
16. MAPS [Matter-Antimatter Propulsion System]

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Abstract:

In this paper we are going to discuss MAPS (Matter-Antimatter Propulsion System). MAPS is a proposed class of rocket propulsion system that can reach near the speed of light and might even exceed it allowing interstellar and intergalactic travels. We'd learn about the history of antimatter like how Dirac's equation allowed for the existence of anti-particles, which are mirror images of normal matter. We are also going to discuss its various advantages and also the disadvantages, such as no ignition temperature or mass criticality requirements, and mass-energy conversion. Storage of bulk antimatter in conventional matter is necessary to increase fuel-to-tank mass ratio. We will be analyzing one of NASA's reports that how the Propulsion Research Center at NASA's Marshall Space Flight Center is developing antimatter propulsion technologies to shorten trip times, increase safety, and reduce cost of space transportation. Gerald Jackson has proposed architecture for an interstellar spacecraft utilizing anti-matter propulsion and power. After that we're going to see all the technical stuff and working principle of different types of MAPS and various concepts regarding proper production of antimatter to be used in space missions. Finally, there's my conclusion which might seem bit philosophical to some readers. So, let's dive deep into it and explore for all it is.

Keywords:

MAPS, Matter, Antimatter, Propulsion, Annihilation, Particles, Specific Impulse, Pions, Muons, Neutrinos, Gamma Rays, Helium, Hydrogen, Fission, Fusion, Penning–Malmberg magnetic traps, Storage, Big Bang, Speed of Light.

16.1 Introduction:

MAPS is a proposed class of rocket propulsion system that use antimatter as its power source. The advantage to this class of rocket engines is that a large fraction of the rest mass of a matter-antimatter mixture can be converted to energy, allowing antimatter rockets to have a far higher energy density and specific impulse than any other proposed class of rocket. As we know light is currently the fastest thing in existence with a speed of 671 million mph Approx., recent research had shown that rockets powered by MAPS could reach near the speed of light and might even exceed it. If we're able to harness this incredible source of energy, then interstellar travel just won't be a matter of sci-fi. Now, there are three biggest problems with these systems which are the availability of antimatter, storage & delivery and technical feasibility of accelerator concepts. So, let us know what exactly the antimatter is. Antimatter is the same as matter but just with an opposite charge.

In 1928, British physicist Paul A.M. Dirac Revised Einstein's famous equation $E=mc^2$. Dirac said that Einstein didn't consider that the mass (m) in the equation could have negative properties as well as positive. Dirac's equation ($E= +$ or $- mc^2$) allowed for the existence of anti-particles in the universe. Scientists have since proven that several anti-particles exist. These anti-particles are, literally, mirror image of normal matter. Each anti-particle has the same mass as its corresponding particle. E.g., Matter has electron with is negatively charged whereas Antimatter has positron which is positively charged.

According to some hypothesis when universe was forming the matter & antimatter were in a constant collision releasing unfathomable amount of energy. Due to some coincidence for every billion particle of antimatter the number of matter particle was billion plus one and this small difference was enough for matter to dominate over antimatter forming our current universe. Therefore, everything we see is made up of matter not antimatter. Antimatter is produced everywhere in the Universe where high-energy particle collisions take place. High-energy cosmic rays impacting Earth's atmosphere produce antimatter in the resulting particle, which is immediately destroyed by contact with nearby matter. Antimatters are also produced in any environment at a sufficiently high temperature (Particle energy should be greater than the pair production threshold. Positrons are produced naturally in β^+ decays of radioactive isotopes and in interactions of gamma quanta with matter. Antineutrinos are another kind of particle that interacts with both the magnetic field and the atmosphere. Now cosmologists through various observations have detected high concentration of antimatter clouds in a few voids which are cold and relatively empty parts of the universe.

16.2 Types of Systems:

Antimatter-matter annihilation is an attractive concept for space propulsion applications because it generates high-energy particles that can be used directly, or indirectly, to produce thrust. This makes antimatter particularly suited to missions that occur far from the Sun where solar power generation is not practical. Various concepts have been proposed, for example, the direct expulsion of annihilation reaction products (so-called beamed core concepts), or heating of a reactor core which then either transfers heat to a working fluid that is expanded from a gas-dynamic nozzle or is used to generate power to run conventional electric propulsion systems (such as gridded ion or Hall thrusters).

Thermal antimatter propulsion concepts can make use of solid, liquid, gaseous, or even plasma cores which generate the highest specific impulses. All these concepts use the most basic antimatter particles: positrons and/or antiprotons. A more recent proposal involves the in-situ formation of antiprotons from dense solid hydrogen with high-powered lasers. Antimatter propulsion is largely speculative due to three major challenges: production, storage and delivery, and technical feasibility of accelerator concepts.

Antimatter production is well-known and studied, but current production rates are orders of magnitude below those needed for practical propulsion applications and costs are exorbitant. Recent space observations have suggested a novel alternative: space-based harvesting of antimatter trapped in planetary magnetic fields. Antimatter storage and delivery pose significant challenges due to the need for electromagnetic fields to isolate antimatter from matter storage vessels. Recent concepts propose the production of charge-neutral anti-hydrogen, which can be stored using electric and magnetic fields.

Antimatter propulsion is essential for space applications, but direct use of positron-electron annihilation is unfeasible due to its high-energy gamma rays and low thrust. A positron radioisotope source has been proposed to catalyze nuclear fusion reactions.

Thermal antimatter propulsion can be differentiated into concepts that heat a reaction mass to produce thrust and heat a working fluid to generate electrical power. The most feasible concepts are propulsion associated with either heating from antimatter annihilation or antimatter catalyzed micro-fission/fusion. The antiproton-driven Inertial confinement fusion (ICF) Rocket concept uses pellets for the D-T reaction, which consists of a hemisphere of fissionable material such as U235 with a hole through which a pulse of antiprotons and positrons is injected. Antiproton annihilation occurs at the surface of the hemisphere, which ionizes the fuel and heats the core of the pellet to fusion temperatures. While the antimatter driven Magnetically Insulated Inertial Confinement Fusion Propulsion (MICF) concept relies on a self-generated magnetic field to insulate the plasma from the metallic shell during the burn. The ICAN-II project employs the antiproton catalyzed micro fission (ACMF) concept which uses pellets with a molar ratio of 9:1 of D-T: U235 for nuclear pulse propulsion.

16.2.1 Some Proposed Types of MAPS:

- A. Positrons: Sanger Photon Rocket: Proposed redirecting energetic γ -rays from the e^-e^+ reaction to produce a thrust, but there is no feasible method to reflect them, resulting in low engine efficiency. An option is to use them to heat a refractory absorber, which then heats a propellant flowing through a heat exchanger. However, the storage density of positrons may be so low that the mass of the e^+ storage facility overwhelms potential benefits.
- B. Proton-Antiproton Solid Core Engine: More than 90% transfer of annihilation energy to tungsten block. Similar performance to an NTP engine (Isp \sim 900 s, high thrust). Typical p^* mass flows \sim several $\mu\text{g}/\text{sec}$ (material temperature limits).
- C. Proton-Antiproton Gas Core Engine: About 35% energy transfer to the high-pressure hydrogen propellant. Specific impulse like chemical engines (\sim 500 s), high thrust. Variants include liquid hydrogen for better transfer efficiency. Typical antiproton mass flow rates $\sim 10^3 \mu\text{g}/\text{sec}$.
- D. Proton-Antiproton Plasma Core Engine: Charged particles trapped and guided by strong magnetic fields. Higher Isp than chemical engines (several 1000 s), moderate thrust. Annihilation energy transferred to hydrogen is only about 1-2%. Typical pulse $\sim 10^{18} p^*$ (depending on rep rate, $\sim 100^3 \mu\text{g}/\text{sec}$). Detailed numerical studies not yet performed for heavier elements.
- E. Proton-Antiproton Beam Core Engine: Charged pions directed by magnetic nozzle; contain 40% of the initial annihilation energy. Very high Isp (\sim 28 million seconds) but low thrust (typically 10^3 N). Typical p^* flow rate $\sim 100^3 \mu\text{g}/\text{sec}$.
- F. Antiproton Catalyzed Micro Fission/Fusion (ACMF): Like the Orion pulsed nuclear engine concept. Spherical fuel pellets (3 g; molar ratio D:U235 = 9:1) coated with 200 g (about 7.05 oz) of lead. Pellet radially compressed with ion drivers; 2-ns pulse of $10^{11} p^*$ injected to initiate fission in U235. High energy fission products rapidly heat target and initiate DD fusion. Releases \approx 300 GJ energy: (83% radiation energy, 15% neutron energy, 2% ion-electron energy). Lead reradiates 1-keV photons, which ablate a SiC plate to produce thrust. 1 Hz rep rate: Thrust $>$ 100,000N, Isp $>$ 10,000 s.

- G. Antimatter Initiated Micro Fusion (AIM) Starship: 10^{11} antiprotons confined in Penning trap (potential well). 42 ng of D-He³ fuel injected into the trap along with a small amount of fissile material. A fraction of the antiprotons annihilates the fissile material; the resulting energetic particles rapidly ionize the D-He³ fuel. Fusion initiated as the fuel is further compressed in the potential well; hot plasma exhausted to produce thrust. Potentially well relaxed, additional p* injected, process repeats. Produces ≈ 2 -N thrust, Isp $\approx 67,000$ s. 200 Hz rep rate over 4-5 years delivers a 100-kg payload to 10,000 AU (Oort cloud) in about 50 years, using 5.7 mg of p*.
- H. Antimatter Driven Sail: Antiprotons directed at uranium sail coating. Resulting fission products traveling $\approx 10^7$ m/s. Isp $\approx 10^6$ s. Preliminary mission analysis: 10 kg instrument payload could be sent to 250 AU in 10 years using 30 mg of H*. A similar probe could be sent to Alpha Centaur in 40 years using grams of H*.

16.3 Practical Difficulties:

Antimatter rockets face two main challenges: creating antimatter and storing it. Creating antimatter requires vast amounts of energy, typically equivalent to the rest energy of the created particle/antiparticle pairs. Storage of antimatter is typically done by trapping electrically charged frozen antihydrogen pellets in Penning or Paul traps, but is expected to be expensive due to current production abilities being limited to small numbers of atoms. Antiproton-induced fission and self-generated magnetic fields may enhance energy localization and efficient use of annihilation energy. A secondary problem is extracting useful energy or momentum from the products of antimatter annihilation, which are primarily in the form of extremely energetic ionizing radiation. The classic rocket equation with its "wet" mass (M0 with propellant mass fraction) to "dry" mass (M1 with payload fraction) (M1/M0), the velocity change (Δv) and specific impulse (Isp) no longer holds due to mass losses occurring in antimatter annihilation.

A proton-antiproton annihilation propulsion system transforms 39% of the propellant mass into an intense high-energy flux of gamma radiation, which can cause heating and radiation damage if not shielded against. Relativistic interstellar rockets propelled by matter-antimatter annihilation or powered by an annihilation reactor cannot be realized without solving the problem of antimatter storage onboard. To do this, liquid or solid antihydrogen should be stored in a compact form, and a container made of conventional matter should be constructed. An energy barrier should be installed to prevent the stored antimatter block from touching the walls of the container.

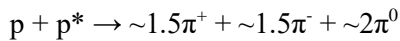
Antihydrogen storage options include plasma of free charged antiparticles in Penning–Malmberg magnetic traps, cooled antihydrogen gas in quadrupole or octupole magnetic traps, neutral antihydrogen atoms in Bose-Einstein condensate, and diamagnetic anti parahydrogen in the form of liquid droplets or solid crystalline micro-icicles levitating in the regions of minimum quadrupole magnetic field. The efficiency of antimatter production and storage is very low. About 1 billion times more energy is required to make antimatter than is finally contained in its mass. Using $E = mc^2$, we find that 1 gram of antimatter contains:

$$0.001 \text{ kg} \times (300,000,000 \text{ m/s})^2 = 90,000 \text{ GJ} = 25 \text{ million kWh}$$

Considering the low production efficiency, it would need 25 million billion kWh to make one single gram. By considering the current amount of production the cost of antimatter stands at \$62.5 trillion (about \$190,000 per person in the US) per gram, which is a whopping 61.54% of world's total GDP (nominal) of 101.56 trillion.

16.4 Working of Maps:

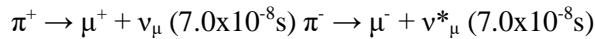
The energy is produced by the collision of matter particles with its counterpart antimatter particles consisting of various stages involving different subatomic particles. Proton-Antiproton Annihilation Products:



Neutral pion reaction: $\pi \rightarrow 2\gamma$ (2.2×10^{-17} s), results in immediate decay.

690 MeV \rightarrow rest mass energy of all the pions (37%) 750 MeV \rightarrow total kinetic energy of the 3 charged pions (40%) 440 MeV \rightarrow total kinetic energy of the 2 neutral pions (23%) 1880 MeV: p-p* annihilation energy (collision at rest). Each π^+ and π^- : Rest mass: 139.6 MeV Kinetic energy: ≈ 250 MeV. Each π^0 : Rest mass: 135 MeV Kinetic energy: ≈ 220 MeV. Neutral pions quickly decay into γ -rays ($E_\gamma \approx 130$ -300 MeV) At rest the charged pions would decay in 22 ns, but at 250 MeV they're traveling 0.93c and last for 70 ns (traveling about 21 m).

Charged pion decay:



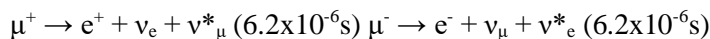
Each charged pion (π^\pm): 139.6 MeV rest mass energy 250 MeV kinetic energy ≈ 390 MeV/pion

Now, where is this 390 MeV/pion energy being utilized?

Charged muon (μ^\pm): 105.7 MeV \rightarrow rest mass energy per muon 192.3 MeV \rightarrow kinetic energy per muon 298 MeV per muon Remaining energy (92 MeV) carried away by the neutrinos (lost from the system; no interactions)

Muons traveling 0.94c; lifetimes extended from 2.2 μ s to 6.2 μ s

Charged muon decay:



Each charged muon (μ^\pm): 105.7 MeV rest mass energy 192.3 MeV kinetic energy 298 MeV/muon

Now, where is this 298 MeV/muon energy being utilized?

Positron or Electron (e^\pm): 0.511 MeV \rightarrow rest mass energy of positron or electron \approx 100 MeV \rightarrow kinetic energy of positron or electron \approx 100.5 MeV per electron or positron Remaining energy (\approx 198 MeV) carried away by the neutrinos (lost from the system; no interactions) Positrons may annihilate with electrons to produce γ -rays Now, the positive pion and positive muon together with negative pion and negative muon produces electron and positron which further combine to release energy in the form of gamma rays:

$$e^- + e^+ \rightarrow 2\gamma$$

p^* – Heavy nucleus reactions

Antiproton annihilation with heavier nuclei results in fragmentation (and fission in very heavy nuclei). Morgan (1986) analyzed theoretical and experimental data of the kinetic energy of charged nuclear fragments emitted after antiproton annihilation with a nucleus: Fraction of annihilation energy available as kinetic energy of heavier nuclear fragments \approx 10% for nuclei as heavy as silicon, and \approx 20% for very heavy nuclei (including release of fission energy, e.g. splitting ^{235}U). Easier to couple the kinetic energy of heavier charged fragments to a working fluid, but charged pions from p - p^* have higher energy fraction (40%) if we can use them.

16.5 Results of the Experiments Form Different Types:

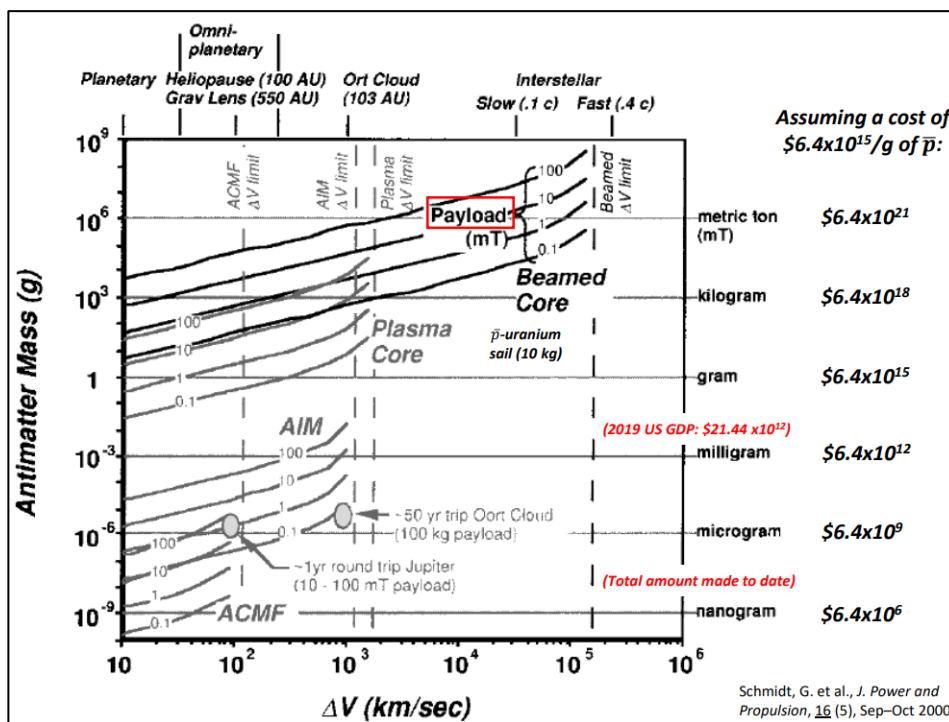


Figure 16.1: Represents a graph showing types of spacecrafts with different velocities with respect to amount of antimatter required in grams and mentioning the individual costs.

Current experimental production:

Antiprotons: Created in high energy particle accelerators (Generally not optimized for antiproton production) Collide a high energy proton beam with a target:

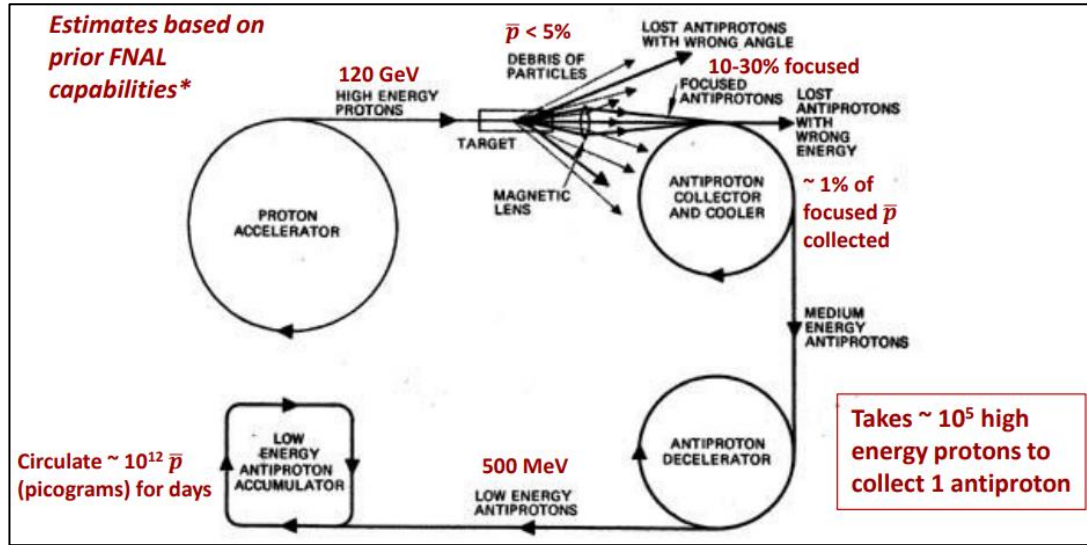


Figure 16.2: Represents a visualization of experimental production of positron inside particle accelerators.

Accurate cost analysis through production: Schmidt et al. estimated the energy cost (K) as: $K = k_{\text{grid}} E_{\text{grid}} / \eta_{\text{tot}}$ where k_{grid} = unit cost of electrical power (\$/kW-hr) $E_{\text{grid}} = M_a c^2 / \eta_{\text{tot}}$ = energy required to create an amount of antimatter M_a with efficiency η_{tot} Efficiency η_{tot} can be expressed as $\eta_{\text{tot}} = \eta_{\text{conv}} \eta_{\text{grid}}$, where: η_{conv} = efficiency of the antimatter production and collection process η_{grid} = electrical efficiency of the accelerator system

Now putting the values in the equation:

$$K = (k_{\text{grid}} M_a c^2) / (\eta_{\text{conv}} \eta_{\text{grid}})$$

Rough estimate based on Fermilab values: k_{grid} = wall plug power \approx \$0.10/kW-hr (2.8×10^{-8} \$/J) M_a = antimatter rest mass collected (kg) η_{grid} = electrical efficiency of the accelerator system $\approx 5 \times 10^{-3}$: 14 MW of power required to deliver 5×10^{12} 120-GeV proton beam every 1.5 s onto production target \rightarrow power in beam $\approx 6.4 \times 10^4$ W; $6.4 \times 10^4 / 14 \times 10^6 \approx 5 \times 10^{-3}$ η_{conv} = efficiency of production and collection process $\approx 7.8 \times 10^{-8}$: Rest mass energy of $p^* = 938$ MeV = 9.38×10^8 eV/ p^* . Energy to create and collect one $p^* = 120$ GeV/proton $\times 10^5$ $p/p^* = 1.2 \times 10^{16}$ eV/ p^* ; $9.38 \times 10^8 / 1.2 \times 10^{16} = 7.8 \times 10^{-8}$

$$K/M_a = (k_{\text{grid}} c^2) / (\eta_{\text{conv}} \eta_{\text{grid}}) = [(\$2.8 \times 10^{-8} / \text{J}) (3 \times 10^8 \text{ m/s})^2 / (5 \times 10^{-3}) (7.8 \times 10^{-8})] \approx \$6.4 \times 10^{18} \text{ per kg} = \$6.4 \times 10^{12} \text{ per mg.}$$

Now, how to improve production?

Researchers then came up with the concept of antiproton factory where high efficiency linear accelerator for proton beams are produced after those positrons sent to decelerator laser-enhanced antihydrogen generator then antiprotons collected using wide angle collecting lenses to stochastic coolers then to the decelerator cooling ring and finally to the antihydrogen generator in sequence which combine with positrons to make H^* atoms.

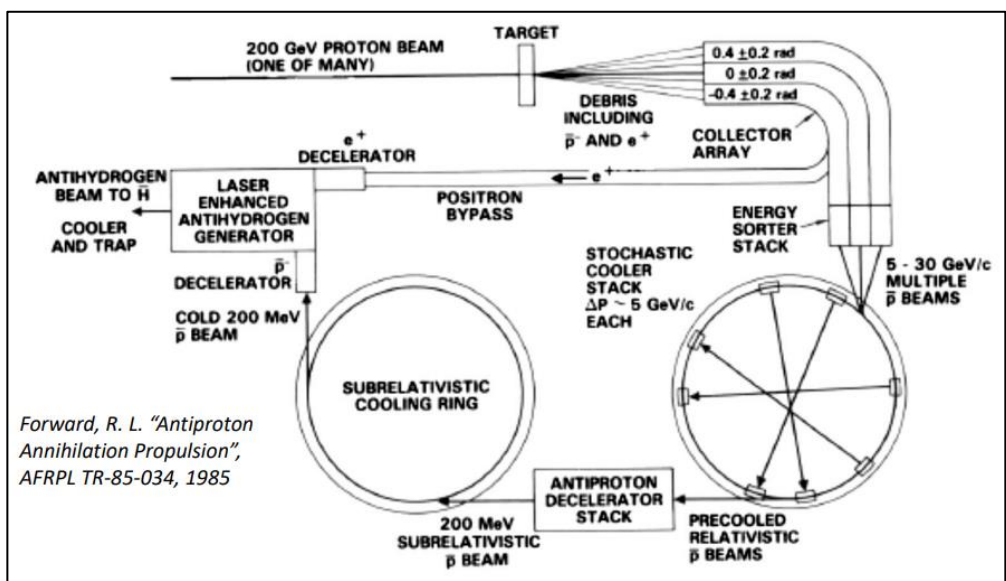


Figure 16.3: Represents a visualization of the Antiproton factory concept.

Idea of Production of Antimatter in Space:

Potential advantages:

Hard vacuum

Abundant solar power

Fuel vehicles in space (vs. launching p^* from ground)

Challenges:

Requires a lot of infrastructure (accelerator, collector, storage rings, etc.) Requires many solar arrays (100's MW) Will be expensive to launch, assemble, operate and repair Then how about capturing antimatter already in space?: A very low-density cloud of antimatter particles trapped in Earth's van Allen radiation belt could be captured for propulsion.

Challenges faced in storage:

Density of antiprotons or positrons limited by space charge: Facility accumulator rings can hold $\approx 10^{12} p^*$ (1.7×10^{-12} g, or picograms) for indefinite periods of time.

A portable High Performance Antiproton Trap (HiPAT) developed by NASA was designed to hold 10^{12} p^* for up to 18 days using a Penning-Malmberg electromagnetic trap (For p^* testing at MSFC, sadly it's built but never used) 1000s of traps required to hold nanograms of antimatter.

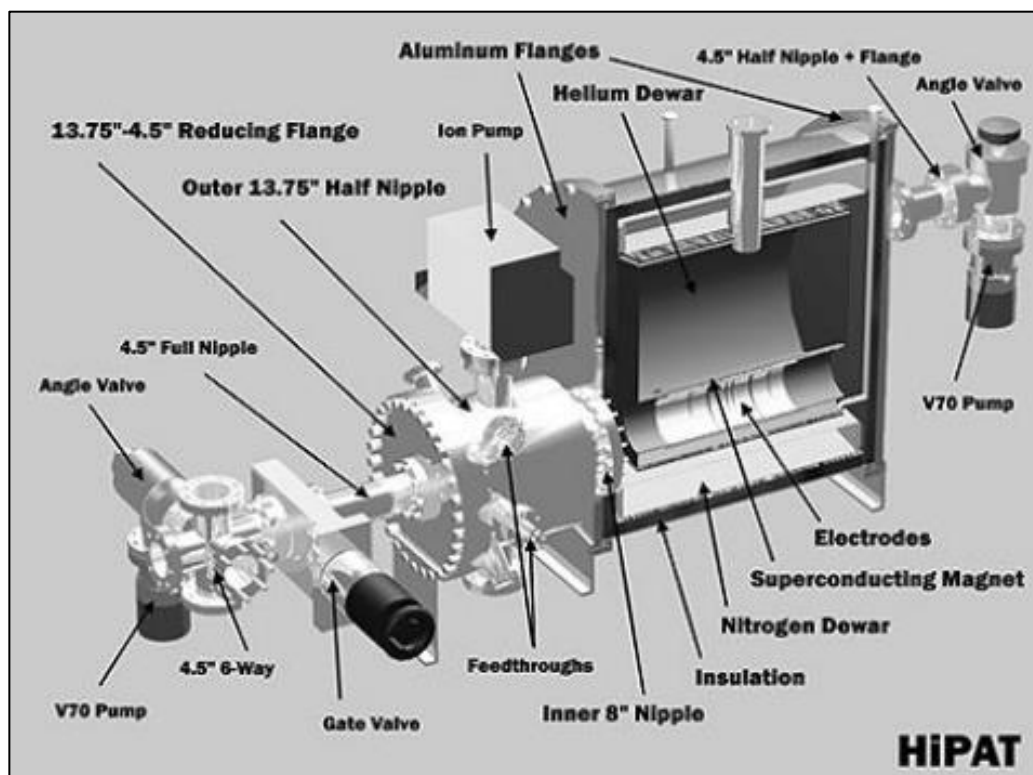


Figure 16.4: Represents a visualization of HiPAT which refers to a portable high energy antiproton trap.

Through meticulous research we came across solid antihydrogen which is a game changing concept. Here nucleation approaches creating solid H^* atoms that can't touch walls, so nucleation approaches need to nucleate directly from cold, trapped antihydrogen atoms.

Laser cooling and cluster ion formation can be used to form and trap H^* molecules, which can then condense into microcrystals. Cluster ion formation, in which large numbers of H^* atoms cluster around a single charged p^* , can also be used to add H^* atoms to form microcrystals of H^* ice.

Now, we need to store it which becomes easier compared to when it was a gas. Since hydrogen and antihydrogen are diamagnetic, a weak magnetic dipole moment is induced in the direction opposite to an applied magnetic field. Solid antihydrogen can be passively trapped in a magnetic bottle. An alternative method being electrostatic levitation between two charged electrodes where weak UV light is used to liberate positrons to provide a

surface charge on the solid H^* ice. Despite these developments we still need to put a lot of effort into it.

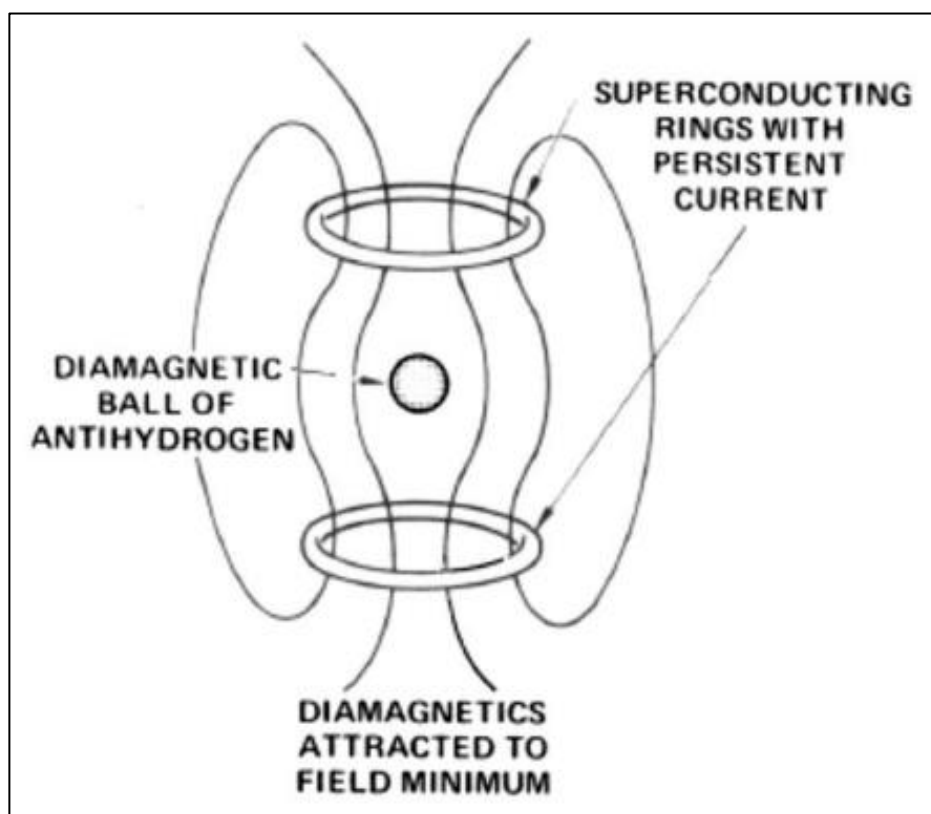


Figure 16.5: Represents a visualization of antihydrogen storage.

Now comes delivering it to the engine where feasible options exist to remove antiprotons and microcrystals of H^* from storage. UV light drives positrons from selected microcrystals then electric fields pull charged particles from the cloud and finally charged microcrystals are electromagnetically channeled to the annihilation engine.

16.5 Designing the Vehicles:

Important things to be taken into consideration:

Depending upon the concept, but generally:

Storage during launch and in-space acceleration; methods to transport p^* or e^+ to the engine

Magnet requirements (magnetic nozzles, etc.)

Radiation shielding (high energy γ -rays will damage material structures, electronics and humans)

Thermal radiators (need to reject a significant amount of waste heat; may be large and massive)

Vehicle support structures (trusses, tanks, etc.)

Payload placement (far away from radiation)

Additional propellant (to interact with antimatter)

Number of launches and in-space assembly.

16.6 Conclusion:

So, in this paper we discussed what's antimatter, its history and the most important thing i.e., MAPS. We discussed its types and categorizing them into different aspects of their use, also mentioning various challenges faced due to its production, storage and cost. We also discussed a few methods to overcome these challenges like the concept of antiproton factory and solid antihydrogen. All over if we could harness this power like normal propulsion systems then space missions would become short and cost effective. We will also be sending humans to explore further deep into the cosmos. Still one thing just keeps bugging me i.e., despite this technology how far could we go coz our current research says that spacecrafts powered by MAPS could only attend 72% of the speed of light at its maximum output. The figure may look massive but if you ask me, it's extremely slow and beyond comparison of what we need coz light, which is currently the fastest thing in existence, takes a whopping 100,000 years to travel from one end of the milky way to the other and even our milky way is smaller than a dot when we discuss about the size of our universe, also with the recent concepts of multiverse and omniverse travelling even with the speed of light to a further destination will take unimaginable time. So, now comes the concept of warp drive which could bend space to reduce the distance to something negligible and worm holes which probably connects a black hole and a white hole to create a hypothetical tunnel through space and time fabric to travel at a negligible amount of time to the desired distance, though these are the topics for later discussion. Finally, I'd like to conclude by saying that we just need to dive deep into what's ahead of us in this field coz from what I've known that the color of the cosmos is dark and according to me, "LIGHT IS AN ILLUSION WHEREAS TRUTH LIES IN THE DARKNESS".

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