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2. Testing of Hypothesis

Dr. R. Narmada Devi and Dr. S. Karpagam

Associate Professor,
Dept. of Mathematics,
Vel Tech Rangarajan Dr. Sgunthala R & D Institute of Science and Technology,
Avadi, Chennai.

2.1 Introduction:

Many problems in engineering require, that whether decide whether to accept or reject a statement about some parameter. The statement is called hypothesis, and the decision-making procedure about the hypothesis is called hypothesis testing.

This is one of the most useful aspects of statistical inference, since many types of decision-making problems, tests, or experiments in the engineering world can be formulated as hypothesis-testing problems.

Furthermore, as we will see, there is a very close connection between hypothesis testing and confidence intervals.

Statistical hypothesis testing and confidence interval estimation of parameters are the fundamental methods used at the data analysis stage of a comparative experiments.

2.1.1 Population:

A population is statistics means a set of objects or mainly the set of numbers which are measurements or observations pertaining to the objects. The population is finite or infinite according to the number of elements of the set is finite or infinite.

2.1.2 Sample:

A part selected from population is called a sample. The process of a sample is called sample.

2.1.3 Parameters and Statistics:

The statistical constants of the population, such as mean (μ), standard deviation (σ) are called parameters.

Parameters are denoted by Greek letters.

The mean $\overline{x}$, standard deviation S of a sample are known as statistics. Statistics are noted by Roman letters.

2.1.4 Symbols for Population and Samples:

Characteristics	Population	Sample
	Parameter	**Statistic**
Symbols	Population size = N	Sample size = n
	Population mean = μ	Sample mean = $\bar{x}$
	Population standard deviation = σ	Sample standard deviation = s
	Population proportion = p	Population proportion = p

2.2 Hypothesis Testing or Significance Testing:

Hypothesis is usually considered as the principal instrument in research. The main aim in many studies of research is to check whether the data collected support certain predictions or conditions. It is very essential to a research worker to understand the meaning and nature of hypothesis. The researcher always plans or formulate a hypothesis in the beginning of the problem. A hypothesis is an assertion or conjecture concerning one or more populations. In short, hypothesis testing enables us to make probability statements about population parameter.

The word hypothesis combination of two words: **Hypo + Thesis = Hypothesis**. That is, “Hypo” means tentative or subject to the verification and “Thesis” means statement about solution of a problem.

Alterative meaning of the word hypothesis which is composed of two words **Hypo + Thesis** – ‘Hypo’ means composition of two or more variables which is to be verified. “Thesis” means position of these variables in the specific frame of reference. In simply, we can say a hypothesis is an assumption about the population parameter.

A process of testing of the significance regarding the parameters of the population on the basis of sample drawn from it is called a “Test of hypothesis” (Or) “Test of Significance. In many cases, researchers may find that the results of an experiment do not support the original hypothesis. When writing up these results, the researchers might suggest other options that should be explored in future studies.

2.2.1 Characteristic of Good Hypothesis:

a. Hypothesis should be specific, clear and precise.
b. Hypothesis should be capable of being tested.
c. Hypothesis should state relationship between variables.
d. Hypothesis should be limited in scope.
e. Hypothesis should be stated as far as possible in most simple terms so that the same is easily understandable by all concerned.
f. Hypothesis should be amenable to testing within a reasonable time.
g. Hypothesis must explain empirical reference.
h. Hypothesis should not be contradictory.

2.2.2 Role of Hypothesis:

The hypotheses play a vital role in the scientific studies. Some of the important role and functions of the hypothesis –

a. Helps in the testing of the theories.
b. Serves as a great platform in the investigation activities.
c. Provides guidance to the research work or study.
d. Hypothesis sometimes suggests theories.

2.2.3 Sources of Hypotheses:

The main sources of hypotheses are as follows:

a. Specialization of an educational field.
b. Published studies, abstracts research journals, hand books, seminars on the issue, current trends on the research area.
c. Instructional programs persuaded.
d. Analyze of the area studied.
e. Considering existing practices and needs.
f. Extension of the investigation.
g. Offshoots of research studies in the field.

2.2.4 Limitations of the Test of Hypotheses:

a. Test do not explain the reasons as to why do the difference exist, say between the means of the two samples. They simply indicate whether the difference is due to fluctuations of sampling or because of other reasons but the tests do not tell us as to which is/are the other reason(s) causing the difference.
b. Results of significance tests are based on probabilities and as such cannot be expressed with full certainty.
c. Statistical inferences based on the significance tests cannot be said to be entirely correct evidences concerning the truth of the hypotheses.

2.3 Types of Hypothesis:

Hypothesis consists of two major types. They are

2.3.1 Research Hypothesis:

A research hypothesis is a tentative claim for the problem being investigated. It motivates the researcher to accomplish future course of action.

In research, the researcher determines whether or not their supposition can be supported through scientific investigation. The following are the types of research hypothesis:

2.3.2 Simple Hypothesis:

This predicts the relationship between a single independent variable (IV) and a single dependent variable (DV). **For example:** Lower levels of exercise postpartum (IV) will be associated with greater weight retention (DV).

2.3.3 Complex Hypothesis:

This predicts the relationship between two or more independent variables and two or more dependent variables.

For Example: The implementation of an evidence-based protocol for urinary incontinence (IV) will result in (DV)

a. decreased frequency of urinary incontinence episodes;
b. decreased urine loss per episode;
c. Decreased avoidance of activities among women in ambulatory care settings.

2.3.4 Directional Hypothesis:

This may imply that the researcher is intellectually committed to a particular outcome. They specify the expected direction of the relationship between variables i.e. the researcher predicts not only the existence of a relationship but also its nature.

For Example: High school students who participate in extracurricular activities have a lower GPA than those who do not participate in such activities.

2.3.5 Non-Directional Hypothesis:

It cannot stipulate the direction of the relationship.

For Example: The academic performance of high school students is related to their participation in extracurricular activities.

2.3.6 Associative Hypothesis:

It can propose relationships between variables, when one variable changes, the other changes. Do not indicate cause and effect.

2.3.7 Causal Hypothesis:

Causal hypotheses propose a cause and effect interaction between two or more variables. The independent variable is manipulated to cause effect on the dependent variable. The dependent variable is measured to examine the effect created by the independent variable.

For example: High school students who participate in extracurricular activities spend less time studying which leads to a low GPA.

2.3.8 Inductive and Deductive Hypotheses:

Inductive hypotheses are formed through inductively reasoning from many specific observations to tentative explanations. Deductive hypotheses are formed through deductively reasoning implications of theory.

2.3.9 Statistical Hypothesis:

Statistical hypothesis is a statement about the population which we want to verify on the basis of sample taken from population. Statistical hypothesis is stated in such a way that they may be evaluated by appropriate statistical techniques. The following are the types of statistical hypotheses:

a. **Null Hypothesis (H0)**: A statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.
b. **Alternative Hypothesis (H1)**: A statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters. Alternative hypothesis is created in a negative meaning of the null hypothesis.

2.4. Basic Concepts of Testing of Hypotheses:

2.4.1 The Level of Significance:

The level of significance (α) is the probability of rejecting a true null hypothesis that is the probability of "Type I error" and is denoted by α. The frequently used values of α are 0.05; 0.01; 0.1 etc.

When, $\alpha = 0.05$ it means that level of significance is 5%.

$\alpha = 0.01$ it means 1% level of significance.

$\alpha = 0.01$ it means 10% level of significance.

In fact, α specifies the critical region. A competed value of the test statistic that falls in the critical region (CR) is said to be significant.

2.4.2 Critical / Rejection Region:

The critical region (CR) or rejection region (RR) is the area under the curve beyond certain limits in which the population value is unlikely to fall by chance only when the null hypothesis is assumed to be true.

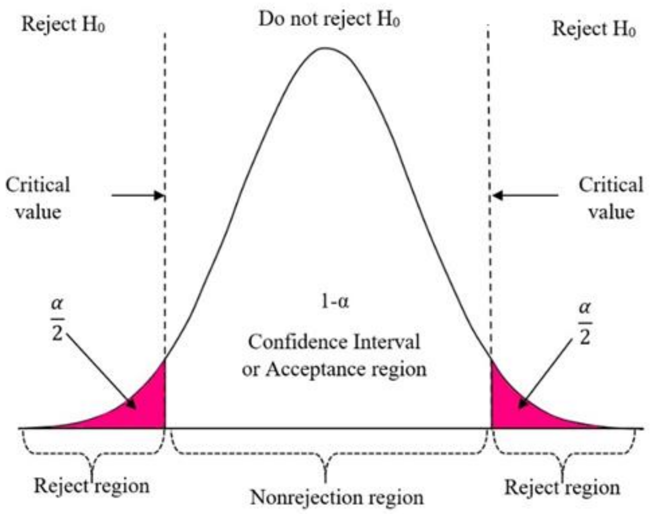
If an observed value falls in this region H0 is rejected and the observed value is said to be significant. In a word, the region for which H0 is rejected is called critical region or rejection region.

2.4.3 Confidence Interval:

The interval which marked by limits within which the population value lies by chance and the hypothesis is consider to be tenable. If an observed value falls in confidence interval H0 is accepted.

2.4.4 Critical Values:

The values of the test statistic which separates critical region from confidence region (acceptance region) are called critical values.



2.4.5 Decision rule or Test of Hypothesis:

A decision rule is a procedure that the researcher uses to decide whether to accept or reject the null hypothesis. The decision rule is a statement that tells under what circumstances to reject the null hypothesis.

The decision rule is based on specific values of the test statistic (e.g., reject H0 if Calculated value > table value at the same level of significance)

2.4.6 Types of Error:

In testing of hypotheses, there are basically, we have two types of errors in testing of

a. **Type I Error:** To reject the null hypothesis when it is true is to make what is known as a type I error. The level at which a result is declared significant is known as the type I error rate, often denoted by α.
b. **Type II Error:** If we do not reject the null hypothesis when in fact there is a difference between the groups, we make what is known as a type II error. The type II error rate is often denoted as β.

In a tabular form,

Truth of the Population	Decision	
	Accept H_0	**Reject H_0**
H_0 (True)	Correct Decision	Type I error (α error)
H_0 (False)	Type II error (β Error)	Correct decision

2.4.7 Standard Error:

The standard error is an estimate of the standard deviation of a statistic. The standard error is used to compute other measures like confidence intervals and margins of error.

The standard error is computed from known sample statistics, and it provides an unbiased estimate of the standard deviation of the statistic. Reciprocal of standard error is known as precision. The following table shows standard error for the respected statistics of the population:

Sr. No.	Statistic	Standard error
1.	$\bar{x}$	$\frac{\sigma}{\sqrt{n}}$
2.	s	$\sqrt{\frac{\sigma^2}{2n}}$
3.	s^2	$\sigma^2\sqrt{\frac{2}{n}}$
4.	$\overline{x_1} - \overline{x_2}$	$\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}$
5.	$s_1 - s_2$	$\sqrt{\frac{\sigma^2}{2n_1} + \frac{\sigma^2}{2n_2}}$
6.	$p_1 - p_2$	$\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$
7.	Observed sample proportion p	$\sqrt{\frac{PQ}{n}}$

2.4.8 Degree of Freedom:

Degree of freedom refers to the number of values which are free to vary after we have given the number of restrictions imposed upon the data. It is commonly abbreviated by df.

2.4.9 One- Tailed Test:

A test of statistical hypothesis, where the region of rejection is on only one side of the sampling distribution, is called a one tailed test and it is represented as

H_0	$\mu \geq \mu_0$	$\mu \leq \mu_0$
H_1	$\mu < \mu_0$	$\mu > \mu_0$

One- tailed Test are classified as

a. Right Tailed Test:
A test in which critical region is located in right tail of the distribution of test statistic is called right tailed test or upper one tailed test.

b. Left Tailed Test:
A test in which critical region is located in left tail of the distribution of test statistic is called left tailed test or lower one tailed test.

For example: Suppose,

H_0 : The population mean $\mu \leq 10$ and
H_1 : The population mean $\mu > 10$.

The region of rejection which consist of a range of numbers come across on the right side of sampling distribution. i.e., a set of numbers greater than 10.

2.4.9 Two-Tailed Test:

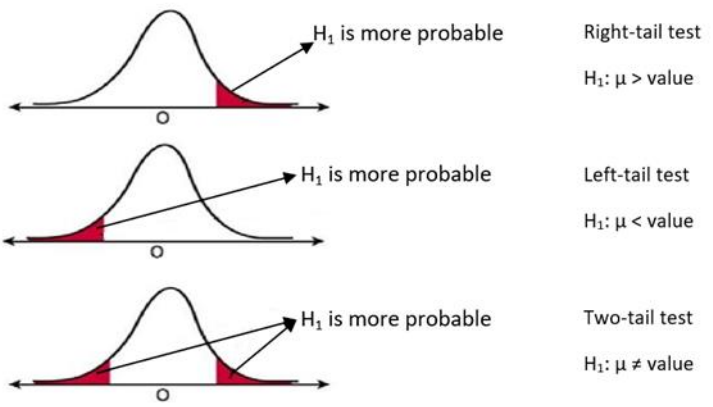
A test of statistical hypothesis, where the region of rejection is on both sides of the sampling distribution, is called a two-tailed test. The hypotheses are represented as

$H_0: \theta = \theta_0; H_1: \theta \neq \theta_0$.

For example: Suppose,

H_0: The population mean $\mu = 10$ and
H_1: The population mean $\mu > 10$ or $\mu < 10$.

The region of rejection which consist of a range of numbers come across on both sides of sampling distribution; i.e., the region of rejection would consist partly of numbers that were less than 10 and partly of numbers that were greater than 10.



Large and Small Samples:

a. A sample is small if its size $n < 30$.
b. A sample is large if its size $n \geq 30$.

2.5 Purpose of Testing of Hypotheses:

The main purposes of testing of hypotheses which can be classified as:

a. Parametric Tests or Standard Tests:

Parametric tests usually assume certain properties of the parent population from which we draw samples. Assumptions like observations come from a normal population, sample size is large, assumptions about the population parameters like mean, variance, etc., must hold good before parametric tests can be used.

b. Non-Parametric Tests or Distribution-Free Test:

Non-parametric tests usually assume only nominal or ordinal data, whereas parametric tests require measurement equivalent to at least an interval scale. As a result, non-parametric tests need more observations than parametric tests to achieve the same size of Type I and Type II errors.

2.5.1 Important Parametric Tests:

All these tests are based on the assumption of normality i.e., the source of data is considered to be normally distributed.

a. **z-test:** It is based on the normal probability distribution and is used for judging the significance of several statistical measures, particularly the mean. This is a most frequently used test in research studies. This test is used even when binomial distribution or t-distribution is applicable on the presumption that such a distribution tends to approximate normal distribution as 'n' becomes larger. z-test is generally used for comparing the mean of a sample to some hypothesized mean for the population in case of large sample, or when population variance is known. z-test is also used for judging his significance of difference between means of two independent samples in case of large samples, or when population variance is known. z-test is also used for comparing the sample proportion to a theoretical value of population proportion or for judging the difference in proportions of two independent samples when n happens to be large. Besides, this test may be used for judging the significance of median, mode, coefficient of correlation and several other measures.

b. **t- test:** It is based on t-distribution and is considered an appropriate test for judging the significance of a sample mean or for judging the significance of difference between the means of two samples in case of small sample(s) when population variance is not known. In case two samples are related, we use paired t-test for judging the significance of the mean of difference between the two related samples. It can also be used for judging the significance of the coefficients of simple and partial correlations.

c. **F-test:** It is based on F-distribution and is used to compare the variance of the two-independent samples. This test is also used in the context of analysis of variance (ANOVA) for judging the significance of more than two sample means at one and the same time. It is also used for judging the significance of multiple correlation coefficients.

2.6. Procedure for Testing of Hypothesis:

2.6.1 Test for a Specified Mean (Population Mean vs Sample Mean) of Large Sample:

A random sample of size n ≥ 30 is drawn from a population. We want to test the population mean has a specified value μ_0

I. Procedure for Two-Tailed Test:

a. State the null hypothesis $H_0 : \mu = \mu_0$ and alternative hypothesis $H_1 : \mu \neq \mu_0$ (Two tailed)
b. Select the level of significance.

A	0.01 or 1% level	0.05 or 5% level
z- table value	2.58	1.96

c. Test statistic $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

d. **Conclusion:** Make the decision to reject or accept the null hypothesis.
  (i). |Test statistic value| <Table value
  null hypothesis (H_0) is accepted.
  (ii). |Test statistic value| > Table value
  null hypothesis (H_0) is rejected.

For example:

1) For α = 0.01 (1% level)
  (i). |Test statistic value| < 2.58
  null hypothesis (H_0) is accepted at 1% level.

(ii). |Test statistic value| > 2.58
null hypothesis (H_0) is rejected at 1% level.

2) For α = 0.05 (5% level)

(i). |Test statistic value| < 1.96
null hypothesis (H_0) is accepted at 5% level.

(ii). |Test statistic value| > 1.96
null hypothesis (H_0) is rejected at 5% level.

e. Summarize the Result:

II. Procedure for Testing (For One-Tailed Test):

a. State the null hypothesis $H_0 : \mu = \mu_0$ and alternative hypothesis $H_1 : \mu < \mu_0$ (left tailed)
b. Select the level of significance.

α	0.01 or 1% level	0.05 or 5% level
z- table value	- 2.33	-1.645

c. Test statistic $z= \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$
d. **Conclusion:** Make the decision to reject or accept the null hypothesis
(i). |Test statistic value| < Table value
null hypothesis (H_0) is accepted.
(ii). |Test statistic value| > Table value
null hypothesis (H_0) is rejected.
e. Summarize the result.

2.6.2 Test for the Equality of Two Mean (Sample Mean Vs Sample Mean) Of Large Sample:

a. State the null hypothesis $H_0 : \mu = \mu_0$ and alternative hypothesis $H_1 : \mu \neq \mu_0$.
b. Select the level of significance.
c. Test statistic $z= \frac{\overline{x_1}-\overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$
d. **Conclusion:** Make the decision to reject or accept the null hypothesis.
(i). |Test statistic value| < Table value
null hypothesis (H_0) is accepted.
(ii). |Test statistic value| > Table value
null hypothesis (H_0) is rejected.
e. Summarize the result.

2.6.3 Test for a Specified Proportion (Population Proportion (vs) Sample Proportion) of Large Sample:

a. State the null hypothesis H_0 :p=P and alternative hypothesis H_1 :$p \neq P$
b. Select the level of significance.
c. Test statistic $z=\frac{p-P}{\sqrt{\frac{PQ}{n}}}$
d. **Conclusion:** Make the decision to reject or accept the null hypothesis.
   (i). |Test statistic value| <Table value
   null hypothesis (H_0) is accepted.
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.

e. Summarize the result.

2.6.4 Test for the Equality of Two Proportions (Sample Proportion (vs) Sample Proportion) of Large Sample:

a. State the null hypothesis H_0 :$p_1 = p_2$ and alternative hypothesis H_1 : $p_1 \neq p_2$
b. Select the level of significance.
c. Test statistic $z=\frac{p_1-p_2}{\sqrt{PQ(\frac{1}{n_1}+\frac{1}{n_2})}}$, $P = \frac{n_1p_1+n_2p_2}{n_1+n_2}$, $Q = 1 - P$
d. **Conclusion:** Make the decision to reject or accept the null hypothesis.
   (i). |Test statistic value| <Table value
   null hypothesis (H_0) is accepted.
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.
e. Summarize the result.

2.6.5 Test for a Specified Mean (Population Mean (vs) Sample Mean) of Small Samples:

a. State the null hypothesis H_0 : $\mu = \mu_0$ and alternative hypothesis H_1 : $\mu \neq \mu_0$
b. Choose the level of significance and the degrees of freedom is n-1
c. Test statistic $t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}}$,
d. **Conclusion:** Make the decision to reject or accept the null hypothesis
   (i). |Test statistic value| <Table value
   null hypothesis (H_0) is accepted.
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.
e. Summarize the result.

2.6.6 Test for the Equality of Two Mean (Sample Mean (vs) Sample Mean) of Small Samples:

a. State the null hypothesis H_0 : $\mu_1 = \mu_2$ and alternative hypothesis H_1 :$\mu_1 \neq \mu_2$
b. Calculate the table value.
c. Test statistic $t = \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, $S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$, S- Standard error
d. Degrees of freedom = $n_1 + n_2 - 2$
e. **Conclusion:** Make the decision to reject or accept the null hypothesis
   (i). |Test statistic value| < Table value
   null hypothesis (H_0) is accepted.
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.
f. Summarize the result.

2.6.7 t - Test for Paired Sample Observations of Small Samples:

a. State the null hypothesis H_0: $\mu_2 - \mu_1 = 0$ and alternative hypothesis H_1 : $\mu_2 - \mu_1 \neq 0$
b. Calculate the table value for given level of significance with n-1 degrees of freedom.
c. Test statistic t = $\frac{\bar{d}}{s/\sqrt{n-1}}$, $\bar{d} = \frac{\sum d}{n}$
d. Degrees of freedom = $n - 1$
e. Conclusion: Make the decision to reject or accept the null hypothesis
   (i). |Test statistic value| <Table value
   null hypothesis (H_0) is accepted.
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.
f. Summarize the result.

2.6.8 F - Test for Equality of Variance of Small Samples:

a. State the null hypothesis H_0 :$\sigma_1^2 = \sigma_2^2$ and alternative hypothesis H_1:$\sigma_1^2 \neq \sigma_2^2$
b. Calculate the table value for given level of significance with $n_1 - 1$, $n_2 - 1$ degrees of freedom.
c. Test statistic
   (i). I $S_1^2 > S_2^2$f then $F = \frac{S_1^2}{S_2^2}$
   (ii). If $S_2^2 > S_1^2$ then $F = \frac{S_2^2}{S_1^2}$
   Where $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$ and $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

d. **Conclusion:** Make the decision to reject or accept the null hypothesis
   (i). |Test statistic value| <Table value
   null hypothesis (H_0) is accepted
   (ii). |Test statistic value| > Table value
   null hypothesis (H_0) is rejected.
e. Summarize the result

2.6.9 χ^2 – test to test the goodness of fit:

A very powerful test for testing the significance of the discrepancy between theory and experiment was given by prof. Karl-Pearon in 1990 and is known as “Chi-square test of goodness of fit.” It enables us to find if the deviation of the experiment from theory is just by chance or is it really due to the inadequacy of the theory of fit the observed data.

By this test, we test whether differences between observed and expected frequencies are significant or not.

χ^2-test statistic of goodness of fit is defined by

$\chi^2 = \sum \frac{(O-E)^2}{E}$, Where O → observed frequencies and E → Expected frequency.

2.7 Illustrations:

Example 1

The mean life time of a sample of 100 light bulbs produced by a company is computed to be 1570 hours with a standard deviation of 120 hours. If μ is the mean life time of all the bulbs produced by the company, test the hypothesis μ = 1600 hours, against the alternative hypothesis μ ≠ 1600 hours with α =0.05 and 0.01.

Solution:

Given: n=100, $\bar{x}$=1570, μ=1600, s= 120, α = 0.05 and α = 0.01.

a. The null hypothesis H_0: $\mu = 1600$
   Alternative hypothesis H_1 : μ ≠ 1600
   (Two tailed)

b. Level of significance

A	0.01 or 1% level	0.05 or 5% level
z- table value	2.58	1.96

c. Test statistic z $= \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{1570-1600}{120/\sqrt{100}}$ = -2.5

d. Conclusion:

(i) |-2.5| ≮ 1.96
So, we reject the null hypothesis H_0

(ii) |-2.5| < 2.58
So, we accept the null hypothesis H_0 at 1% level of significance.

Thus, the mean life time of all the bulbs produced by the company is 1600 hours.

Example 2

The mean breaking strength of the cables supplied by a manufacturer is 1800 with S.D. of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. In order to test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance?

Solution:

Given: n=50, $\bar{x}$=1850, μ=1800, s= 100, α = 0.01.

a. The null hypothesis H_0 : $\mu = 1800$
   Alternative hypothesis H_1: μ > 1800 (Use one tailed test (right))
b. Level of significance:

A	0.01 or 1% level
z- table value	2.33

c. Test statistic $z=\frac{\bar{x}-\mu}{s/\sqrt{n}}=\frac{1850-1800}{100/\sqrt{50}}$ = 3.54
d. Conclusion: |2.33| ≮ 3.54.

   So, we reject the null hypothesis H_0 at 1% level of significance.

Hence, the mean breaking strength is greater than 1800.

Example 3

The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation 2.5 inches at 5 % level of significance?

Solution:

Given: n_1=1000, $\overline{x_1}$ =67.5, σ_1=σ_2= 2.5,

n_2=2000, $\overline{x_1}$=68.

a. The null hypothesis $H_0 : \mu_1 = \mu_2$ [No significant difference]
   Alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ (Use one tailed test (right))
b. Level of significance:

A	0.05 or 5% level
z- table value	1.96

c. Test statistic $z=\frac{\overline{x_1}-\overline{x_2}}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} = \frac{67.5-68}{\sqrt{\frac{(2.5)^2}{1000}+\frac{(2.5)^2}{2000}}} = -5.16$

d. Conclusion: $|5.16| \nless 1.96$.

So, we reject the null hypothesis H_0 at 5% level of significance.

∴ there is a significant difference between them.

Example 4

The mean lifetime of a sample of 25 bulbs is found as 1550 hours with a S.D. of 120 hours. The company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% level of significance?

Solution:

Given: n=25, $\bar{x}$=1550, s = 120, μ=1600

a. The null hypothesisH_0: μ $= 1600$
alternative hypothesis H_1: μ < 1600 (Use one tailed test (left))

b. Level of significance: 5%
degrees of freedom = n -1= 24
table value =1.711

c. Test statistic $t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{1550-1600}{120/\sqrt{24}} = -2.04$

d. Conclusion: $|-2.08| \nless 1.711$.

So, we reject the null hypothesis H_0 at 5% level of significance.

∴ The claim of the company cannot be accepting at 5% level of significance.

Example 5

The average number of articles produced by two machines per day are 200 and 250 with S.D 20 and 25 respectively on the basis of records of 25 days production. Can you regard both the machines equally efficient at 1% level of significance?

Solution:

Given: n_1=25, $\overline{x_1}$ =200, s_1=s_1=20, $s_2 = 25$, n_1=25, $\overline{x_1}$ =250.

a. The null hypothesis H_0: μ $= 1600$
alternative hypothesis H_1: μ < 1600 (Use one tailed test (left))

b. Level of significance: 1%
   degrees of freedom = $n_1 + n_2 - 2$ = 48
   table value =2.58
c. Test statistic t $= \frac{\overline{x_1} - \overline{x_2}}{S\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{200-250}{(23.11)\sqrt{\left(\frac{1}{25}+\frac{1}{25}\right)}} = -0.33,$

   Where S= $\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}}$, S - Standard error
d. Conclusion: |-0.33| < 2.58.
   So, we accept the null hypothesis H_0 at 1% level of significance.
   ∴ The machines are equally efficient.

Example 6

An IQ test was administrated to 5 persons before and after they were trained. The results are given below:

Candidates:	I	II	III	IV	V
IQ before training	110	120	123	132	125
IQ after training	120	118	125	136	121

Test whether there is any change in IQ after the training programme.

Solution:

a. the null hypothesis H_0 : $\mu_2 - \mu_1 = 0$ and alternative hypothesis H_1 : $\mu_2 - \mu_1 \neq 0$.
b. Calculate the table value:
   The table value for degrees of freedom = n-1= 4 at α=0.005 is 4.604
c. Test statistic t $= \frac{\bar{d}}{s/\sqrt{n-1}}$, $\bar{d} = \frac{\sum d}{n}$

Candidates	**IQ Before Training (x)**	**IQ after Training (y)**	**d=$y - x$**	d^2
I	110	120	10	100
II	120	118	-2	4
III	123	125	2	4
IV	132	136	4	16
V	125	121	-4	16
			$\sum d = 10$	$\sum d^2 = 140$

$\bar{d} = \frac{\sum d}{n} = \frac{10}{5}$=2,

Standard deviation (s)= $\sqrt{\frac{\sum d^2}{n} - \left[\frac{\sum d}{n}\right]^2} = \sqrt{\frac{140}{5} - \left[\frac{10}{5}\right]^2} = \sqrt{28 - 4} = \sqrt{24}$ = 4.899

$$t = \frac{\bar{d}}{s/\sqrt{n-1}} = \frac{2}{4.899/\sqrt{3}} = 0.7071$$

d. **Conclusion:** |0.7071| < 4.899

So, we accept the null hypothesis H_0 is accepted.

∴ there is no significant change in IQ due to the training program.

Example 7

The following date gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.

Days:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of. accidents:	14	16	8	12	11	9	14

Solution:

The parameter of interest is to test the accidents are uniformly distributed.

a. The null hypothesis H_0: The accidents are uniformly distributed over the week.
   alternative hypothesis H_1 : The accidents are not uniformly distributed.
b. Level of significance: 5%

O	**E**	**O-E**	$\frac{(O-E)^2}{E}$
14	12	2	0.333
16	12	4	1.333
8	12	-4	1.333
12	12	0	0
11	12	-1	0.083
9	12	-3	0.75
14	12	2	0.333
		Total	4.165

degrees of freedom = n-1 = 7-1 = 6

Table value = 12.592

c. Test statistic $\chi^2 = \sum \frac{(O-E)^2}{E}$.

d. Total number of accidents = 84.

On the assumptions of H_0, the expected number of accidents on any day = $\frac{84}{7} = 12.$

e. Conclusion: |4.165| <12.592

So, we accept the null hypothesis H_0 at 5% level of significance.

∴ Thus, the accidents are uniformly distributed over the week.

Example 8

A sample analysis of examination results of 500 students was made. It was found that 220 students have failed, 170 have secured a third class, and 90 have secured a second class and the rest, a first class. So, do these figures support the general belief that the above categories are in the ration 4: 3: 2: 1 respectively?

Solution:

The variable of interest is the results in the four categories.

a. H_0 : The results in the four categories are in the ratio 4: 3: 2: 1.

H_1: The results in the four categories are in the ratio 4: 3: 2: 1

b. Level of significance: 5% degrees of freedom = n-1 = 4-1 = 3

Table value = 7.815

	O	**E**	$\boldsymbol{\frac{(O-E)^2}{E}}$
Failures	220	220	2.000
III	170	150	2.667
II	90	100	1.000
I	20	50	18.000
			23.667

c. Test statistic $\chi^2 = \sum \frac{(O-E)^2}{E} = 23.667$

d. **Conclusion:** |23.667| <7.815

So, we reject the null hypothesis H_0 at 5% level of significance.

Hence, the results of the four categories are not in the ration 4 : 3 : 2 : 1.

2.8 Conclusion:

A hypothesis is an educated guess about something in the world around us. Hypotheses are theoretical guesses based on limited knowledge; they need to be tested. Thus, hypothesis testing is a decision-making process for evaluating claims about a population. We use various statistical analysis to test hypotheses and answer research questions. In formal hypothesis testing, we test the null hypothesis and usually want to reject the null because rejection of the null indirectly supports the alternative hypothesis to the null, the one we deduce from theory as a tentative explanation. Thus, a hypothesis test mutually exclusive statements about a population to determine which statement is best supported by the sample data.

2.9 References:

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