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9. On Neutrosophic *i*-open Set

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Abstract:

The purpose of this article is to introduce neutrosophic i-open set, neutrosophic i-closed set and study some basic properties. We also define neutrosophic i-interior and neutrosophic i-closure of a neutrosophic set and then investigate some properties involving them.

Keywords:

Neutrosophic i-open set; Neutrosophic i-closed set; Neutrosophic i-interior; Neutrosophic i- closure.

9.1 Introduction:

The notion of a neutrosophic set was coined and studied by Florentin Smarandache (Smarandache, 1999, Smarandache, 2002 Smarandache, 2005). A neutrosophic set is knotted with three membership functions which are the truth-membership function, falsity-membership function, and indeterminacy-membership function and it is notable that all these three neutrosophic factors are unbiased to one another. After Smarandache had added the thought of neutrosophy, several researchers (Salama and Alblowi, 2012, Salama and Alblowi, 2012, Wang *et al*. 2010) throughout the globe studied and contributed to the upliftment of this theory. With the help of neutrosophic theory, many real-life problems can be dealt with in a more general and appropriate way. Different kinds of practical-based works (Abdel-Basset *et al.* 2020, Dey and Ray, 2022, Guo and Cheng, 2009), had been carried out in a neutrosophic environment.

In the year 2012, Salama & Alblowi (Salama and Alblowi, 2012), introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space developed by D. Coker (Coker, 1997), in 1997. Later, various concepts related to neutrosophic topological spaces were developed by many researchers (Arokiarani *et al.* 2017, Dey and Ray, 2022, Karatas and Kuru, 2016, Salama and Alblowi 2012, Salama *et al.* 2014). Different types of open and closed sets in connection with neutrosophic topological spaces were also introduced by the researchers. Iswaraya and Bageerathi (Ishwarya and Bageerathi, 2016), explored the concept of neutrosophic semi-closed and semi-open sets in 2016. Rao and Rao (Rao and Rao, 2017)in their work introduced neutrosophic pre-open and pre-closed sets in neutrosophic topologicalspaces. Puvaneswari & Bageerathi (Puvaneswari and Bageerathi, 2017) developed the idea of neutrosophic feebly open sets and closed sets. The study by Al-Omeri and Jafari (Al-Omari and Jafari, 2018) focused on generalized neutrosophic pre-closed sets and generalised pre-open sets. Ebenanjar *et al.* (Ebenanjar *et al.* 2018), in 2018, introduced the conceptof neutrosophic *b*-open sets and *b*-closed sets. After that, Maheswari *et al.* (Maheswari *et al.* 2018) extended the research to neutrosophic generalized *b*-open sets and generalized *b*-closed sets. D. Jayanthi (Jayanthi, 2018) developed idea of neutrosophic α generalized closed sets. The author studied and analyzed their properties thoroughly. Das and Pramanik (Das and Pramanik, 2020) introduced generalized neutrosophic *b*-open sets and investigated some of their properties. The concept of neutrosophic simply *b*-open set inneutrosophic topological spaces was revealed by Das and Tripathy (Das and Tripathy, 2021). The authors established various results with some illustrative examples. Recently, V. Banu Priya *et al.* (Priya *et al.* 2022) introducedneutrosophic α-generalized semi closed sets and neutrosophic α-generalized semi open sets and discussed some of its basic properties. The authors also inspected their connections with the existing neutrosophic closed and open sets.

In this article, we introduce a new kind of open set which we call neutrosophic *i*-open set and study some basic properties. We also define neutrosophic *i*-interior and neutrosophic *i*-closure of a neutrosophic set and try to investigate some properties involving them. The article is organized by conferring some basic concepts in the second section. In third section, we focus on the main points of this study. In the last section, we bestow a conclusion.

9.2 Preliminaries:

Definition 9.2.1: (Smarandache, 2002**)**

Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, where the functions T_A, I_A, F_A are real standard or non- standard subsets of $]^-0, 1^+[$, i.e., $T_A : X \to\,]^-0, 1^+[$, $I_A : X \to\,]^-0, 1^+[$, $F_A : X \to\,]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function T_A, indeterminacy-membership function I_A, falsehood-membership function F_A.

Definition 9.2.2: (Wang *et al.* 2010)

Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$, where T_A, I_A, F_A are functions from X to $[0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The set of all single valued neutrosophic sets over X is denoted by $N(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

Definition 9.2.3: (Karatas and Kuru, 2016**)**

Let $A, B \in N(X)$. Then

(i) (Inclusion): If $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for all $x \in X$ then A issaid to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $T_A(x) = 1, I_A(x) = 0, F_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $T_A(x) = 0, I_A(x) = 1, F_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

Definition 9.2.4: (Salama and Alblowi, 2012**)**

Let $\{A_i : i \in \Delta\} \subseteq N(X)$, where Δ is an index set. Then

(i) $\cup_{i\in\Delta} A_i = \{\langle x, \vee_{i\in\Delta} T_{A_i}(x), \wedge_{i\in\Delta} I_{A_i}(x), \wedge_{i\in\Delta} F_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\Delta} A_i = \{\langle x, \wedge_{i\in\Delta} T_{A_i}(x), \vee_{i\in\Delta} I_{A_i}(x), \vee_{i\in\Delta} F_{A_i}(x)\rangle : x \in X\}$.

Definition 9.2.5: (Karatas and Kuru, 2016**)**

Let $\tau \subseteq N(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) An arbitrary union of neutrosophic sets in τ is in τ.

(iii) The intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic open sets in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic closed set in X.

Definition 9.2.6: (Karatas and Kuru, 2016**)**

Let (X, τ) be a NTS and $A \in N(X)$. Then the neutrosophic

(i) interior of A, denoted by $int(A)$, is defined as $int(A) = \cup\{G : G \in \tau$ and $G \subseteq A\}$.
(ii) closure of A, denoted by $cl(A)$, is defined as $cl(A) = \cap\{G : G^c \in \tau$ and $G \supseteq A\}$.

9.3 Main Results:

Definition 9.3.1:

Let (X, τ) be an NTS. A non-empty NS $A \in N(X)$ is called a neutrosophic *i*-open set (*NiO* set, for short) in X iff $A = \tilde{\emptyset}$ or there exists a neutrosophic open set $G \neq \tilde{\emptyset}$ such that $G \subseteq int(A)$.

An NS G is called a neutrosophic *i*-closed set (*NiC* set, for short) iff G^c is neutrosophic *i*-open set.

The collection of all the neutrosophic *i*-open sets of the NTS (X, τ) will be denoted by $NiO(X)$.

Example 9.3.1: Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0.5, 0.4, 0.3\rangle\}$. Obviously (X, τ) is an NTS. Let us consider the NSs $C = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$ and $D = \{\langle a, 1, 0, 0\rangle, \langle b, 0.7, 0.3, 0.2\rangle\}$ over X. Now $int(C) = \tilde{\emptyset}$ and $int(D) = A$. Clearly D is an NiO set but C is not an *NiO* set in X.

Proposition 9.3.1:

(i) Every neutrosophic open set in an NTS is an *NiO* set.
(ii) Every neutrosophic closed set in an NTS is an *NiC* set.

Proof:

(i) Let (X, τ) be an NTS and $A \in \tau$. If $A = \tilde{\emptyset}$ then by definition, A is an *NiO* set. If $A \neq \tilde{\emptyset}$ then as $A = int(A)$, so, there exists anon-empty neutrosophic open set A such that $A \subseteq int(A)$. Therefore, A is an *NiO* set. Hence proved.
(ii) Let (X, τ) be an NTS and A be a neutrosophic closed set. Then A^c is a neutrosophicopen set and so, A^c is an *NiO* set [by (i)] and therefore, A is an *NiC* set.

Remark 9.3.1: Converse of the proposition 9.3.1 is not true. We show it by the following counterexample.

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0.5, 0.4, 0.3\rangle\}$. Obviously (X, τ) is an NTS. Let us consider the NSs $E = \{\langle a, 1, 0, 0\rangle, \langle b, 0.7, 0.3, 0.2\rangle\}$ and $F = \{\langle a, 0, 1, 1\rangle, \langle b, 0.2, 0.7, 0.7\rangle\}$ over X. Now $int(\text{E}) = A$. Clearly E is an NiO set in X but it is not a neutrosophic open set. Again $F^c = E$, which is an NiO set. Therefore, F is an *NiC* set. But F is clearly not a neutrosophic closed set.

Proposition 9.3.2: In an NTS, union of an arbitrary collection of *NiO* sets is an *NiO* set.

Proof: Let (X, τ) be an NTS and $A = \{G_\lambda : \lambda \in \Delta\}$ be an arbitrary collection of *NiO* sets in X, where Δ is an index set. If $\cup_{\lambda \in \Delta} G_\lambda = \tilde{\emptyset}$ then by definition, $\cup_{\lambda \in \Delta} G_\lambda$ is an *NiO* set. If $\cup_{\lambda \in \Delta} G_\lambda \neq \tilde{\emptyset}$ then we take the set $B = \{G_{\lambda_k} \in A : G_{\lambda_k} \neq \tilde{\emptyset}\}$. Since G_{λ_k} is a non-empty *NiO* set, so there exists a non-empty neutrosophic open set H_{λ_k} for each G_{λ_k}, $\lambda_k \in \Delta$, such that $H_{\lambda_k} \subseteq int(G_{\lambda_k})$. Now $H_{\lambda_k} \subseteq int(G_{\lambda_k}) \Rightarrow \cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \subseteq \cup_{\lambda_{k\in\Delta}} int(G_{\lambda_k}) \Rightarrow \cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \subseteq int(\cup_{\lambda_{k\in\Delta}} G_{\lambda_k}) \Rightarrow \cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \subseteq int(\cup_{\lambda \in \Delta} G_\lambda)$. Since for each $\lambda_k \in \Delta$, $H_{\lambda_k} \neq \tilde{\emptyset}$, so $\cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \neq \tilde{\emptyset}$. Also $\cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \in \tau$. Thus, there exists a non-empty neutrosophic open set $\cup_{\lambda_{k\in\Delta}} H_{\lambda_k}$ such that $\cup_{\lambda_{k\in\Delta}} H_{\lambda_k} \subseteq int(\cup_{\lambda \in \Delta} G_\lambda)$. Therefore $\cup_{\lambda \in \Delta} G_\lambda$ is an *NiO* set. Hence proved.

Remark 9.3.2: In an NTS, intersection of two *NiO* sets may not be an *NiO* set. We establish it by thefollowing counterexample.

Let us consider the NTS (X, τ), where $X = \{a, b\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Let us take the neutrosophic sets C $= \{\langle a, 1, 0, 0\rangle, \langle b, 0.7, 0.3, 0.2\rangle\}$ and $D = \{\langle a, 0.3, 0.2, 0.1\rangle, \langle b, 1, 0, 0\rangle\}$ over X. Clearly C and D are NiO sets in X. Now $C \cap D = \{\langle a, 0.3, 0.2, 0.1\rangle, \langle b, 0.7, 0.3, 0.2\rangle\} \Rightarrow int(C \cap D) = \tilde{\emptyset}$. Clearly there is no non-empty neutrosophic open set G in X such that $G \subseteq int(C \cap D)$. Therefore, $C \cap D$ is not an *NiO* set in X.

Proposition 9.3.3: In an NTS, intersection of an arbitrary collection of *NiC* sets is an *NiC* set.

Proof: Let (X, τ) be an NTS and $\{G_\lambda : \lambda \in \Delta\}$ be an arbitrary collection of *NiC* sets in X, where o is an index set. Then G_λ^c is an *NiO* set for each $\lambda \in \Delta \Rightarrow \cup_{\lambda \in \Delta} G_\lambda^c$ is an *NiO* set [by 9.3.2] $\Rightarrow (\cap_{\lambda \in \Delta} G_\lambda)^c$ is an *NiO* set $\Rightarrow \cap_{\lambda \in \Delta} G_\lambda$ is an *NiC* set. Hence proved.

Remark 9.3.3:

In an NTS, union of two *NiC* sets may not be an *NiC* set. We establish it by the following counterexample.

Let us consider the NTS (X, τ), where $X = \{a, b\}$, $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, $A =$

$\{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Let us take the neutrosophic sets E = $\{\langle a, 0, 1, 1\rangle, \langle b, 0.2, 0.7, 0.7\rangle\}$ and F = $\{\langle a, 0.1, 0.8, 0.3\rangle, \langle b, 0, 1, 1\rangle\}$ over X. Clearly E and F are NiC sets in X. Now $E \cup F = \{\langle a, 0.1, 0.8, 0.3\rangle, \langle b, 0.2, 0.7, 0.7\rangle\} \Rightarrow (E \cup F)^c = \{\langle a, 0.3, 0.2, 0.1\rangle, \langle b, 0.7, 0.3, 0.2\rangle\} \Rightarrow int((E \cup F)^c) = \tilde{\emptyset}$. Clearly there is no non-empty neutrosophic open set G in X such that $G \subseteq int((E \cup F)^c)$. Therefore $(E \cup F)^c$ is not an NiO setin X and so, $E \cup F$ is not an NiC set in X.

Definition 9.3.2: Let (X, τ) be an NTS and $A \in N(X)$. Then the neutrosophic *i*-closure of A, denoted by $Nicl(A)$, is defined as $Nicl(A) = \cap\{G : G$ is an NiC set in X and $G \supseteq A\}$.

Proposition 9.3.4: Let (X, τ) be an NTS and $A \in N(X)$. Then the following hold.

(i) $Nicl(A)$ is an NiC set
(ii) $A \subseteq Nicl(A)$.
(iii) A is an NiC set iff $A = Nicl(A)$.
(iv) $Nicl(\tilde{\emptyset}) = \tilde{\emptyset}$
(v) $Nicl(\tilde{X}) = \tilde{X}$
(vi) $Nicl(Nicl(A)) = Nicl(A)$

Proof:

(i) Since $Nicl(A)$ is the intersection of NiC sets, so by prop. 9.3.3, $Nicl(A)$ is an NiC set.

(ii) Since $Nicl(A)$ is the intersection of all NiC sets containing A, so $A \subseteq Nicl(A)$.

(iii) We know $Nicl(A) = \cap\{G : G$ is an NiC set in X and $G \supseteq A\}$. Suppose that A is an NiC set. Since $A \supseteq A$ and A is an NiC set, so $Nicl(A) \subseteq A$. Again $A \subseteq Nicl(A)$ [by (ii)]. Therefore $A = Nicl(A)$. Conversely if $A = Nicl(A)$ then A is an NiC set as $Nicl(A)$ is anNiC set [by (i)]. Hence proved.

(iv) Since every neutrosophic closed set is an NiC set, so $\tilde{\emptyset}$ is an NiC set and therefore by (iii), $Nicl(\tilde{\emptyset}) = \tilde{\emptyset}$.

(v) Since every neutrosophic closed set is an NiC set, so $\tilde{X}$ is an NiC set and therefore by (iii), $Nicl(\tilde{X}) = \tilde{X}$.

(vi) Since by (i), $Nicl(A)$ is an NiC set, so by (iii), $Nicl(Nicl(A)) = Nicl(A)$.

Proposition 9.3.5:

Let (X, τ) be a NTS and $A, B \in N(X)$. Then the following hold.

(iii) $A \subseteq B \Rightarrow Nicl(A) \subseteq Nicl(B)$
(iv) $Nicl(A \cup B) \supseteq Nicl(A) \cup Nicl(B)$.
(v) $Nicl(A \cap B) \subseteq Nicl(A) \cap Nicl(B)$.

Proof: (i) $A \subseteq B$ and $B \subseteq Nicl(B)$, so $A \subseteq Nicl(B)$. Since $Nicl(B)$ is an NiC set suchthat $A \subseteq Nicl(B)$, so $Nicl(A) \subseteq Nicl(B)$.

(ii) $A \subseteq A \cup B \Rightarrow Nicl(A) \subseteq Nicl(A \cup B)$. Similarly, $Nicl(B) \subseteq Nicl(A \cup B)$. Therefore $Nicl(A \cup B) \supseteq Nicl(A) \cup Nicl(B)$.

(iii) $A \cap B \subseteq A \Rightarrow Nicl(A \cap B) \subseteq Nicl(A)$. Similarly $Nicl(A \cap B) \subseteq Nicl(B)$. Therefore $Nicl(A \cap B) \subseteq Nicl(A) \cap Nicl(B)$.

Definition 9.3.3: Let (X, τ) be an NTS and $A \in N(X)$. Then the neutrosophic *i*-interior of A, denoted by $Niint(A)$, is defined as $Niint(A) = \cup\{G : G$ is an NiO set in X and $G \subseteq A\}$.

Proposition 9.3.6:

Let (X, τ) be an NTS and $A, B \in N(X)$. Then the following hold.

(i) $Niint(A)$ is an NiO set.
(ii) $Niint(A) \subseteq A$
(iii) A is an NiO set iff $A = Niint(A)$.
(iv) $Niint(\tilde{\emptyset}) = \tilde{\emptyset}$
(v) $Niint(\tilde{X}) = \tilde{X}$
(vi) $Niint(Niint(A)) = Niint(A)$

Proof:

i) Since $Niint(A)$ is the union of NiO sets, so by 9.3.2, $Niint(A)$ is an NiO set.
ii) Since $Niint(A)$ is the union of all NiO sets contained in A, so $Niint(A) \subseteq A$.
iii) We know $Niint(A) = \cup\{G : G$ is a NiO set in X and $G \subseteq A\}$. Suppose that A is an NiO set. Since $A \subseteq A$ and A is an NiO set, so $A \subseteq Niint(A)$. Again $Niint(A) \subseteq A$ [by (ii)]. Therefore $A = Niint(A)$. Conversely if $A = Niint(A)$ then A is an NiO set as $Niint(A)$ is NiO set [by (i)]. Hence proved.
iv) Since every neutrosophic open set is an NiO set, so $\tilde{\emptyset}$ is a NiO set and therefore by (iii), $Niint(\tilde{\emptyset}) = \tilde{\emptyset}$.
v) Since every neutrosophic open set is an NiO set, so $\tilde{X}$ is a NiO set and therefore by (iii), $Niint(\tilde{X}) = \tilde{X}$.
vi) Since by (i), $Niint(A)$ is an NiO set, so by (iii), $Niint(Niint(A)) = Niint(A)$.

Proposition 9.3.7:

Let (X, τ) be an NTS and $A, B \in N(X)$. Then the following hold.

(i) $A \subseteq B \Rightarrow Niint(A) \subseteq Niint(B)$
(ii) $Niint(A \cup B) \supseteq Niint(A) \cup Niint(B)$.
(iii) $Niint(A \cap B) \subseteq Niint(A) \cap Niint(B)$.

Proof:

(i) Since $A \subseteq B$ and $Niint(A) \subseteq A$, so $Niint(A) \subseteq B$. Since $Niint(A)$ is an NiO set such that $Niint(A) \subseteq B$ and since $Niint(B)$ is the largest NiO set contained in B, so $Niint(A) \subseteq Niint(B)$.

(ii) $A \subseteq A \cup B \Rightarrow Niint(A) \subseteq Niint(A \cup B)$. Similarly, $Niint(B) \subseteq Niint(A \cup B)$. Therefore $Niint(A \cup B) \supseteq Niint(A) \cup Niint(B)$.

(iii) $A \cap B \subseteq A \Rightarrow Niint(A \cap B) \subseteq Niint(A)$. Similarly, $Niint(A \cap B) \subseteq Niint(B)$. Therefore, $Niint(A \cap B) \subseteq Niint(A) \cap Niint(B)$.

Proposition 9.3.8:

Let (X, τ) be an NTS and $A, B \in \mathrm{N}(X)$. Then the following hold.

(i) $(Nicl(A))^c = Niint(A^c)$

(ii) $Nicl(A^c) = (Niint(A))^c$.

Proof:

(i) We have $Nicl(A) = \cap\{G : G$ is an NiC set in X and $A \subseteq G\} \Rightarrow (Nicl(A))^c = (\cap\{G : G$ is an NiC set in X and $A \subseteq G\})^c \Rightarrow (Nicl(A))^c = \cup\{G^c : G^c$ is an NiO set in X and $G^c \subseteq A^c\} \Rightarrow (Nicl(A))^c = Niint(A^c)$.

(ii) Replacing A *by* A^c in (i), we get $(Nicl(A^c))^c = Niint((A^c)^c) \Rightarrow (Nicl(A^c))^c = Niint(A) \Rightarrow Nicl(A^c) = (Niint(A))^c$.

Hence proved.

9.4 Conclusion:

In this article, we have introduced the concepts of neutrosophic *i*-open sets and neutrosophic *i*-closed sets within the context of neutrosophic topological spaces and examine their fundamental properties. We have shown that every neutrosophic open (respectively, neutrosophic closed) set qualifies as a neutrosophic *i*-open (respectively, neutrosophic *i*-closed) set. Our study also reveals that the intersection (respectively, union) of two neutrosophic *i*-open sets (respectively, *i*-closed sets) is not necessarily neutrosophic *i*-open (respectively, *i*-closed) set, challenging traditional notions in neutrosophic topology. Additionally, we have defined the neutrosophic *i*-interior and neutrosophic *i*-closure of a neutrosophic set and explore some of their properties.

In future research, we aim to explore new concepts building on these foundations. We hope that the findings in this article will encourage further advancements in various aspects of neutrosophic topology, supporting the progress of research in this field.

Conflict of Interest: We certify that there is no conflict of interest in connection with this article.

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