

It is obvious that $0 \leq \pi_A(x) \leq 1$, for each $x \in X$.

Smaller $\pi_A(x)$ indicates more certain knowledge about x certain and vice versa. Obviously, when $\pi_A(x) = 0$, fuzzy set concept is resulted (Shu et al. 2006). If A and B are two intuitionistic fuzzy sets, then:

$$A_1.A_2 = \{(x, \mu_A(x).\ \mu_B(x), \upsilon_A(x) + \upsilon_B(x) - \upsilon_A(x).\ \upsilon_B(x)) \mid x \in X\} \quad (2)$$

$$A_1 + A_2 = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x).\ \upsilon_B(x), \upsilon_A(x).\ \upsilon_B(x)) \mid x \in X\} \quad (3)$$

$$\bar{A} = (\upsilon_A(x), \mu_A(x)) \quad (4)$$

$$\lambda A = \{(1 - (1 - \mu_A(x))^{\lambda}, \upsilon_A(x)^{\lambda}) \mid x \in X\} \quad (5)$$

score function S of an intuitionistic fuzzy value shown as follows [Xu, 2007]:

$$S(A) = \mu_A(x) - \upsilon_A(x) \quad (6)$$

The larger S indicates the greater the intuitionistic fuzzy value A.

3.3.2 Linear Programming Model:

This model is proposed to determine suppliers and to calculate the optimum order quantities among the selected suppliers. In order to formulate the model, notations of the model are defined as follow: Indices:

$i = 1, 2, \ldots, an$	Index of suppliers
$j = 1, 2, \ldots, e_1$	Index of satisfaction factors
$k = 1, 2, \ldots, e_2$	Index of flexibility factors

Parameters:

D	Anticipated demand
C	Supplier capacity
SI	Satisfaction index
FI	Flexibility index
w_{1j}	Relative important of j^{th} element of SI
w_{2k}	Relative important of k^{th} elements FI
S_{ij}	Value of the jth factor of SI for supplier i
F_{ij}	Value of the element j for FI of supplier i

Decision variables:

X_i	Ordered amount to supplier i
Y_i	1 if supplier i is selected, 0 otherwise

The objective functions and the constraints of this model are described as follow:

$$\max f_1 = \sum_{n} SI_i X_i \quad (7)$$

$$\max f_2 = \sum_{i=1}^{n} FI_i Y_i \quad (8)$$

$$SI_i = \sum_{i=1}^{n} \sum_{j=1}^{e_1} w_{1i} S_{ij} \quad (9)$$

$$FI_i = \sum_{i=1}^{n} \sum_{j=1}^{e_2} w_{2i} f_{ij} \quad (10)$$

$$\sum_{i=1}^{n} X_i \geq D \quad (11)$$

$$X_i \leq C_i \quad \forall i \quad (12)$$

$$X_i \leq MY_i \quad \forall i \quad (13)$$

$$Y_i \leq X_i \quad \forall i \quad (14)$$

$$X_i \geq 0 \quad \forall i \quad (15)$$

$$y_i \in \{0, 1\} \quad \forall i \quad (16)$$

Objective 7 maximizes satisfaction index of the suppliers and objective 8 maximizes flexibility index.

Constraints 9 and 10 show satisfaction and flexibility indices of supplier I , respectively. Constraint 11 says that ordered values to suppliers have to support buyer demand. Constraint 12 shows that ordered values to each supplier have to be less than its capacity.

Constraints 13 and 14 relate two variables X and Y, and if $x_i > 0$ then $Y_i = 1$.

M in constraint 13 is a large number.

3.3.3 To Solve Multi-Objective Model:

Zimmermann (1978) used the following steps to solve multi-objective linear programming (MOLP) model, by applying fuzzy logic approach:

Step 1. Solve the MOLP of as a single-objective linear programming model by using only one objective at a time and ignoring the others.

Step 2. By using optimal solutions calculated from previous step, values for other functions are obtained and pay-off matrix of developed as follows:

f_1 …

x^*_1 (f^*_{11} …

. . .

. . .

. . .

x^*_n (f_{n1} …

Here x^*_1, … are the optimal solutions of the objective functions $f_1(x)$, …

Step 3. Obtain lower bound (L) and upper bound (U) for each objective:

. $Lr = \min\{f_r(x_1),$ $\}$ r = 1, 2, …, n

. $Ur = \max\{f_r(x_1), \ldots$ $\}$ r = 1, 2, …, n

Step 4. Define membership function for each objective.

$$\mu_{fr}(x) = \begin{cases} 1 & f_r(x) \le L_r \\ \dfrac{U_r - f_r(x)}{U_r - L_r} & L_r \le f_r(x) \le U_r \\ 0 & f_r(x) \le L_r \end{cases} \quad (17)$$

Step 5. Convert multi-objective problem into a single objective problem.

$\max \lambda$ (18)

$\lambda (U_r - L_r) \le U_r - f_r(x)$ r = 1, …, n (19)

$g(x) \le b$ = (20)

$x \ge 0, \lambda \square [0, 1]$ (21)

3.4 The Proposed Approach:

This method uses IFSs to explain two indices, satisfaction and flexibility. Satisfaction Index (SI) is a measure of the extent to which a buyer is satisfied by a supplier capability and it is calculated by 3 importance factors of the quality, price, and lead-time. Flexibility Index (FI) shows the additional capability of the supplier to respond when buyer requirement changes and is calculated by 2 factors: extra production volume and product variety.

FI in the product volume (FLvo) shows extra capacity percent what supplier can allocate buyer contrast changes in demand. FI in the product variety (FIva) show the ability to create different products.

The proposed approach includes the following steps:

Step 1- Evaluate SI and FI.

Satisfaction functions are defined for satisfaction factors. Satisfaction functions for any factor show the buyer satisfaction's measurement for the related factor.

Step 2- Determine relative weight of each element of index.

Relative score these indexes are determined by IFSs which is described as follows:

2-1- Determine a group of the decision makers and their weights ($w_D = w_{D1}, w_{D2}, \ldots, w_{D\upsilon}$) which is expressed by linguistic variables. These linguistic variables are shown in table 1.

2-2- Construct intuitionistic fuzzy preference relations to determine score for each factor of each index by each DM.

Preference relations are expressed by linguistic variables (table 1). The DMs provides his/her intuitionistic preference for each pair of criteria. To calculate score, the following steps are implemented:

Table 3.1: Linguistic variables for importance of each criterion and DMs

Linguistic Values	IFNs
Very Low (VL)	(0.1, 0.9)
Low (L)	(0.15, 0.25)
Medium Low (ML)	(0.25, 0.32)
Medium (M)	(0.5, 0.4)
Medium High (MH)	(0.55, 0.25)
High (H)	(0.85, 0.1)
Very High (VH)	(0.9, 0.1)

2-2-1- Calculate score of each factor, by each DM:

$$W^{\upsilon}_{ci} = \frac{1}{n} \sum_{j=1}^{n} W^{\upsilon}_{ij}, \qquad i = 1, 2, \ldots, n \qquad (22)$$

W^{υ}_{ci} is Averaged intuitionistic fuzzy value i of the criterion over all the other criteria which is concluded from v^{th} DM. W^{υ}_{ij} is intuitionistic preference relation of the criterion i on j that is determined by v^{th} DM. Sum of the intuitionistic fuzzy numbers obtain by equation 3. Also, multiply a constant number in intuitionistic fuzzy number compute equation 5.

2-2-2- Obtain final score by using equations 2 and 3.

$$w_{ci} = \sum_{\upsilon=1}^{m} w_{D\upsilon} W^{\upsilon}_{ci}, \qquad i = 1, 2, \ldots, n \qquad (23)$$

where w_{ci} is intuitionistic fuzzy weight criterion i and w_{Dk} is weight of v^{th} DM.

3-2- Obtain score by using equation 6.

Step 3- Putting the results of the previous step in the linear programming problem.

3.5 Numerical Example:

A hair drier manufacturer wants to produce a new model. He needs by purchasing engines with power 2000w from external suppliers. after primary evaluations, 5 suppliers are selected as the qualified suppliers. suppliers' data is represented in the table 2.

Table 3.2: Suppliers' information

Supplier	Price	Leadtime	Quality	Variety	Volume
1	15-0.5	11-.8	3-.78	3/10	.2
2	12-.8	8-1	1.5-.94	2/10	.15
3	14-.6	6-.5	4-.67	4/10	.24
4	16-.4	10-1	3.5-.72	6/10	.12
5	11-.9	12-.6	2.5-.83	5/10	.09

Supplier selection process is shown below:

Step 1 – Evaluate SI and FI.

1-1- Determine satisfaction functions for any factor. Satisfaction function, indeed, shows desirable level of customers for each factor. These functions are shown in the table 3. Amount of the satisfaction for each factor is shown in table 4.

2-1- Determine flexibility index.

FI in the product volume (FI-vo), shows the capacity of suppliers for answering extra demand. Proportion of this potential capability is shown in the 6^{th} column of table 4. FI in the product variety (FI-va) shows the ability to create different products. We assume that the ten versions of the desired product. We assume that the ten versions of the desired product are produced by different companies. Suppliers Ability to produce different products is shown in Table 4.

Step 2- Determine relative weight of each element of index.

2-1- Determine decision makers and their weights by linguistic variables. Intuitionistic weights of DMs are: (0.85, 0.1), (0.9, 0.1) and (0.85, 0.1).

Table 3.4: Suppliers' information

Supplier	Price	Leadtime	Quality	variety	Volume (%)
1	0.5	0.8	0.78	3/10	0.2
2	0.8	1	0.94	2/10	0.15
3	0.6	0.5	0.67	4/10	0.24
4	0.4	1	0.72	6/10	0.12
5	0.9	0.6	0.83	5/10	0.09

1- 2-2- Construct intuitionistic fuzzy preference relations to determine score of each factor index by each DM. Score of each factor of the index by each DM is calculated by equations 13. The results are presented in Table 5 and 6 for SI and FI. 3-2- Obtain intuitionistic fuzzy weights (IFWs). To calculate IFWs, equations 22 and 23 are used. The results are presented in Table 3.7.

4-2-Final score: Final score for factors of SI and FI are calculated by using equation 6, and the results are shown in tables 7 and 8, respectively. 4-Put weights in the linear programming model and solve it. Weights are put in the linear programming model and order quantities to each supplier identified that are shown in table 9. As you can see from table suppliers 1, 4 and 5, with values 300, 500 and 700, respectively, are selected.

3.6 Conclusion and Future Research:

This article outlined a new method by using intuitionistic fuzzy set (IFS) and linear programming for supplier selection problem. To order select suppliers, two indices were introduced: satisfaction index (SI) and flexibility index (FI). For SI, 3 factors quality, price and leadtime are defined. In contrast to the previous works, in this book flexibility is determined as a factor with a detailed definition. Two factors, production volume and product variety, is developed for FI.

The relative importance of the factors of SI and FI are calculated by IFS method.

By Using IFSs, decision-making process is more realistic.

In IFS, DM may provide his/her preferences with a degree of doubt in a more realistic form. A linear programming model uses the relative weights of each factor to determine the most suitable suppliers. There are a number of opportunities for expanding the research, including defining further factors or other indices or considering the inter-dependency between the evaluation factors.

Table 3.5: Determined score by DM for SI

Element FI	**DM_1**	**DM_2**	**DM_3**
Price	0.42,0.31	0.42,0.31	0.65,0.23
Leadtime	0.43,0.31	0.43,0.3	0.32,0.27
Quality	0.39,0.52	0.3,0.4	0.3,0.41

Table 3.6 Determined score by DM for FI

Element FI	**DM_1**	**DM_2**	**DM_3**
Variety	0.73,0.22	0.39,0.42	0.53,0.35
Volume	0.33,0.65	0.43,0.35	0.32,0.27

Table 3.3. Satisfaction functions

$$U_p = \begin{cases} 1 & c \leq 10 \\ \dfrac{20 - c}{20 - 10} & c \geq 10 \end{cases}$$

Satisfaction function for price: satisfaction measurement for buyer is desirable when goods price is lower or equal than 10$.

$$U_l = \begin{cases} \dfrac{r - 5}{7 - 5} & 5 \leq r \leq 7 \\ 1 & 7 < r \leq 10 \\ \dfrac{15 - r}{15 - 10} & 10 < r \leq 15 \end{cases}$$

Satisfaction function for lead time: The product is desirable when its lead-time is between 7 and 10 days. Also, for lead-time less than a predefined limit (i.e. 7 days), due to problems related to inventory capacity, satisfaction of buyer reduced.

$$U_q = \begin{cases} 1 & r \leq 1 \\ \dfrac{10 - r}{10-1} & 1 \leq r \leq 10 \\ 0 & r > \end{cases}$$

Satisfaction function for quality: An order is desirable when defect rate of the product (r, as the defect percentage) is lower or equal than 1%.

Table 3.8: Final score FI

H, VH, H	Variety	Volume
IFW	0.86, 0.06	0.68, 0.1
Score	0.8	0.58
Crisp weight	0.58	0.42

Table 3.9. Allocated values each supplier

Supplier	Ordered Value
1	300
2	0
3	0
4	500
5	700

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ISBN: 978-81-19149-24-7

Chapter 4

Intuitionistic Fuzzy Multi-Objective Non-Linear Programming Problem

4.1 Introduction:

The concept of maximizing decision was initially proposed by Bellman and Zadeh [3], Zadeh [10-12]. By adopting this concept of fuzzy sets was applied in mathematical programs firstly by Zimmerman [12]. In the past few years, many researchers have come to the realization that a variety of real-world problems which have been previously solved by non-linear programming techniques are in fact more complicated. Frequently, these problems have multiple goals to be optimized rather than a single objective. Moreover, many practical problems cannot be represented by non-linear programming model. Therefore, attempts were made to develop more general mathematical programming methods and many significant advances have been made in the area of multi-objective non-linear programming. Several authors in the literature have studied the fuzzy multi-objective quadratic programming problems.

Out of several higher order fuzzy sets, intuitionistic fuzzy sets (IFS) [1,2] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives. Certainly, fuzzy sets theory is not appropriate to deal with such problems; rather intuitionistic fuzzy sets (IFS) theory is more suitable. The Intuitionistic fuzzy set was introduced by Atanassov. K. T. [1] in 1986. For the fuzzy multiple criteria decision-making problems, the degree of satisfiability and non-satisfiability of each alternative with respect to a set of criteria is often represented by an intuitionistic fuzzy number (IFN). This Intuitionistic fuzzy mathematics is very little studied subject.

In a recent review, Toksari [4] gave a Taylor series approach to fuzzy multi-objective fuzzy quadratic programming problem. A. Nagoorgani, R. Irene Hepzibah et al., [6] proposed a method to solve multi-objective fuzzy quadratic programming problems are available in the literature [4, 7, 8, 13]. In this book, membership and non-membership functions, which are associated with each objective of intuitionistic fuzzy multi-objective quadratic programming problem (IFMOQPP) are transformed by using first order Taylor polymial series [4, 8]. Then the IFMOQPP can be reduced to a single objective linear programming. The book is organized as follows:

The formulation of the problem is given in Section 2 and Section 3 deals with an algorithm for solving a intuitionistic fuzzy multi-objective quadratic programming problem. Finally, in Section 4, the effectiveness of the proposed method is illustrated by means of an example. Some concluding remarks are provided in section 5.

4.2 Formation of the Problem:

The multi-objective linear programming problem and the multi-objective intuitionistic fuzzy linear programming problem are described in this section.

A. Multi-objective quadratic programming problem (MOQPP)

A linear multi-objective optimization problem is stated as

Maximize or Minimize: $[z_1(x), z_2(x), ski(x)]$

Subjecti to $Ax (\leq = \geq) b, x \geq 0$

Where $z_j(x)$, j = 1, 2, n is an N vector of cost coefficients, An an m x N – Coefficients matrix of constraints and b an m vector of demand (resource) availability.

A. Intuitionistic fuzzy multi-objective quadratic programming problem (IFMOQPP)

If an imprecise aspiration level is introduced to each of the objectives of MOQPP,

then these intuitionistic fuzzy objectives are termed as intuitionistic fuzzy goals.

Let g^l_k be the aspiration level assigned to the k^{th} objective $z_k(x)$.

Then the intuitionistic fuzzy objectives appear as

a. $Z_k(x) \geq g_k^l$ (for maximizing $Z_k(x)$);
b. $Z_k(x) \leq g_k^l$ (for minimizing $Z_k(x)$);

Where $\geq$ and $\leq$ indicate the fuzziness of the aspiration levels,

and is to be understood as “essentially more than” and “essentially less than in the sense of Zimmerman [12].

Hence, the intuitionistic fuzzy multi-objective linear programming problem can be stated as follows:

Find X

So as to satisfy $Z_i(x) \leq g_i^l$, i=1, 2, ………………i_l, $Z_i(x) \geq g_i^l$, i=i_{l+1}, i_{l+2}, k

Subject to x□ X, $X \geq 0$.

Now, in the field of intuitionistic fuzzy programming, the intuitionistic fuzzy objectives are characterized by their associated membership functions and non-membership functions. They can be expressed as follows:

$$\text{If } Z_i(x) \geq g_i^l,\ \mu_i^l(x) = \begin{cases} 1, & \text{if } z_i(x) \geq g_i \\ \dfrac{z_i(x) - t_i}{g_i - t_i}, & \text{if } t_i \leq z_i(x) \leq g_i \\ 0, & \text{if } z_i(x) \leq t_i \end{cases}$$

And

$$\upsilon_i^l(x) = \begin{cases} 0, & \text{if } z_i(x) \geq g_i \\ \dfrac{z_i(x) - g_i}{t_i - g_i}, & \text{if } t_i \leq z_i(x) \leq g_i \\ 1, & \text{if } z_i(x) \leq t_i \end{cases}$$

$$\text{If } Z_i(x) \leq g_i^l,\ \mu_i^l(x) = \begin{cases} 1, & \text{if } z_i(x) \leq g_i \\ \dfrac{t_i - z_i(x)}{t_i - g_i}, & \text{if } g_i \leq z_i(x) \leq t_i \\ 0, & \text{if } z_i(x) \leq t_i \end{cases}$$

And

$$\upsilon_i^l(x) = \begin{cases} 0, & \text{if } z_i(x) \geq g_i \\ \dfrac{g_i - z_i(x)}{t_i - g_i}, & \text{if } t_i \leq z_i(x) \leq g_i \\ 1, & \text{if } z_i(x) \leq t_i \end{cases}$$

Where t_i and t_i are the upper tolerance limit and the lower tolerance limit respectively, for the i^{th} intuitionistic fuzzy objective. Now, in a intuitionistic fuzzy decision environment, the achievement of the objective goals to their aspired levels to the extent possible are actually represented by the possible achievement of their respective membership values and non-membership values to the highest degree. The relationship between constraints and the objective functions in the intuitionistic fuzzy environment is fully symmetric, that is, there is no longer a difference between the former and the latter. This guarantees the maximization of both objectives' membership values and non-membership values simultaneously.

4.3 Algorithm for Intuitionistic Fuzzy Multi-Objective Quadratic Programming Problem:

Toksari [4] proposed a Taylor series approach to fuzzy multi-objective linear fractional programming. Here, in the intuitionistic fuzzy multi-objective quadratic programming problem, membership functions and non-membership functions associated with each objective are transformed by using Taylor series at first and then a satisfactory value(s) for the variable(s) of the model is obtained by solving the intuitionistic fuzzy model, which has a single objective function. Based on this idea, an algorithm for solving intuitionistic fuzzy multi-objective quadratic programming problem is developed here.

Step 1. Determine $x^*I = (x^*_{i1}, x^*_{i2}, x^*_{in})$, that is used to maximize or minimize the i^{th} membership function $\mu_i^l(x)$ and non-membership function $\upsilon_i^l(x)$ (i=1, 2, k) where n is the number of variables. Step 2. Transform membership and non-membership functions by using first-order Taylor polynomial series

$$\mu_i^l(x) = \mu_i^l(x) = \mu_i^l(x^*) + [(x_1 - x_{i1}^*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_1} + (x_2 - x_{i2}^*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_2} + \ldots$$

$$\ldots\ldots + (x_n - x_{in}^*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_n}]$$

$$\mu_i^l(x) = \mu_i^l(x) = \mu_i^l(x^*) + \sum_{j=1}^{n} (x_j - x_{ij}^*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_j}$$

$$\upsilon_i^l(x) = \upsilon_i^l(x) = \upsilon_i^l(x^*) + [(x_1 - x_{i1}^*) \frac{\partial \upsilon_i^l(x_i^*)}{\partial x_1} + (x_2 - x_{i2}^*) \frac{\partial \upsilon_i^l(x_i^*)}{\partial x_2} + \ldots$$

$$\ldots\ldots + (x_n - x_{in}^*) \frac{\partial \upsilon_i^l(x_i^*)}{\partial x_n}]$$

$$\frac{\partial \upsilon_i^l(x_i^*)}{\partial x_j}$$

$\upsilon_i^l(x) = \upsilon_i^l(x) = \upsilon_i^l(x^*) + \sum_{j=1}^{n} (x_j - x_{ij}*)$ Step 3.

Find satisfactory $x^* = (x^*_1, x^*_2, \ldots\ldots\ldots, x^*_n)$

by solving the reduced problem to a single objective for membership function and non-membership function respectively.

$$p(x) = \sum_{i=1}^{k} \mu_i^l (x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_j}$$

and $q(x) = \sum_{i=1}^{k} \mu_i^l (x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}*) \frac{\partial \upsilon_i^l(x_i^*)}{\partial x_j}$

thus, IFMOQPP is converted into a new mathematical model and is given below:

Maximize or minimize

$$\sum_{i=1}^{k} \mu_i^l (x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}*) \frac{\partial \mu_i^l(x_i^*)}{\partial x_j}$$

And maximize or minimize $q(x) = \sum_{i=1}^{k} \mu_i^l (x_i^*) + \sum_{j=1}^{n} (x_j - x_{ij}*) \frac{\partial \upsilon_i^l(x_i^*)}{\partial x_j}$

were

$$\mu_i^l(x) = \begin{cases} 1, & \text{if } z_i(x) \geq g_i \\ \dfrac{z_i(x) - t_i}{g_i - t_i}, & \text{if } t_i \leq z_i(x) \leq g_i \\ 0, & \text{if } z_i(x) \leq t_i \end{cases}$$

And

$$\upsilon_i^l(x) = \begin{cases} 0, & \text{if } z_i(x) \geq g_i \\ \dfrac{z_i(x) - g_i}{t_i - g_i}, & \text{if } t_i \leq z_i(x) \leq g_i \\ 1, & \text{if } z_i(x) \leq t_i \end{cases}$$

$$\mu_i^I(x) = \begin{cases} 1, & \text{if } z_i(x) \le g_i \\ \dfrac{t_i - z_i(x)}{t_i - g_i}, & \text{if } g_i \le z_i(x) \le t_i \\ 0, & \text{if } z_i(x) \le t_i \end{cases}$$

And

$$\upsilon_i^I(x) = \begin{cases} 0, & \text{if } z_i(x) \ge g_i \\ \dfrac{g_i - z_i(x)}{t_i - g_i}, & \text{if } t_i \le z_i(x) \le g_i \\ 1, & \text{if } z_i(x) \le t_i \end{cases}$$

respectively. consider the following MOQPP:

$$\left.\begin{aligned} &\text{minimize } z_1(x) = -4x^1 + x_2{}^1 - 2x_1x_2 + 2x^2{}_2 \\ &\text{minimize } z_2(x) = -3x_1 + x^2{}_1 + 2x_1x_2 + 3x_2{}^2 \end{aligned}\right\} \quad (4.1)$$

subject to the constraints $2x_1 + x_2 \le 6$

$x_1 + 4x_2 \le 12$ and $x_1, x_2 \ge 0$.

a. The membership and non-membership functions were considered to be intuitionistic triangular (see figure 1) [5]. When they depend on three scalar parameters (a_1, b_1, c_1). z_1 depend on intuitionistic fuzzy aspiration levels (-113, -6.5, 100) when z_2 depends intuitionistic fuzzy aspiration levels (-240, 5, 250). The membership and non-membership functions of the goals are obtained as follows:

$$\mu_1^I(x) = \begin{cases} 0, & \text{if } z_1(x) \ge c_1 \\ \dfrac{c_1 - z_1(x)}{c_1 - b_1} & \text{if } b_1 \le z_1(x) \le c_1 \\ \dfrac{z_1(x) - a_1}{b_1 - a_1} & \text{if } a_1 \le z_1(x) \le b_1 \\ 0, & \text{if } z_1(x) \le a_1 \end{cases}$$

That implies

$$\mu_1^I(x) = \begin{cases} 0, & \text{if } z_1(x) \geq 100 \\ \dfrac{100-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{100-(-6.5)} & \text{if } -6.5 \leq z_1(x) \leq 100 \\ \dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-(-113)}{-6.5-(-113)} & \text{if } -113 \leq z_1(x) \leq -6.5 \\ 0, & \text{if } z_1(x) \leq -113 \end{cases}$$

In the similar way,

$$\mu_2^I(x) = \begin{cases} 0, & \text{if } z_2(x) \geq 250 \\ \dfrac{250-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{250-5} & \text{if } 5 \leq z_2(x) \leq 250 \\ \dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-(-240)}{5-(-240)} & \text{if } -240 \leq z_2(x) \leq 5 \\ 0, & \text{if } z_2(x) \leq -240 \end{cases}$$

If $\mu_1^I(x) = \mu_1^I(x) = \mu_1^I(2.46, 1.08) + [(x_1 - 2.46) \dfrac{\partial \mu_i^l(2.46,1.08)}{\partial x_1} + (x_2 - 1.08) \dfrac{\partial \mu_i^l(2.46,1.08)}{\partial x_2}]$

$$\mu_1^I(x) = 0.012x_1 - 0.006x_2 + 0.974 \tag{4.2}$$

If $\mu_2^I(x) = \mu_2^I(x) = \mu_2^I(0.89, 0.22) + [(x_1 - 0.89) \dfrac{\partial \mu_2^l(0.89,0.22)}{\partial x_1} + (x_2 - 0.22) \dfrac{\partial \mu_2^l(0.89,0.22)}{\partial x_2}]$

$\mu_2^I(x) = 0.003x_1 - 0.013x_2 + 0.974$ (4.3)

then the objective of the IFMOQPP is obtained by adding (4.2) and (4.3), that is $p(x) = \mu_1^I(x) + \mu_2^I(x) = 0.015x_1 - 0.019x_2 + 1.948$

subject to the constraints

$$2x_1 + x_2 \leq 6$$

$$x_1 + 4x_2 \leq 12$$

The problem is solved and the solution is obtained is as follows: $x_1 = 2.67$; $x_2 = 0.67$; $z_1(x) = 6.2231$; $z_2(x) = -4.0043$ and the membership values are $\mu_1 = 0.881$ and $\mu_2 = 0.963$. The membership function values show that both goals z_1 and z_2 are satisfied with 88.1% and 96.3% respectively for the obtained solution which is $x_1 = 2.67$; $x_2 = 0.67$.

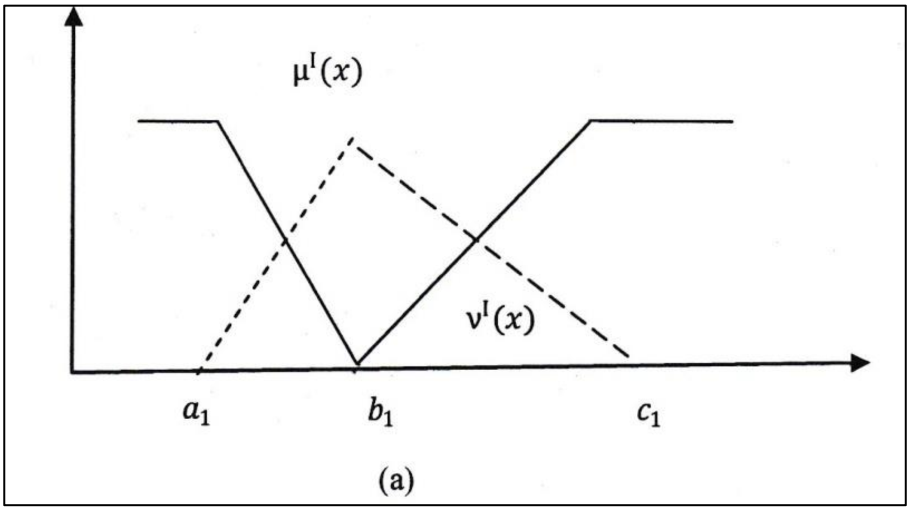

Figure 4.1: Membership and Non-Membership Functions Defined as Intuitionistic Triangular (a)

The non-membership functions of the goals are as obtained as follows:

$$\upsilon_1^I(x) = \begin{cases} 1, & \text{, if } z_1(x) \geq c_1 \\ \dfrac{z_1(x) - b_1}{c_1 - b_1} & \text{, if } b_1 \leq z_1(x) \leq c_1 \\ \dfrac{b_1 - z_1(x)}{b_1 - a_1} & \text{, if } a_1 \leq z_1(x) \leq b_1 \\ 1, & \text{, if } z_1(x) \leq a_1 \end{cases}$$

$$= \begin{cases} 1, & \text{if } z_1(x) \geq 100 \\ \dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-(-6.5)}{100-(-6.5)} & \text{if } -6.5 \leq z_1(x) \leq 100 \\ \dfrac{(-6.5)-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{-6.5-(-113)} & \text{if } -113 \leq z_1(x) \leq -6.5 \\ 1, & \text{if } z_1(x) \leq -113 \end{cases}$$

In the similar way,

$$\upsilon_2^I(x) = \begin{cases} 1, & \text{if } z_2(x) \geq 250 \\ \dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-5}{250-5} & \text{if } 5 \leq z_2(x) \leq 250 \\ \dfrac{5-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{5-(-240)} & \text{if } -240 \leq z_2(x) \leq 5 \\ 1, & \text{if } z_2(x) \leq -240 \end{cases}$$

If $\upsilon_1^I((x)=\max\left(\min\left(\dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-(-6.5)}{100-(-6.5)}, \dfrac{(-6.5)-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{-6.5-(-113)}\right), 1\right)$ and

If $\upsilon_2^I((x)=\max\left(\min\left(\dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-5}{250-5}, \dfrac{5-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{5-(-240)}\right), 1\right),$

Then $\upsilon_1^{I*}(2.46,1.08)$ and $\upsilon_2^{I*}(0.89,0.22)$. The non-membership functions are transformed by using first-order Taylor polynomial series

If $\upsilon_1^I((x) = \upsilon_1^I(x) = \upsilon_1^I(2.46, 1.08) + [(x_1 - 2.46)\dfrac{\partial\upsilon_i^I(2.46,1.08)}{\partial x_1} +$

$(x_2-1.08)\dfrac{\partial\upsilon_i^I(2.46,1.08)}{\partial x_2}]$

$$\upsilon_1^I(x) = 0.012x_1 - 0.006x_2 + 0.977 \quad (4.4)$$

$$\upsilon_2^I(x) = \upsilon_2^I(x) = \upsilon_2^I(0.89, 0.22) + [(x_1 - 0.89)\frac{\partial \upsilon_2^I(0.89,0.22)}{\partial x_1} + (x_2 - 0.22)\frac{\partial \upsilon_2^I(0.89,0.22)}{\partial x_2}]$$

$$\upsilon_2^I(x) = 0.003x_1 - 0.013x_2 + 1 \quad (4.5)$$

then the objective of the IFMOQPP is obtained by adding (4.4) and (4.5), that is $q(x) = \upsilon_1^I(x) + \upsilon_2^I(x) = 0.015x_1 - 0.019x_2 + 1.977$

subject to the constraints

$$2x_1 + x_2 \le 6$$

$$x_1 + 4x_2 \le 12$$

The problem is solved and the solution is obtained is as follows: $x_1 = 2.67$; $x_2 = 0.67$; $z_1(x) = 6.2231$; $z_2(x) = -4.0043$ and the membership values are $\upsilon_1 = 0.119$ and $\upsilon_2 = 0.037$.

The non-membership function values show that both goals z_1 and z_2 are satisfied with 11.9% and 3.7% respectively for the obtained solution which is $x_1 = 2.67$; $x_2 = 0.67$.

b. The membership and non-membership functions were considered to be intuitionistic trapezoidal (see figure 2) [5] when they depend on four scalar parameters (a_1, b_1, c_1, d_1). z_1 depends on intuitionistic fuzzy aspiration levels (5, 6.5, 8, 9.5) when z_2 depends on intuitionistic fuzzy aspiration levels (-8.5, -4, 0.5, 5).

The membership and non-membership functions of the goals are obtained as follows:

$$\mu_1^I(x) = \begin{cases} 0, & \text{if } z_1(x) \ge d_1 \\ \dfrac{d_1 - z_1(x)}{d_1 - c_1} & \text{if } c_1 \le z_1(x) \le d_1 \\ 1 & \text{if } b_1 \le z_1(x) \le c_1 \\ \dfrac{z_1(x) - a_1}{b_1 - a_1} & \text{if } a_1 \le z_1(x) \le b_1 \\ 0, & \text{if } z_1(x) \le a_1 \end{cases}$$

$$= \begin{cases} 0, & \text{if } z_1(x) \geq 9.5 \\ \dfrac{9.5-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{9.5-8} & \text{if } 8 \leq z_1(x) \leq 9.5 \\ 1 & \text{if } 6.5 \leq z_1(x) \leq 8 \\ \dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-(5)}{6.5-5} & \text{if } 5 \leq z_1(x) \leq 6.5 \\ 0, & \text{if } z_1(x) \leq 5 \end{cases}$$

In the similar way,

$$\mu_2^I(x) = \begin{cases} 0, & \text{if } z_1(x) \geq 5 \\ \dfrac{5-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{5-0.5} & \text{if } 0.5 \leq z_1(x) \leq 5 \\ 1 & \text{if } -4 \leq z_1(x) \leq 0.5 \\ \dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-(-8.5)}{4-(-8.5)} & \text{if } -8.5 \leq z_1(x) \leq -4 \\ 0, & \text{if } z_1(x) \leq -8.5 \end{cases}$$

If $\mu_1^I((x)=\max(\min(\dfrac{9.5-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{9.5-8}, \dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-(5)}{6.5-5}), 0)$

and

If $\mu_2^I((x)=\max(\min(\dfrac{5-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{5-0.5}, \dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-(-8.5)}{4-(-8.5)}), 0),$

Then $\mu_1^{I*}(2.46,1.08)$ and $\mu_2^{I*}(0.89,0.22)$. The membership and non-membership functions are transformed by using first-order Taylor polynomial series

If $\mu_1^I((x) = \mu_1^I(x) = \upsilon_1^I(2.46, 1.08) + [(x_1 - 2.46) \dfrac{\partial \mu_i^I(2.46,1.08)}{\partial x_1}$

$(x_2 - 1.08) \dfrac{\partial \mu_i^I(2.46,1.08)}{\partial x_2}$]

$\mu_1^I(x) = 0.83x_1 - 0.4x_2 + 0.61$ (4.6)

$$\mu_2^I(x) = \mu_2^I(x) = \mu_2^I(0.89, 0.22) + \left[(x_1 - 0.89)\frac{\partial \mu_2^I(0.89,0.22)}{\partial x_1} + (x_2 - 0.22)\frac{\partial \mu_2^I(0.89,0.22)}{\partial x_2}\right]$$

$\mu_2^I(x) = 0.71x_1 - 0.69x_2 + 1$ (4.7)

then the objective of the FMOLPP is obtained by adding (4.6) and (4.7), that is $p(x) = \mu_1^I(x) + \mu_2^I(x) = x_1 - 1.09x_2 + 0.39$

subject to the constraints

$$2x_1 + x_2 \leq 6$$

$$x_1 + 4x_2 \leq 12$$

The problem is solved and the solution is obtained is as follows: $x_1 = 2.67$; $x_2 = 0.67$; $z_1(x) = 6.2231$; $z_2(x) = -4.0043$ and the membership values are $\mu_1 = 0.8154$ and $\mu_2 = 1$. The membership function values show that both goals z_1 and z_2 are satisfied with 81.54% and 100% respectively for the obtained solution which is $x_1 = 2.67$; $x_2 = 0.67$.

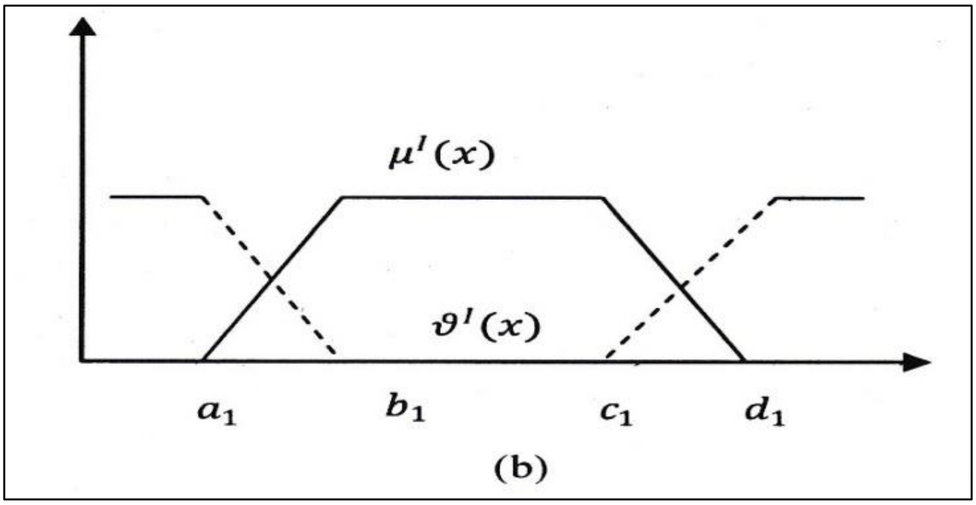

Figure 4.2: Membership and Non-Membership Functions Defined as Intuitionistic Trapezoidal (B)

The non-membership functions of the goals are obtained as follows:

$$\upsilon_1^I(x) = \begin{cases} 1, & \text{if } z_1(x) \geq d_1 \\ \dfrac{z_1(x)-c_1}{d_1 - c_1} & \text{if } c_1 \leq z_1(x) \leq d_1 \\ 0 & \text{if } b_1 \leq z_1(x) \leq c_1 \\ \dfrac{b_1-z_1(x)}{b_1 - a_1} & \text{if } a_1 \leq z_1(x) \leq b_1 \\ 1, & \text{if } z_1(x) \leq a_1 \end{cases}$$

$$= \begin{cases} 1, & \text{if } z_1(x) \geq 9.5 \\ \dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-8}{9.5-8} & \text{if } 8 \leq z_1(x) \leq 9.5 \\ 0 & \text{if } 6.5 \leq z_1(x) \leq 8 \\ \dfrac{6.5-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{6.5-5} & \text{if } 5 \leq z_1(x) \leq 6.5 \\ 1, & \text{if } z_1(x) \leq 5 \end{cases}$$

In the similar way,

$$\upsilon_2^I(x) = \begin{cases} 1, & \text{if } z_1(x) \geq 5 \\ \dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-0.5}{5-0.5} & \text{if } 0.5 \leq z_1(x) \leq 5 \\ 0 & \text{if } -4 \leq z_1(x) \leq 0.5 \\ \dfrac{-4-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{4-(-8.5)} & \text{if } -8.5 \leq z_1(x) \leq -4 \\ 1, & \text{if } z_1(x) \leq -8.5 \end{cases}$$

If $\upsilon_1^I((x)=\max\left(\min\left(\dfrac{(-4x_1+x^2_1-2x_1x_2+2x^2_2)-8}{9.5-8}, \dfrac{6.5-(-4x_1+x^2_1-2x_1x_2+2x^2_2)}{6.5-5}\right), 1\right)$

and

If $\upsilon_2^I((x)=\max\left(\min\left(\dfrac{(-3x_1+x^2_1+2x_1x_2+3x^2_2)-0.5}{5-0.5}, \dfrac{-4-(-3x_1+x^2_1+2x_1x_2+3x^2_2)}{4-(-8.5)}\right), 0\right)$, Then $\upsilon_1^{I*}(2.46,1.08)$ and $\upsilon_2^{I*}(0.89,0.22)$. The membership and non-membership functions are transformed by using first-order Taylor polynomial series

If $\upsilon_1^I((x) = \upsilon_1^I(x) = \upsilon_1^I(2.46, 1.08) + [(x_1 - 2.46) \frac{\partial \upsilon_i^l(2.46,1.08)}{\partial x_1} +$

$(x_2 - 1.08) \frac{\partial \upsilon_i^l(2.46,1.08)}{\partial x_2}]$

$\upsilon_1^I(x) = 0.83x_1 - 0.4x_2 + 1.61$ (4.8)

$\upsilon_2^I(x) = \upsilon_2^I(x) = \upsilon_2^I(0.89, 0.22) + [(x_1 - 0.89) \frac{\partial \upsilon_2^l(0.89,0.22)}{\partial x_1} +$

$(x_2 - 0.22) \frac{\partial \upsilon_2^l(0.89,0.22)}{\partial x_2}]$

$\upsilon_2^I(x) = 0.17x_1 - 0.69x_2$ (4.9)

then the objective of the FMOLPP is obtained by adding (4.6) and (4.7), that is $q(x) = \upsilon_1^I(x) + \upsilon_2^I(x)) = x_1 - 1.09x_2 + 1.61$

subject to the constraints

$$2x_1 + x_2 \leq 6$$

$$x_1 + 4x_2 \leq 12$$

The problem is solved and the solution is obtained is as follows: $x_1 = 2.67$; $x_2 = 0.67$; $z_1(x) = 6.2231$; $z_2(x) = -4.0043$ and the membership values are $\upsilon_1 = 0.8146$ and $\upsilon_2 = 0$. The non-membership function values show that both goals z_1 and z_2 are satisfied with 18.46% and 0% respectively for the obtained solution which is $x_1 = 2.67$; $x_2 = 0.67$.

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ISBN: 978-81-19149-24-7

Chapter 5

Mathematical Programming on Reliability Optimization Model

5.1 Introduction:

The theory of fuzzy sets [1] has a very rich literature and many modified and generalized forms of the theory have been developed. Intuitionistic fuzzy set (IFS) is one of the generalizations of fuzzy set theory. IFS was first introduced by Atanassov [2, 3, 4] and has been found to be well suited for dealing with problems concerning vagueness. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in a situation where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [5].

Some researchers have used the technique of fuzzy set theory for solving multi-objective reliability optimization problems. Sakawa and Yano [6] introduced the fuzzy sequential proxy optimization technique for multi-objective decision making and applied it to the reliability design of a standby system.

Dhingra [7] used goal programming and goal attainment formulations under fuzziness in multi-objective reliability apportionment problem subject to several design constraints. Rao and Dhingra [8] presented the problem of crisp and fuzzy multi-objective optimization in the context of reliability and redundancy apportionment of multistage, multi-components systems subject to several resource constraints. Huang [9] presented a fuzzy multi-objective optimization decision-making method and its application in reliability optimization. Ravi et al. [10] presented redundancy allocation on a multistage series-parallel system as a fuzzy multi-objective optimization problem where apart from the system reliability, system cost, weight and volume are all considered as fuzzy goals/objectives.

Mahapatra and Roy [11] used fuzzy multi-objective optimization method for the decision making of series and complex system cost simultaneously. Pandey et al. [12] presented enhanced particle swarm optimization algorithm and it is applied to the reliability optimization problem of a multistage mixed system with four different value functions.

Zio et al. [13] and Kishor et al. [14] used genetic algorithms for multi-objective system reliability design optimization.

Now intuitionistic fuzzy optimization (IFO) is an open field for research work. Very little research work has been carried out on IFO. Angelov [15] proposed a framework of the optimization problem under uncertainty in an intuitionistic fuzzy environment.

Xu and Yager [16], Li [17] investigated multi-objective intuitionistic fuzzy linear programming with its application in transportation model. Liu and Wang [19], Lin et al. [20] presented methods for solving multi-criteria environment. Pramanik and Roy [21] introduced an intuitionistic fuzzy goal programming approach to vector optimization problem. However, it seems that so far there has been little research on multi-criteria/multi-objective optimization using IFS, which is indeed one of the most important areas in decision analysis as most real-world decision problems involve multi-objective optimization problem. Use of IFO technique in reliability optimization model is very rare in literature.

In this book, a solution procedure of a multi-objective nonlinear programming problem using IFO technique is considered. Here we consider the problem of finding the optimum reliability and cost of system subject to retain the goal of system space. This multi-objective non-linear programming problem is solved by IFO technique. This book envisages the application of IFO in the context of reliability apportionment for a multistage, multi-component system.

5.2 Multi-Objective Reliability Optimization Model Description:

Notations

Reliability optimization model in intuitionistic fuzzy environment is developed and worked out under the following notations:

$R = (R_1, \dots, R_n)^T$: decision vector

$R_s(R)$: function of system reliability

$C_s(R)$: function of system cost

$V_s(R)$: function of system space

Parameters for i^{th} (= 1, …, n) item are,

R_i: reliability of ith component as decision variables

C_i: cost of i^{th} component

V_i: space of i^{th} component

V_s: system space goal.

The size of the system, the intricacy of the specific functions, cost of the components, and the degree of hostility of the system’s environment all influence the reliability. Consider a complex system where the system reliability has to be maximized and the system cost is to be minimized simultaneously subject to system goal of space.

Therefore, we have to find maximum system reliability $R_s(R)$ and minimum system cost $C_s(R)$ subject to the system space $V_s(R)$ as a target goal. So, the model becomes a multi-objective reliability optimization problem as follows

$$(1)\quad \begin{aligned} &\text{Maximize } R_s(R) \\ &\text{Minimize } C_s(R) \end{aligned}$$

Subject to $V_s(R) \leq V_s$

$R = (R_1, \ldots, R_n)^T$ and $R_i > 0.5$ for $i = 1, \ldots, n$.

To avoid mathematical complexity, we transfer above minimization problem in to maximization problem to form one type optimization problem.

Therefore, the reliability model becomes

$$(2)\quad \begin{aligned} &\text{Maximize } R_s(R) \\ &\text{Maximize } C'_s(R) = - C_s(R) \end{aligned}$$

Subject to $V_s(R) \leq V_s$

$R = (R_1, ..., R_n)^T$ and $R_i > 0.5$ for $i = 1, \ldots, n$.

The complex system is modelled as a five-stage combination reliability model.

The objective is to determine an optimal reliability of the system and minimize the system cost simultaneously such as size of the system is restricted within the system space goal.

5.2.1 Reliability of Reliability Model of an LCD Display Unit:

We are looking at the overall reliability for a LCD display unit [22] that consists of a display, backlighting panel, and a number of circuit boards with the following setup:

S_1 LCD panel with hardware reliability R_1.

S_2 A backlighting board with 10 bulbs with individual bulb reliability R_2 such that the board functions with at most one bulb failure.

S_3 Two microprocessor boards A and B hooked up in parallel, each of reliability R_3.

S_4 Dual power suppliers in a standby redundancy, with reliability R_4 for each power supply, perfect switching being assumed.

S_5 EMI board reliability R_5 hooked up in series with common input of the power supply A.

The schematic diagram [22] is constructed as shown in figure 5.1.

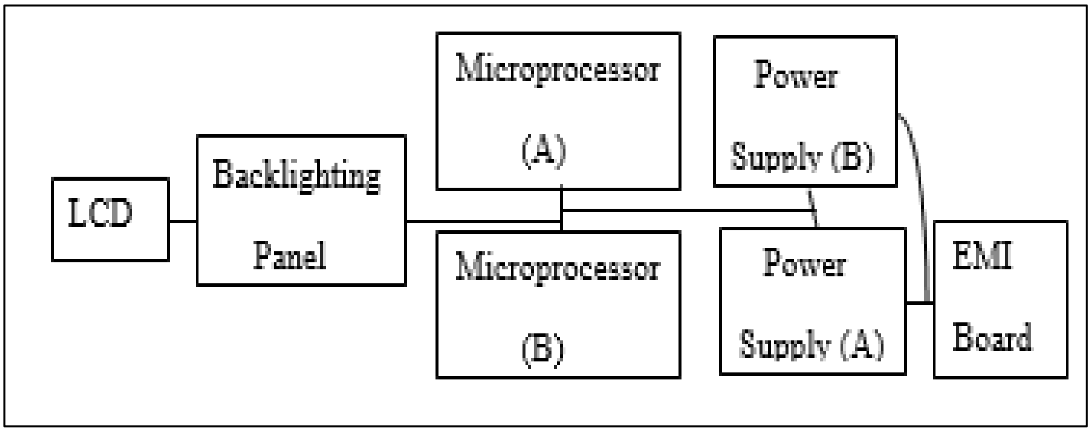

Figure 5.1: Combination Reliability Model of an LCD Display Unit.

There are five basic pieces in this model, and a generalized formula for the reliability of this model can be viewed somewhat in a serial fashion as

$R_s = S_1 \times S_2 \times S_3 \times S_4 \times S_5$, where S_i, i = 1, …, 5 are as follows:

$S_1 = R_1$ (LCD circuit board)

$S_2 = R^{10}_2 + 10R^9_2 (1 – R_2)$ (Backlighting panel)

$S_3 = 1 – (1- R_3)^2$ (Microprocessors A and B)

$S_4 = R_4 + R_4 \, In(1/R_4)$ (Power supply A and B)

$S_5 = R_5$ (EMI board)

We now multiply each of the terms listed for $R_s(R)$

Thus, we get

$$R_s(R) = R_1 (R^{10}_2 + 10R^9_2(1 – R_2)) (1 – (1-R_3)^2) (R_4+R_4In(1/R_4)) R_5 \quad (3)$$

Combined models get quite complicated, as can be seen above;

more complex models can arise when there are conditional situations or changing states.

5.2.2 Cost Function of Reliability Model of An LCD Display Unit:

Generally, the rise in cost of a system is likely to increase sharply with the increase in reliability and can be expected to be especially steep after a certain stage. With this idea in view, we model the cost function as an increasing function of reliability as follows:

$$C_s(R) = \sum_{i=1}^{5} C_i \left[\tan((\pi/2)\, Ri\right]^{\alpha i} \quad (4)$$

$R = (R_1, R_5)^T$, $0.5 \le R_i \le 1$, $0.5 \le R_s \le 1$ for i = 1, …,5.

5.2.3 Space Function of Reliability Model of a LCD Display Unit:

In is natural to expect the space occupied by a system to be more if it is more sophisticated and reliable (except in the event of some major technological breakthrough).

So, one can consider the space function as an increasing function of reliability. Here we considered space constraint as follows:

$$V_s(R) = \sum_{i=1}^{5} ViR_i^{ai} \le V_s \quad (5)$$

$R = (R_1, R_5)^T$, $0.5 \le R_i \le 1$, $0.5 \le R_s \le 1$ for i = 1, …,5.

5.3 Mathematical Analysis

Definition 1: Let a set X be fixed. An IFS $\tilde{A}^i$ in X is an object having the form

$\tilde{A}^i = \{(x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x)): x \in X\}$ where $\mu_{\tilde{A}}(x): X \to [0, 1]$ and $\upsilon_{\tilde{A}}(x): X \to [0, 1]$ define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set $\tilde{A}^i$, which is a subset of X, for every element of $x \in X$, $0 \le \mu_{\tilde{A}}(x) + \upsilon_{\tilde{A}}(x) \le 1$.

5.3.1 Intuitionistic Fuzzy Non-Linear Programming Technique to Solve Multi-Objective Non-Linear Programming Problem:

A multi-objective non-linear programming (MONLP) may be taken in the following form:

$$\text{Maximize } f(x) = [f_1(x), f_2(x), \ldots, f_k(x)]^T \quad (6)$$

Subject to $x \in X = \{x \in R^n: g_j(x) \le \text{ or } = \text{ or } \ge b_j$ for $j = 1, \ldots, m$; $1_i \le x_i \le u_i$ for $i = 1, 2, \ldots, n\}$.

To solve the MONLP (6) problem, following Zimmermann [23] and Angelov [15], we have presented a solution procedure to solve the MONLP problem by IFO technique, and the following steps are used:

Step 1: Solve the MONLP (6) as a single objective non-linear problem k time for each problem by taking one of the objectives at a time and ignoring the others.

These solutions are known as ideal solutions. Let x^i be the respective optimal solution for the i^{th} different objective and evaluate each objective values for all these i^{th} optimal solutions. It is assumed that at least two of these solutions are different for which the i^{th} objective function has different bounded values.

Step 2: From the result of Step 1, determine the corresponding values for every objective for each derived solution. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c} \\ x^1 \\ x^2 \\ \ldots \\ x^k \end{array} \begin{array}{c} \begin{array}{cccc} f_1(x) & f_2(x) & \ldots & f_k(x) \end{array} \\ \begin{pmatrix} f^*_1(x^1) & f_2(x^1) & \ldots & f_k(x^1) \\ f_1(x^2) & f^*_2(x^2) & \ldots & f_k(x^2) \\ \ldots & \ldots & \ldots & \ldots \\ f_1(x^k) & f_2(x^k) & & f^*_k(x^k) \end{pmatrix} \end{array}$$

Here x^1, …, xk are the ideal solutions of the objectives $f_1(x)$,$f_k(x)$ respectively.

For each objective $f_i(x)$, find lower bound (minimum) L_i^{acc} and the upper bound (maximum) U_i^{acc}. But in IFO the degree of non-membership (rejection) and degree of membership (acceptance) are considered so that the sum of both values is less than one [2]. To define the non-membership function of MONLP problem, let L_i^{rej} and U_i^{rej} be the lower and upper bond of the objective function $f_i(x)$ where $L_i^{acc} \leq L_i^{rej} \leq U_i^{rej} \leq U_i^{acc}$.

Find the worst (U_i) and the best (L_i) values of each objective for the degree of acceptance and rejection corresponding to the set of solutions as $U_i^{acc} = \max\{f^*_i(x^k)\}$ and $L_i^{acc} = \min_{r=1,2,,i-1,i+1,\ldots,k}\{f_i(x^r)\}$ for i=1, 2, …,k for degree of acceptance of objectives. It is known from the theorem that

Theorem 1: For objective function of maximization problem, the upper bound for non-membership functions (rejection) is always less than that the upper bound of membership functions (acceptance).

Proof: (See Appendix-I)

So, one can take the lower and upper bound for the non-membership function as follows

$U_i^{rej} = U_i^{acc} - \varepsilon_i$ where $\varepsilon i = (U_i^{acc} - U_i^{rej})$ for i = 1, …, k based on the decision maker choice.

And $L_i^{rej} = L_i^{acc}$.

Step 3: The initial intuitionistic fuzzy model with aspiration levels of objectives becomes

Find $\{x_i, i = 1, \ldots, k\}$
(7)

So as to satisfy

$f_i(x) \leq L_i^{acc}$ with tolerance $(U_i^{acc} - L_i^{rej})$ for the degree of acceptance for i = 1, …, k.

$f_i(x) \geq U_i^{rej}$ with tolerance $(U_i^{rej} - L_i^{rej})$ for the degree of rejection for i = 1, …, k.

define the membership and non-membership functions of above uncertain objectives as follow:

for the i^{th} (i = 1,…,k) objectives functions the membership function $\mu_i(f_i(x))$ are taken as following linear functions

$$\mu_i(f_i(x)) = \begin{cases} 0 & \text{if } f_i(x) \leq L_i^{acc} \\ \dfrac{f_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} & \text{if } L_i^{acc} \leq f_i(x) \leq U_i^{acc} \\ 1 & \text{if } f_i(x) \geq U_i^{acc} \end{cases}$$

And

$$\upsilon_i(f_i(x)) = \begin{cases} 0 & \text{if } f_i(x) \geq U_i^{rej} \\ \dfrac{U_i^{rej} - f_i(x)}{U_i^{rej} - L_i^{rej}} & \text{if } L_i^{rej} \leq f_i(x) \leq U_i^{rej} \\ 1 & \text{if } f_i(x) \geq L_i^{rej} \end{cases}$$

Rough sketch of the membership function and non-membership function for maximization type objective function are shown in figure 2.

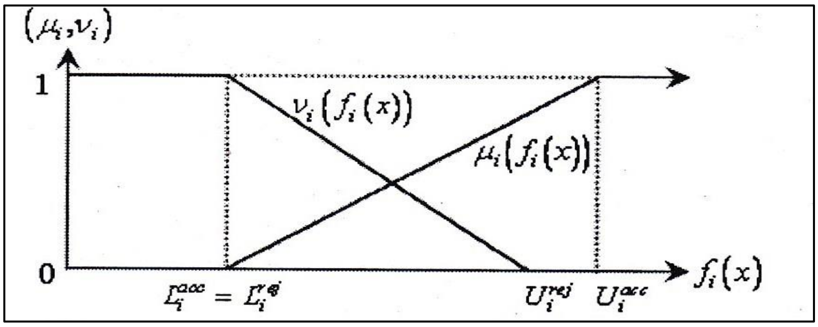

Figure 5.2: Rough Sketch of Membership and Non-Membership Functions of Objective Function.

Step 4: Now an IFO technique [15] for MONLP problem with the membership and non-membership functions can be written as

$$\text{Maximize } \mu_i(f_i(x)) \tag{10}$$

$$\text{Minimize } \upsilon_i(f_i(x))$$

Subject to

$$\mu_i(f_i(x)) \geq \upsilon_i(f_i(x))$$

$$\mu_i(f_i(x)) + \upsilon_i(f_i(x)) < 1$$

$$\upsilon_i(f_i(x)) \geq 0$$

$$g_j(x) \leq b_j,\ x \geq 0$$

for i = 1, 2,,k; j= 1,2,…,m.

In IFO technique we have to maximize $\mu_i(f_i(x))$ and minimize $\upsilon_i(f_i(x))$ simultaneously. These optimize membership function and non-membership functions provide solution of objective function $f_i(x)$. However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and hence the Pareto optimality concept arises. Following Sakawa [24, 25], the IFO is applied for multi-objective programming problem the notion of Pareto optimal solution determined in terms of objective functions is not applicable. Hence the concept of intuitionistic fuzzy Pareto or M-N refers to membership and non-membership function. Some basic definitions on Pareto optimal solutions in IFO environment are introduced below.

Definition 2: M-N Pareto Optimal Solution

x^*□ X is said to be an M-N Pareto optimal solution to the intuitionistic fuzzy non-linear programming (IFNLP) (10) if and only if there does not exist another xεX such that $\mu_i(f_i(x)) \geq \mu_i(f_i(x^*))$, $\upsilon_i(f_i(x)) \geq \upsilon_i(f_i(x^*))$ for all i=1, 2,…,k and $\mu_i(f_i(x)) \neq \mu_i(f_i(x^*))$, $\upsilon_i(f_i(x)) \neq \upsilon_i(f_i(x^*))$ for at least one j, j □ {1, 2,…,k}.

Definition 3: Weak M-N Pareto Optimal Solution

x^*□ X is said to be an M-N Pareto optimal solution if and only if there does not exist another xεX such that $\mu_i(f_i(x)) > \mu_i(f_i(x^*))$, $\upsilon_i(f_i(x)) < \upsilon_i(f_i(x^*))$ for all i=1, 2, k.

If the decision-maker selects the additive operator, the problem to be solved is an equivalent crisp model by using the membership and non-membership functions of objectives by IFO as follows:

$$\text{Maximize} \sum_{i=1}^{k} \{\mu_i(f_i(x)) - \upsilon_i(f_i(x))\} \qquad (11)$$

Subject to same constraint and restriction as in (10)

Step 5: Solve the above (11) crisp model by an approximate mathematical programming algorithm to get M-N Pareto optimal solution. Some basic theorems on M-N Pareto optimal solutions are introduced below.

Theorem 2: The solution of (7) based on max-min operator of IFNLP problem (11) is weakly M-N Pareto optimal.

Proof: Let x^*□ X be a solution of the IFNLP problem. Let us suppose that it is not weakly M-N Pareto optimal. In this case, there exists a solution x□ X such that $f_i(x) > f_i(x^*)$ for all i = 1, 2, …,k. Observing that $\mu_i(f_i(x))$ is a strictly monotone increasing function with respect to $f_i(x)$ this implies $\mu_i(f_i(x)) > \mu_i(f_i(x^*))$ and $\upsilon_i(f_i(x))$ is a strictly monotone decreasing function with respect to $f_i(x)$, which implies $\upsilon_i(f_i(x)) < \upsilon_i(f_i(x^*))$. Thus, we have $\min\mu_i(f_i(x)) > \min\mu_i(f_i(x^*))$ and $\max\upsilon_i(f_i(x)) < \max\upsilon_i(f_i(x^*))$ and. This is a contradiction to the assumption that x* is a solution of the IFNLP problem. Thus x* is weakly M-N Pareto optimal.

Theorem 3: The unique solution of IFNLP problem (10) is M-N Pareto optimal.

Proof: Let x^*□ X be a unique solution of the IFNLP problem. Let us suppose that it is not M-N Pareto optimal. In this case, there exists a solution x□ X such that $\mu_i(f_i(x)) \geq \mu_i(f_i(x^*))$, $\upsilon_i(f_i(x)) \leq \upsilon_i(f_i(x^*))$ for all i=1, 2, …,k and $\mu_j(f_j(x)) > \mu_j(f_j(x^*))$, $\upsilon_j(f_j(x)) < \upsilon_j(f_j(x^*))$ for at least one j. Noting that $\mu_i(f_i(x))$ is a strictly monotone increasing function with respect to $f_i(x)$, this implies $\mu_i(f_i(x)) < \mu_i(f_i(x^*))$ for some i=1, 2,…,k and $\upsilon_i(f_i(x))$ is a strictly monotone decreasing function with respect to $f_i(x)$, which implies $\upsilon_i(f_i(x)) < \upsilon_i(f_i(x^*))$ for some i=1, 2,..,k.

Thus we have $\min_{i=1,2,\ldots,k}\mu_i(f_i(x)) \geq \min_{i=1,2,\ldots,k}\mu_i(f_i(x^*))$ and $\max_{i=1,2,\ldots,k}\upsilon_i(f_i(x)) \leq \max_{i=1,2,\ldots,k}\upsilon_i(f_i(x^*))$ and $\max_{i=1,2,\ldots,k}\upsilon_i(f_i(x)) \leq \max_{i=1,2,\ldots,k}\upsilon_i(f_i(x^*))$. On the other hand, the uniqueness of x* meaning that $\min_{i=1,2,\ldots,k}\mu_i(f_i(x^*)) > \min_{i=1,2,\ldots,k}\mu_i(f_i(x'))$ and $\max_{i=1,2,\ldots,k}\upsilon_i(f_i(x)) < \max_{i=1,2,\ldots,k}\upsilon_i(f_i(x^*))$ and for all x'□ X. The two sets inequalities above are contradictory and thus, x* must be M-N Pareto optimal.

5.4 Intuitionistic Fuzzy Optimization Technique on Reliability Optimization Model:

We have to find maximum system reliability $R_s(R)$ and minimum system cost $C_s(R)$ (which is equivalent to $C'_s(R) = -C_s(R)$) subject to the system space $V_s(R)$ goal V_s. So, the problem is a multi-objective reliability optimization problem as follows

Maximize $R_s(R)$

Maximize $C'_s(R)$

(12)

Subject to $V_s(R) \leq V_s$ $\quad R= (R_1, Rn)^T$ and $R_i > 0.5$ for i=1,n.

To solve the above MOROP (12), step 1 of (6.1) is used. After that according to step 2 pay-off matrix is formulated as follows:

$$\begin{array}{c} \\ R^1 \\ R^2 \end{array}\begin{array}{c} \begin{array}{cc} R_s(R) & C'_s(R) \end{array} \\ \begin{pmatrix} R^*_s(R^*) & C'_s(R) \\ R_s(R) & C'^*_s(R^*) \end{pmatrix} \end{array}$$

5

Now U_1^{acc}, L_1^{acc}, U_2^{acc}, L_2^{acc} (where $L_1^{acc} \leq R_s(R)^{acc} \leq U_1^{acc}$ and $L_2^{acc} \leq C'_s(R)^{acc} \leq U_2^{acc}$) and U_1^{rej}, L_1^{rej}, U_2^{rej}, L_2^{rej} (where $L_1^{acc} \leq R_s(R)^{rej} \leq U_1^{rej}$ and $L_2^{acc} = L_2^{acc} \leq C'_s(R)^{rej} \leq U_2^{rej}$) Uirej = $U_i^{acc} - \varepsilon_i$ for i=1, 2; where $0 < \varepsilon_i < (U_i^{acc} - U_i^{rej})$ are identified. Here for simplicity linear membership function $\mu_{Rs}(R_s(R))$, $\mu_{C's}(C'_s(R))$ and linear non-membership function $\upsilon_{Rs}(R_s(R))$, $\upsilon_{C's}(C'_s(R))$ for the objective functions $R_s(R)$ and $C'_s(R)$ respectively, are defined as follows:

$$\mu_{Rs}(R_s(R))= \begin{cases} 0 & \text{if } R_s(R) \leq L_1^{acc} \\ \dfrac{R_s(R) - L_1^{acc}}{U_1^{acc} - L_1^{acc}} & \text{if } L_1^{acc} \leq R_s(R) \leq U_1^{acc} \\ 1 & \text{if } R_s(R) \geq U_1^{acc} \end{cases}$$

$$\mu_{C's}(C'_s(R)) = \begin{cases} 0 & \text{if } C'_s(R) \leq L_2^{acc} \\ \dfrac{C'_s(R) - L_2^{acc}}{U_2^{acc} - L_2^{acc}} & \text{if } L_2^{acc} \leq C'_s(R) \leq U_2^{acc} \\ 1 & \text{if } C'_s(R) \geq U_2^{acc} \end{cases}$$

$$\upsilon_{Rs}(R_s(R)) = \begin{cases} 1 & \text{if } R_s(R) \leq L_1^{rej} \\ \dfrac{U_1^{rej} - R_s(R)}{U_1^{rej} - L_1^{rej}} & \text{if } L_1^{rej} \leq R_s(R) \leq U_1^{rej} \\ 0 & \text{if } R_s(R) \geq U_1^{rej} \end{cases}$$

And

$$\upsilon_{C's}(C'_s(R)) = \begin{cases} 1 & \text{if } C'_s(R) \leq L_2^{rej} \\ \dfrac{U_2^{rej} - C'_s(R)}{U_2^{rej} - L_2^{rej}} & \text{if } L_2^{rej} \leq C'_s(R) \leq U_2^{rej} \\ 0 & \text{if } C'_s(R) \geq U_2^{rej} \end{cases}$$

According to IFO technique, having elicited the above membership and non-membership function for MOROP (12) crisp nonlinear programming problem is formulated as follows

$$\text{Maximize } (\mu_{Rs}(R_s(R)) + \mu_{C's}(C'_s(R)) - \upsilon_{Rs}(R_s(R)) - \upsilon_{C's}(C'_s(R))) \tag{13}$$

Subject to

$\upsilon_{Rs}(R_s(R)),\ \upsilon_{C's}(C'_s(R)) \geq 0,$

$\mu_{Rs}(R_s(R)) \geq \upsilon_{Rs}(R_s(R)),$

$\mu_{C's}(C'_s(R)) \geq \upsilon_{C's}(C'_s(R)),$

$\mu_{Rs}(R_s(R)) + \upsilon_{Rs}(R_s(R)) < 1,$

$\mu_{C's}(C'_s(R)) + \upsilon_{C's}(C'_s(R)) < 1,$

$V_s(R) \leq V_s,$

$R = (R_1, \ldots, Rn)^T$ and $R_i > 0.5$ for i=1, n.

Solve the above crisp model by an appropriate mathematical programming algorithm to get M-N Pareto optimal solution of objective functions i.e., system reliability and system cost.

Numerical Solution of Reliability Model of LCD Display Unit:

The system reliability, cost, and space are constrained by the design. So, the reliability allocation for five stages combine system is given by

Maximize $R_s(R) = R_1 (R^{10}_2 + 10R^9_2 (1-R_2)) \times (1 - (1-R_3)^2) (R_4 + R_4 \text{In}(1/R_4)) R_5$

$$\text{Maximize } C'_s(R) = - \sum_{i=1}^{5} C_i [\tan((\pi/2) Ri)]^{\alpha i} \qquad (14)$$

$$\text{Subject to } Vs(R) = \sum_{i=1}^{5} V_i R_i^{ai} \leq V_s$$

For $R=(R_1, R_5)^T$ and $R_i > 0.5$ for i=1, …,5.

The input data for the MOROP (14) is given as follows: The cost and space of each component is $C_1 = 40$, $C_2 = 30$, $C_3 = 35$, $C_4 = 36$, $C_5 = 32$ and $V_1 = 6$, $V_2 = 4.75$, $V_3 = 2$, $V_4 = 3$, $V_5 = 7$ respectively.

The shape parameters are $a_i = 1$ and $\alpha_i = 0.75$ for all i=1,5.

Let the space goal of the system is $V_s = 22$ units.

Solution: According to step 2 pay-off matrix is formulated as follows:

	Rs(R)	C's(R)
R^1	0.9457139	-6564.526
R^2	0.001705135	-172.8623

Here, $U_1^{acc} = 0.9457139$, $L_1^{acc} = L_1^{rej} = 0.001705135$, $U_2^{acc} = -172.8623$, $L_2^{acc} = L_2^{rej} = -6564.526$, $U_1^{rej} = 0.9457139 - \varepsilon_1$ and $U_2^{rej} = -172.8623-\varepsilon_2$.

Here linear membership and non-membership functions for the objective functions $R_s(R)$ and $C'_s(R)$ respectively, are define as follows:

$$\mu_{R_s}(R_s(R))= \begin{cases} 0 & \text{if } R_s(R) \leq 0.001705135 \\ \dfrac{R_s(R) - 0.001705135}{0.944008765} & \text{if } 0.001705135 \leq R_s(R) \leq 0.9457139 \\ 1 & \text{if } R_s(R) \geq 0.9457139 \end{cases}$$

$$\mu_{C'_s}(C'_s(R))= \begin{cases} 0 & \text{if } C'_s(R) \leq -6564.526 \\ \dfrac{C'_s(R) - (-6564.526)}{-172.8623 - (-6564.526)} & \text{if } -6564.526 \leq C'_s(R) \leq -172.8623 \\ 1 & \text{if } C'_s(R) \geq -172.8623 \end{cases}$$

$$\upsilon_{R_s}(R_s(R))= \begin{cases} 1 & \text{if } R_s(R) \leq 0.001705165 \\ \dfrac{0.9457139 - \varepsilon_1 - R_s(R)}{0.944008765 - \varepsilon_1} & \text{if } 0.001705135 \leq R_s(R) \leq 0.9457139-\varepsilon_1 \\ 0 & \text{if } R_s(R) \geq 0.9457139 - \varepsilon_1 \end{cases}$$

And

$$\upsilon_{C'_s}(C'_s(R))= \begin{cases} 1 & \text{if } C'_s(R) \leq -6564.526 \\ \dfrac{-172.8623 - \varepsilon_2 - C'_s(R)}{6391.6637-\varepsilon_2} & \text{if } -172.8623-\varepsilon_2 \leq C'_s(R) \leq -6564.526 \\ 0 & \text{if } C'_s(R) \geq -172.8623-\varepsilon_2 \end{cases}$$

Now IFO technique for MOROP (14) with this membership and non-membership functions can be solving for different value of ε_1 and ε_2. The M-N Pareto optimal solution of the MONLP model (14) using intuitionistic fuzzy multi-objective nonlinear programming (IF-MONLP) technique is given in table 1. The solution obtained by IFMONLP technique is compared with solution obtained by fuzzy multi-objective nonlinear programming (FMONLP) technique of the same MOROP (14) model.

Table 5.1: Comparison of optimal solution of MOROP (14) based on different method.

Method	R^*_1	R^*_2	R^*_3	R^*_4	R^*_5	$R^*_5(R^*)$	$C^*_5(R^*)$ $
FMONLP	0.92151	0.95833	0.87714	0.82318	0.92958	0.77357	1338.35
IFMONLP	0.93213	0.96262	0.87069	0.83953	0.93914	0.80533	1514.9

Here we get best solution for the tolerance $\varepsilon_1 = 0.125$ and $\varepsilon_2 = 1340$ for non-membership function of the objective functions. Form the Table 1, it shows that IFMONLP technique gives better Pareto optimal result in the perspective of system reliability.

Appendix

Theorem 1: For objective function of maximization problem, the upper bound for non-membership function (rejection) is always less that the upper bound of membership function.

Proof: Form definition of intuitionistic fuzzy set, sum of the degree of rejection and acceptance is less than unity.

$$\mu_i(f_i(x)) + \upsilon_i(f_i(x)) < 1 \text{ for all } i=1, 2, \ldots, k.$$

or

$$\frac{f_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} + \frac{U_i^{rej} - f_i(x)}{U_i^{rej} - L_i^{rej}} < 1$$

Case I. If possible, let $U_i^{rej} = U_i^{acc}$ then we have

$$\frac{f_i(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} + \frac{U_i^{acc} - f_i(x)}{U_i^{acc} - L_i^{rej}} < 1$$ this gives $L_i^{rej} < L_i^{acc}$ which is contradicting

the fact that lower bound of the membership and non-membership function is equal. Hence $U_i^{rej} \neq U_i^{acc}$

Case II. Let us consider $L_i^{rej} = L_i^{acc}$ then we have

$$\frac{f(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} + \frac{U_i^{rej} - f(x)}{U_i^{rej} - L_i^{acc}} < 1 \text{ which imply that } U_i^{rej} < U_i^{acc}$$

Case III. Let us consider $L_i^{rej} = L_i^{acc} + \varepsilon_i$, $\varepsilon_i > 0$ for all i=1, 2,k then we have

$$\frac{f(x) - L_i^{acc}}{U_i^{acc} - L_i^{acc}} + \frac{U_i^{rej} - f(x)}{U_i^{rej} - L_i^{acc}} < 1 \text{ which imply that } U_i^{rej} > U_i^{acc}$$

$$+ \varepsilon_i \quad \frac{U_i^{rej} - f_i(x)}{f_i(x) - L_i^{acc} - \varepsilon_i}$$

i.e., $U_i^{acc} > U_i^{rej}$. Hence $U_i^{acc} > U_i^{rej}$ (proof)

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ISBN: 978-81-19149-24-7

Chapter 6

Problem with Intuitionistic Fuzzy Numbers

6.1 Introduction:

Modelling a financial or a production planning problem needs some prior information about its feasibility and possible outcome. In many situations, it also needs financial analysis about resource utilization and optimal profit or gain. Such analysis needs the complete information about various parameters such as profit coefficients, resource limitations, constraints as well as its objectives and other goals. As a matter of fact, in real life production planning problems, it is often difficult to get discrete and exact information for various parameters affecting the process. Even in many situations the information available are imprecise or vague. Under such situations it is difficult to have the mathematical formulation to solve the mathematical programming problem using a linear programming technique. For such situations, fuzzy set developed by Zadeh [22] played a vital role in modelling the optimization problem having imprecision in parameters and was initiated by Zimmermann [23, 24, 25] as fuzzy linear programming problem.

One of the major difficulties to study such fuzzy linear programming problems with fuzzy coefficients is how to compare these fuzzy numbers. Thus, an important issue of ranking of fuzzy numbers and its approximation method took considerable interest amongst the researchers. Some of the authors who made significant contributions in the area are Dubois and Prade [10], Heliperrn[15], Adrian[1,2]. This growing discipline attracted many authors to extend the theory of fuzzy sets to various application areas of industrial planning, production planning, agricultural production planning, economics etc. Atanossov [4, 5] extended the fuzzy set theory to intuitionistic fuzzy sets.

This extended new set, name as intuitionistic fuzzy set, has a feature to accommodate hesitation factor of including an element in a fuzzy set apart from the feature of degree of belonging and non-belonging. This extension of fuzzy set to intuitionistic fuzzy set attracted research workers as well as planners to apply this new set in the field of decision sciences.

Thus, an extension of deterministic optimization to intuitionistic fuzzy optimization was initiated by Angelov [3]. The Angelov study was motivated by Zimmermann visualization of a fuzzy set to explain the degree of satisfaction of respective condition and was expressed by their membership function.

Angelov [3] in his study extended the Bellman and Zadeh [6] approach of maximizing the degree of (membership function) acceptance of the objective functions and constraints to maximizing the degree of acceptance and minimizing the degree of rejection of objective functions and constraints.

In view of its suitability of intuitionistic fuzzy set-in modelling systems having imprecise parameters, a considerable research work has been carried out in the direction of ranking of intuitionistic fuzzy numbers.

Further development of approximation methods is needed for development of intuitionistic optimization techniques (please see Hassan [14], Grzegorzewski [13], Parvathi and Malathi [21]. Nishad et al. [7, 20] have also worked on developing the ranking method for intuitionistic fuzzy numbers and have applied it on intuitionistic fuzzy optimization.

There are many more authors, who worked on the ranking methods and approximation of intuitionistic fuzzy number (please see Inuiguchi and Tanaka [16]).

Recently Dubey et al [11, 12] have studied fuzzy linear programming with intuitionistic fuzzy numbers. The present work is a motivation towards the application of intuitionistic fuzzy numbers to optimization problem and develops a computational method for solution of such optimization problems.

The study is presented in the following sections: Section 2 is preliminaries to intuitionistic fuzzy set and intuitionistic fuzzy numbers needed for consequent sections. Section 3 comprise of modelling of an intuitionistic fuzzy optimization problem and its solution algorithm.

Section 4 illustrates the implementation of the theory developed in section 3 to a linear programming problem as well as to a multi objective linear programming problem. Last section presents the results of the undertaken problem and provides a brief discussion on the developed method.

6.2 Preliminaries:

Definition 1. Fuzzy Set

Let X is a collection of objects denoted by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$, where $\mu_{\tilde{A}}(x)$ and $\upsilon_A(x)$ are called the membership and non-membership functions of x in $\tilde{A}$ respectively.

Where $\mu_A: X \rightarrow [0, 1]$ and $\upsilon_A: X \rightarrow [0, 1]$ and $\mu_{\tilde{A}}(x) + \upsilon_A(x) \leq 1$

Definition 3. Trapezoidal Intuitionistic Fuzzy Number (TIFN)

An intuitionistic fuzzy set (IFS), $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \upsilon_A(x)) \mid x \in X\}$ on R is said to be an intuitionistic fuzzy number, if $\mu_A(x)$ and $\upsilon_A(x)$ are membership and non-membership function respectively and $\upsilon_A \leq \mu_A^c$ where μ_A^c denotes the complement of μ_A.

A trapezoidal intuitionistic fuzzy number with parameters $a' \leq a \leq b \leq c \leq d \leq d'$ denoted by $\tilde{A} = \langle (a, b, c, d, \mu_A), (a', b, c, d', \upsilon_A) \rangle$ is an intuitionistic fuzzy set on real line R whose membership and non-membership functions are defined as follows:

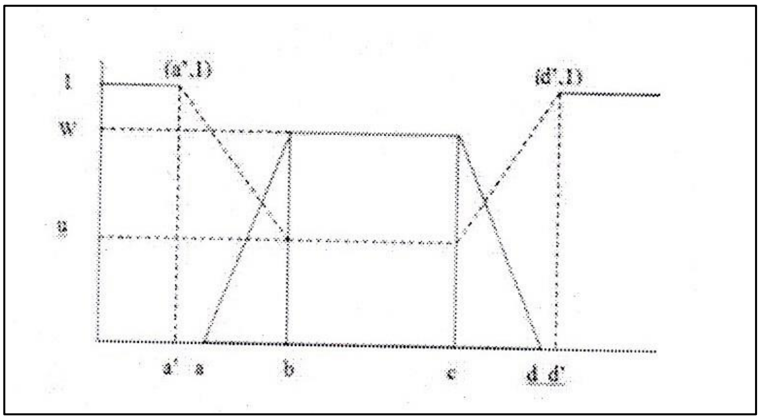

Figure 6.1: Membership and non-membership function of Trapezoidal Intuitionistic Fuzzy Number

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x - a)}{w} & \text{if } a \le x < b \\ w & \text{if } b \le x < c \\ \frac{(d - x)\, w}{(d - c)} & \text{if } c \le x < d \end{cases}$$

$$\upsilon_{\tilde{A}}(x) = \begin{cases} \frac{(x - a')\, u + (b - x)}{(b - a')} & \text{if } a' \le x < b \\ u & \text{if } b \le x < c \\ \frac{(x - d')\, u + (c - x)}{(c - d')} & \text{if } c \le x < d' \end{cases}$$

Here, the values w and u represent the maximum degree of membership and the minimum degree of non-membership function, respectively, such that $0 \le w + u \le 1$.

Definition 4. Triangular Intuitionistic Fuzzy Number (TrIFN)

A trapezoidal Intuitionistic fuzzy number, becomes a triangular intuitionistic fuzzy number by setting b = c and hence parameters become $a' \le a \le b \le d \le d'$ and is denoted by $\tilde{A} = \langle (a, b, d, \mu_A), (a', b, d', \upsilon_A) \rangle$

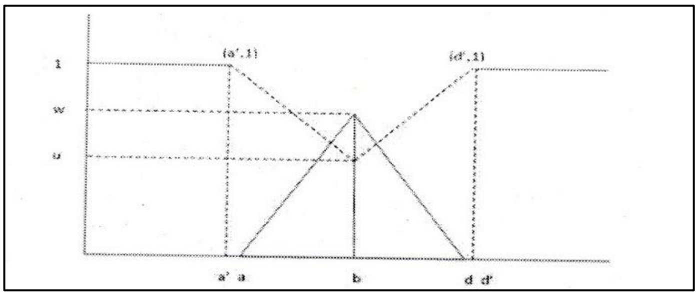

Figure 6.2: Membership and Non-Membership Function of Triangular Intuitionistic Fuzzy Number

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)w}{(b-a)} & \text{if } a \le x < b \\ w & \text{if } x = b \\ \dfrac{(d-x)\,w}{(d-b)} & \text{if } b \le x < d \end{cases}$$

and

$$\upsilon_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a')\,u + (b-x)}{(b-a')} & \text{if } a' \le x < b \\ u & \text{if } \quad x = b \\ \dfrac{(x-d')\,u + (c-x)}{(c-d')} & \text{if } b \le x < d' \end{cases}$$

Definition 5. Expected Interval of Fuzzy Number

One approach to approximate a fuzzy number in deterministic form is expected interval EI(Ã) value. The theory of expected interval of a fuzzy number was introduced by Dubois, Prade and Heilpern. Dubois and Prade [11] considered the approximation of a fuzzy number as a mean value of fuzzy number and give a rigorous definition for mean value of a fuzzy number and give a rigorous definition for mean value is preserved in possibilistic frame work. Later on, Heilpern [15] in his study defined the expected value of a fuzzy number vai a random set and introduced two notations, the expected interval and the expected value of the fuzzy number. He defined the expected value of a fuzzy number as a centre of the expected interval of such a number. A fuzzy number $\tilde{A} = < (a_1, a_2, a_3, a_4) >$ as interval fuzzy number can be written as

$$EI(\tilde{A}) = [E_*(\tilde{A}), E^*(\tilde{A})],$$

Where $E^*(\tilde{A}) = a_2 - \int_{a_1}^{a_2} f_{ar}(x)\,dx$ and $E^*(\tilde{A}) = a_3 - \int_{a_3}^{a_4} g_{\tilde{A}}(x)\,dx$.

Here, the two function $f_{\tilde{A}}(x)$ and $g_{\tilde{A}}(x)$ are defined as

$$f_{\tilde{A}}(x) = \frac{x - a_1}{a_2 - a_1}, \quad g_{\tilde{A}}(x) = \frac{x - a_4}{a_3 - a_4}$$

Definition 6. Expected Interval for Intuitionistic fuzzy number

Let there exist number: $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in R$ such that $a_1 \leq a_2 \leq a_3 \leq a_4 \leq b_1 \leq b_2 \leq b_3 \leq b_4$ with four functions $f_{\tilde{A}}, g_{\tilde{A}}, h_{\tilde{A}}, k_{\tilde{A}} : R \rightarrow [0, 1]$, out of which $f_{\tilde{A}}$ and $k_{\tilde{A}}$ are non-decreasing and $g_{\tilde{A}}, h_{\tilde{A}}$ are non-increasing functions, then an intuitionistic fuzzy number $\tilde{A} = \{x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x) \mid x \in X\}$ is defined by its membership and non-membership function as given as

$$\mu_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}(x) & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x < a_3 \\ g_{\tilde{A}}(x) & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mu_{\tilde{A}}(x) = \begin{cases} h_{\tilde{A}}(x) & \text{if } b_1 \le x \le b_2 \\ 0 & \text{if } b_2 \le x < b_3 \\ k_{\tilde{A}}(x) & \text{if } b_3 \le x \le b_4 \\ 1 & \text{otherwise} \end{cases}$$

The expected interval of the intuitionistic fuzzy number $\tilde{A} = <(a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4)>$ introduced by Grzewski [13] is a crisp interval and is defined as $EI(\tilde{A}) = [E_*(\tilde{A}), E^*(\tilde{A})]$,

$$E_*(\tilde{A}) = \frac{a_2 + b_1}{2} + \frac{1}{2}\int_{b_1}^{b_2} h_{\tilde{A}}(x)\,dx - \frac{1}{2}\int_{a_1}^{a_2} f_{\tilde{A}}(x)\,dx$$

$$E^*(\tilde{A}) = \frac{b_4 + a_3}{2} + \frac{1}{2}\int_{a_3}^{a_4} g_{\tilde{A}}(x)\,dx - \frac{1}{2}\int_{b_3}^{b_4} k_{\tilde{A}}(x)\,dx$$

and

$$h_{\tilde{A}}(x) = \frac{x - b_1}{b_2 - b_1}, \quad f_{\tilde{A}}(x) = \frac{x - a_1}{a_2 - a_1}, \quad g_{\tilde{A}}(x) = \frac{x - a_4}{a_3 - a_4}, \quad k_{\tilde{A}}(x) = \frac{x - b_4}{b_3 - b_4}$$

Definition 7. Expected Interval for a triangular Intuitionistic fuzzy number

If $\tilde{A} = <(a_1, a, a_3; \mu_{\tilde{A}}), (b_1, a, b_2; \upsilon_{\tilde{A}})>$ is triangular intuitionistic fuzzy number then the above definition of expected interval of triangular intuitionistic fuzzy number produces $EI(\tilde{A}) = [E_*(\tilde{A}), E^*(\tilde{A})]$

Were

$$E_*(\tilde{A}) = \frac{a + b_1}{2} + \frac{1}{2}\int^{a} h_{\tilde{A}}(x)\,dx - \frac{1}{2}\int_{a_1}^{a} f_{\tilde{A}}(x)\,dx$$

$$= \frac{3a + b_1 + (a - b_1)\upsilon_{\tilde{A}} - (a - a_1)\mu_{\tilde{A}}}{4}$$

$$E^*(\tilde{A}) = \frac{b_2 + a}{2} + \frac{1}{2}\int_{a}^{a_2} g_{\tilde{A}}(x)\,dx - \frac{1}{2}\int_{a}^{b_2} k_{\tilde{A}}(x)\,dx$$

$$= \frac{3a+b_2+(a_2-a)\,\mu_{\tilde{A}}+(a-a_1)\upsilon_{\tilde{A}}}{4}$$

Definition 8. Ranking of Intuitionistic fuzzy number with expected interval

For any pair of intuitionistic fuzzy number Ã and B with respective expected intervals EI(Ã) = [E_*(Ã), E*(Ã)] and EI(B) = [E_*(B), E*(B)]

(i) Ã > B iff $E_*(\tilde{A}) > E^*(B)$
(ii) Ã = B iff $E_*(\tilde{A}) = E_*(B)$ and $E^*(\tilde{A}) = E^*(B)$

In a situation where the above definition fails, then degree satisfactory method at which Ã is bigger than or equal to B is defined as

$$\mu_E(\tilde{A}, B) = \begin{cases} 1 & \text{if } E_*(\tilde{A}) - E^*(B) > 0 \\ \dfrac{E_*(\tilde{A}) - E^*(B)}{(E^*(\tilde{A})-E_*(B)-(E_*(\tilde{A})-E^*(B))} & \text{if } 0 \square \; [E_*(\tilde{A})-E^*(B),\; E^*(\tilde{A})-E_*(B)] \\ 0 & \text{if } E^*(\tilde{A}) - E_*(B) < 0 \end{cases}$$

(1)

Here, $\mu E(\tilde{A}, B) \geq \alpha$, represented as Ã is bigger than or equal to B at least to a degree α.

6.3 Problem Formulation:

Consider a multi-objective linear programming problem with k objectives and m constraints in n decision variables and is given as

Optimize $Z_k(X) = C_k(X)$

Subject to

$AX\ (\geq, =, \leq)\ b$

$X \geq 0$
(2)

Where X□ R^n, b^T □ R^m, $C^T{}_R$□ R^n and A be a m x n technological matrix. (≥, =, ≤) denotes that the constraints may be of any of the tree types or may be of all the three types.

6.3.1 Multi-Objective Linear Programming Problem with The Intuitionistic Fuzzy Parameters:

Let us consider a multi-objective optimization problem with a decision variable, m constraints and k functions,

Maximize $Z(X) = \{C_1, C_2, C_3, \ldots, C_k\}X$

s.t. $\tilde{A}_i X_j$ (≥, =, ≤) b_i $\quad$ i=1, 2, 3, …, m

$X_j \geq 0 \quad j = 1, 2, 3, n$ $\quad$ (3)

Where $X = \{x_1, x_2, x_3, \ldots, x_n\}$, C_k (k = 1, 2, …k) and b_i (i=1, 2, 3, m) are n dimensional and m dimensional vectors respectively, Ã is a m x n matrix with intuitionistic fuzzy parameter and bi and ck are intuitionistic fuzzy numbers. Since the above problem (3) have intuitionistic fuzzy numbers. Since the above problem (3) have intuitionistic fuzzy coefficients which have possibilistic distribution in an uncertain interval and hence may be approximated in terms of its expected intervals.

Let EI(Ã) be expected interval of intuitionistic fuzzy number Ã defined by the definition (7) and is given as

$EI(\tilde{A}) = [E_*(\tilde{A}), E^*(\tilde{A})]$

Where $E_*(\tilde{A})$ and $E^*(\tilde{A})$ are the lower and upper bound of the expected interval EI(Ã) of intuitionistic fuzzy number.

Since Ck the coefficients of the objective function are intuitionistic fuzzy numbers, expected interval of Ck can be defined as $EI(C_k) = [E_*(C_k), E^*(C_k)]$ where $E_*(C_k)$ and $E^*(C_k)$ is given as in definition of expected interval. Thus $EI(C_k)$ is given as in definition of expected interval. Thus $EI(C_k)$ can be represented as a closed interval $[E_*(C_k), E^*(C_k)]$, such that C_k □ $[E^*(C_k), E^*(C_k)]$

Now the lower and upper bound for the respective expected intervals of the objective function are defined as

$$[Z_k(x)]^L = \sum_{j=1}^{n} E_*(C_{kj})X_j \quad (4)$$

$$[Z_k(x)]^U = \sum_{j=1}^{n} E^*(C_{kj})X_j \quad (5)$$

In the next step, we construct a membership function for minimization type objective function $Z_k(X)$, and then replace by the lower bound of its expected interval that is

$[Z_k(x)]^U = \sum_{j=1}^{n} E^*(C_{kj})X_j$ (6)

Similarly, we construct a membership function for minimization type objective function Zk(x), and then replace by the lower bound of its expected interval is

$[Z_k(x)]^L = \sum_{j=1}^{n} E_*(C_{kj})X_j$ (7)

And the constraint inequalities

$\sum_{j=1}^{n}.(\tilde{A}_{ij})X_j \geq B_i$ $(i = 1, 2, \ldots m_1)$ (8)

$\sum_{j=1}^{n}.(\tilde{A}_{ij})X_j \leq B_i$ $(i = m_1 + 1, m_1 +2, \ldots .m_2)$ (9)

Which can be written in feasibility degree relation in terms of α-parameteric constraints as

$\sum_{j=1}^{n}.[(1-\alpha) E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})] X_j \geq (1-\alpha) E_*(B_i) + \alpha E^*(B_i)$ $(i=1, 2, \ldots m_1)$ (10)

$\sum_{j=1}^{n}.[(1-\alpha) E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})] X_j \leq (1-\alpha) E^*(B_i) + \alpha E_*(B_i)$ $(i=m_1+1, m_1+2, \ldots m_2)$ (11)

And the intuitionistic fuzzy equality constraint

$\sum_{j=1}^{n}.(\tilde{A}_{ij})X_j \geq B_i$ $(i=m_2+1, m_2+2, \ldots m)$ (12)

Can be transformed into two intuitionistic fuzzy inequalities as

$\sum_{j=1}^{n}.(\tilde{A}_{ij})X_j \geq B_j \leq (i=m_2+1, m_2+2, \ldots m)$ (13)

$\sum_{j=1}^{n}.(\tilde{A}_{ij})X_j \geq B_i \geq (i=m_2+1, m_2+2, \ldots m)$ (14)

The above equations can be also written in α-parametric form as equation (10) and (11) using ranking function defined in equation (1).

Thus, the undertaken maximization problem (3) is transformed to an equivalent multi objective linear programming problem (MOLPP) as

$[Z_k(x)]^U = \sum_{j=1}^{n} E^*(C_{kj})X_j$ $(k=1, 2, 3, \ldots ..K)$

Subject to

$\sum_{j=1}^{n}.[(1-\alpha) E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})] X_j \geq (1-\alpha) E_*(B_i) + \alpha E^*(B_i)$ $(i=1, 2, \ldots m_1)$

$\sum_{j=1}^{n}$.[(1-α) E*(Ãij) + αE*(Ãij)] $X_j \leq$ (1-α) $E^*(B_i)$ + α$E_*(B_i)$ (i=m_1+1, m_1+2, …m_2)

$\sum_{j=1}^{n}$.[(1-α) E*(Ãij) + αE*(Ãij)] $X_j \leq$ (1-α) $E^*(B_i)$ + α$E_*(B_i)$ (i=m_1+1, m_1+2, …m_2)

$\sum_{j=1}^{n}$.[(1-α) E*(Ãij) + αE*(Ãij)] $X_j \geq$ (1-α) $E^*(B_i)$ + α$E_*(B_i)$ (i=m_1+1, m_1+2, …n)

$X_j \geq 0$ (j= 1, 2, 3, n) (15)

Now, the problem (15) can be reduced to a deterministic linear programming problem for a prescribed value of α and can proceed to solve by applying the fuzzy programming techniques.

Thus, is need to construct a membership function for maximizing type objective function by using the best and worst acceptable solution defined as:

$$\mu_{Zk}(x) = \begin{cases} 1 & \text{if } Z_x(X) \geq g_k \\ \dfrac{Z_k(X) - l_k}{(g_k - l_k)} & \text{if } l_k \leq Z_k(X) \leq g_k \\ 0 & Z_k(X) \leq l_k \end{cases}$$

Where g_k is aspiration level for the k[th] objective function and the highest acceptable level for the k[th] objective function and the lowest acceptable level l_k are ideal and anti-ideal solutions and are computed as

$g_k = \text{Max} \sum_{j=1}^{n} E^*(C_{kj})X_j$ (k=1, 2, 3, K) (16)

with respect to constraints of the problem (14) for value of α = 1.

Similarly, for maximizing type objective function, an ideal and anti-ideal solution can be also defined.

Now using fuzzy max-min model, the above linear programming problem is converted in to single objective linear programming problem and then can be solved for different value of α as follows.

Maximize λ

Subject to

$$\lambda \leq \frac{Z_k(X) - l_k}{(g_k - l_k)}$$

Subject to

$\sum_{j=1}^{n}.[(1-\alpha)\, E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})]\, X_j \geq (1-\alpha)\, E_*(B_i) + \alpha E^*(B_i)$ (i=1, 2, …m$_1$)

$\sum_{j=1}^{n}.[(1-\alpha)\, E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})]\, X_j \leq (1-\alpha)\, E^*(B_i) + \alpha E_*(B_i)$ (i=m$_1$+1, m$_1$+2, …m$_2$)

$\sum_{j=1}^{n}.[(1-\alpha)\, E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})]\, X_j \leq (1-\alpha)\, E^*(B_i) + \alpha E_*(B_i)$ (i=m$_1$+1, m$_1$+2, …m$_2$)

$\sum_{j=1}^{n}.[(1-\alpha)\, E^*(\tilde{A}_{ij}) + \alpha E^*(\tilde{A}_{ij})]\, X_j \geq (1-\alpha)\, E^*(B_i) + \alpha E_*(B_i)$ (i=m$_1$+1, m$_1$+2, …n)

$X_j \geq 0$ (j= 1, 2, 3, n) (18)

6.4 Illustration:

Example 1 Consider a linear programming problem in intuitionistic fuzzy environment as

Maximize $Z = 25x_1 + 48x_2$

subject to $15x_1 + 30x_2 \leq 45000$

$24x_1 + 6x_2 \leq 24000$

$21x_1 + 14x_2 \leq 28000$ (19)

$x_1, x_2 \geq 0,$

having intuitionistic fuzzy coefficients given as

25 = < (19, 25, 33; 0.9), (18, 25, 34; 0.1) >, 48 = <(44, 48, 54; 0.9), (43, 48, 56;0.1), >, 15 = < (14, 15, 17; 0.9), (10, 15, 18; 0) >, 30 = <(25, 30, 34; 0.9), (23, 30, 38; 0)>, 24 = < (21, 24, 26; 0.9), (20, 24, 33; 0)>, 6 = < (4, 6, 8; 0.9), (2, 6, 11; 0)>, 21 = <(17, 21, 22; 0.9), (16, 21, 26; 0)>, 14 = <(12, 14, 19; 0.9), (8, 14, 22; 0)>, 45000 = <(44980, 45000, 45030; 0.9), (44970, 45000, 45070; 0)>, 24000 = <(23980, 24000, 24060; 0.9), (23940, 24000, 24060; 0)>, 28000 = <(27990, 28000, 28030; 0.9), (27950, 28000, 28040; 0)>, respectively,

Now approximating the above intuitionistic fuzzy numbers by its interval value as described in section 2, are written in terms of its expected intervals as

EI (25) = [22.075, 28.825], EI (48) = [45.975, 51.15], EI (15) = [13.525, 16.2], EI (30) = [27.125, 32.9], EI (24) = [22.325, 26.7], EI (6) = [4.55, 7.7], EI (21) = [18.85, 22.475], EI (14) = [12.05, 17.125], EI(45000) = [44988, 45024.25], EI(24000) = [23980.5, 24026.25], EI(28000) = [27985.25, 28016.75]

With these expected interval values of intuitionistic fuzzy numbers, the problem (19) is transformed in to a α – parametric linear problem defined as.

Maximize $Z = 28.82\, x_1 + 51.15\, x_2$

Subject to
$[(1-\alpha)13.525 + 16.2\alpha]x_1 + [(1-\alpha)27.125 + 32.9\alpha]x_2 \leq [(1-\alpha)45024.25 + 44988\alpha]$

$[(1-\alpha)22.325 + 26.7\alpha]x_1 + [(1-\alpha)4.55 + 7.7\alpha]x_2 \leq [(1-\alpha)24026.25 + 23980.5\alpha]$

$[(1-\alpha)18.85 + 22.475\alpha]x_1 + [(1-\alpha)12.05 + 17.125\alpha]x_2 \leq [(1-\alpha)28016.75 + 27985.25\alpha]$

$x_1, x_2 \geq 0$
(20)

To solve this problem, we compute ideal and anti-ideal solution of objective functions as described in section 4 and thus computed values comes out to be g_1 = 86975.449, l_1 = 62866.215. Now implementing the above developed computational algorithm, the problem (20) can be written to an equivalent linear programming problem as

Maximize λ

Subject to

$\lambda - 0.142x_1 - 0.064x_2 \leq -0.529$

$[(1-\alpha)13.525 + 16.2\alpha]x_1 + [(1-\alpha)27.125 + 32.9\alpha]x_2 \leq [(1-\alpha)45024.25 + 44988\alpha]$

$[(1-\alpha)22.325 + 26.7\alpha]x_1 + [(1-\alpha)4.55 + 7.7\alpha]x_2 \leq [(1-\alpha)24026.25 + 23980.5\alpha]$

$[(1-\alpha)18.85 + 22.475\alpha]x_1 + [(1-\alpha)12.05 + 17.125\alpha]x_2 \leq [(1-\alpha)28016.75 + 27985.25\alpha]$

$x_1, x_2 \geq 0$
(21)

This linear programming problem (21) has been solved by MATLAB® for different value of α and solution obtained is given in table 1.

Table 6.1: Optimal solution for different feasibility degree of α

α	x_1	x_2	Z
0	624.2	1348.7	86978.57
0.1	586.9	1333	85100.342
0.2	551.5	1317.6	83292.227
0.3	518	1302.7	81564.455
0.4	486.1	1288.1	79898.147
0.5	455.9	1273.9	78301.302
0.6	427.1	1259.9	76755.042
0.7	399.8	1246.3	75272.48
0.9	348.9	1220	72460.042

Example. 2 Consider a multi-objective intuitionistic fuzzy linear programming problem as

Maximize $Z_1 = 4x_1 + 2x_2$

Maximize $Z_1 = 2x_1 + 6x_2$

Subject to

$$1\,x_1 + 4\,x_2 \leq 28$$

$$1\,x_1 + 1\,x_2 \leq 10$$

$$3\,x_1 + 1\,x_2 \leq 24 \qquad (22)$$

$$x_1, x_2 \geq 0$$

Here, we assume that each of coefficients are triangular intuitionistic fuzzy numbers and are given as

1 = <($\frac{1}{2}$, 1, $\frac{3}{2}$; 0.9), ($\frac{1}{2}$, 1, $\frac{3}{2}$; 0)>, 2 = <(1, 2, $\frac{5}{2}$; 0.9), (1, 2, $\frac{5}{2}$; 0)>, 3 = <(2, 3, 5; 0.9), (2, 3, 5; 0)>, 4 = <(2, 4, 6; 0.9), (2, 4, 6; 0)>, 6 = <(4, 6, 8; 0.9), (4, 6, 8; 0)>, 10 = <(8, 10, 11; 0.9), (8, 10, 11; 0.9), (8, 10, 11; 0)>, 24 = <(20, 24, 26; 0.9), (20, 24, 26; 0)>, 28 = <(23, 28, 31; 0.9), (23, 28, 31; 0)>, respectively,

Approximating the above intuitionistic fuzzy numbers by its interval value as given in section II,

their respective expected intervals are given as

EI (1) = [0.762, 1.237], EI (2) = [1.525, 2.237], EI (3) = [2.525, 4], EI (4) = [3.05, 4.95], EI (6) = [5.05, 6.95], EI (10) = [9.05, 10.47], EI (24) = [22.1, 24.95], EI (28) = [25.625, 29.425],

Using these approximated expected intervals of intuitionistic fuzzy numbers, the problem (22) is transformed in to an equivalent multi-objective α-parametric linear programming problem defined as.

Maximize $Z1 = 4.95\,x_1 + 2.237x_2$

Maximize $Z2 = 2.237x_1 + 6.95x_2$

Subject to

$$[(1-\alpha)0.762 + 1.237\alpha]x_1 + [(1-\alpha)3.05 + 4.95\alpha]x_2 \leq [(1-\alpha)29.425 + 25.625\alpha]$$

$$[(1-\alpha)0.762 + 1.237\alpha]x_1 + [(1-\alpha)0.762 + 1.234\alpha]x_2 \leq [(1-\alpha)10.47 + 9.05\alpha]$$

$$[(1-\alpha)2.525 + 4\alpha]x_1 + [(1-\alpha)0.762 + 1.237\alpha]x_2 \leq [(1-\alpha)24.95 + 22.1\alpha]$$

$x_1, x_2 \geq 0$

(23)

Now, we calculate ideal and anti-ideal solutions for each of objective functions of the above MOLP as described in section III and thus computed values comes out to be

$g_1 = 53.019$, $l_1 = 18.359$, $g_2 = 69.781$, $l_2 = 26.892$, respectively.

Now implementing our developed computational algorithm, the problem (23) can be written to an equivalent linear programming problem as

Maximize λ

Subject to

$\lambda - 0.142x_1 - 0.064x_2 \leq -0.529$

$\lambda - 0.052x_1 - 0.161x_2 \leq -0.626$

$[(1-\alpha)0.762 + 1.237\alpha]x_1 + [(1-\alpha)3.05 + 4.95\alpha]x_2 \leq [(1-\alpha)29.425 + 25.625\alpha]$

$[(1-\alpha)0.762 + 1.237\alpha]x_1 + [(1-\alpha)0.762 + 1.237\alpha]x_2 \leq [(1-\alpha)10.47 + 9.05\alpha]$

$[(1-\alpha)2.525 + 4\alpha]x_1 + [(1-\alpha)0.762 + 1.237\alpha]x_2 \leq [(1-\alpha)24.95 + 22.1\alpha]$

$x_1, x_2 \geq 0$

(24)

This linear programming problem has been solved by MATLAB® for different value of α and solutions obtained are given in table 2.

Table 6.2: Optimal solution for different α-feasibility degree

α	x_1	x_2	Z_1	Z_2
0	6.608	7.131	48.661	64.34
0.1	6.103	6.663	45.114	59.960
0.2	5.646	6.239	41.904	55.991
0.3	5.244	5.866	39.080	52.499
0.4	4.876	5.524	36.493	49.29
0.5	4.549	5.220	34.194	46.155
0.6	4.246	4.939	32.066	43.824
0.7	3.974	4.687	30.156	41.415
0.8	3.721	4.452	28.378	39.265
0.9	3.491	4.239	26.763	37.270

6.5 Result and Discussion:

The developed algorithm uses the α-degree feasibility of linear intuitionistic fuzzy programming problem. We compare the results obtained in table 1 and table 2 with that of results of Dubey and Kuwano method. Clearly the level of satisfaction of each objective function by the proposed method is higher than the previous results. Thus, for modelling the optimization problems having vegueness and imprecision in information with intuitionistic fuzzy optimization approach may be considered as an alternative method to see optimal values. the proposed algorithm is more suitable to find the optimal solutions of the problems having intuitionistic fuzzy coefficients arising in production planning problems, financial planning problems, agricultural production planning problems and many other real world multi-objective programming problems. One of the interesting features of the α feasible solutions in both case of linear programming problem as well as in multi-objective linear programming problem, the values of the objective functions decrease with increase of α values of α. Thus, solution give an insight on the degree of vagueness and the possible feasible solutions. Hence, the present study provides solution to the problem with various degree of feasibility in the situation of imprecision in parameters. Thus, the decision maker has enough information about the feasible solutions ranging from best to worst to take appropriate decision according to the situation.

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ISBN: 978-81-19149-24-7

Chapter 7

Conclusion

In many real-life optimization problems, the parameters are often imprecise and are difficult to be represented in discrete quantity. One of the approaches to model such situation is considering these imprecise parameters as intuitionistic fuzzy numbers and then approximating these by its expected interval value. Further in process of solution, membership function for each objective function is constructed by computing best and worst acceptable solutions and deal the constraints of the problem with ranking of intuitionistic fuzzy number with a concept of feasibility degree. The paper presents a computational algorithm for solution of objective functions at different feasibility degree. The developed algorithm has been illustrated by implementing on a linear programming problem as well as on a multi objective linear programming problem (MOLPP) in intuitionistic fuzzy environment.

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ISBN: 978-81-19149-24-7